Lecture 12 Notes: 07 / 19

Simple Magnifier

The simple magnifier consists of a single convergent lens. If an object is placed slightly inside the focal point of the lens, the image becomes much larger, but farther away. The observer can then position the eye so that the image is somewhere within the range of normal vision, between the near point and the far point:



The image can be much larger than the object (infinite size if the object is at the focal point), but it is also farther away, so it will not necessarily subtend a much bigger angle with the eye and appear that much bigger. However, the lens allows the object to be held much closer to the observer than the observer's near point, since the lens will cause the virtual image to form beyond the near point; this causes the angle subtended by the image to in fact be larger than the angle subtended by the object at the near point.

The *angular magnification* is defined as the angle subtended by the image, θ , divided by the angle, θ_0 , subtended by the object when it is at the near point without a lens:

$$m = \frac{\theta}{\theta_0}$$

The angular magnification will depend on the focal length of the lens, the distance between the lens and the object p, the observer's near point p_{NP} , and the distance between the lens and the observer's eye d. It can be calculated in a straightforward manner for any combination of these parameters. Here are some examples:

Example: A lens with focal length *f* is held close to the eye (d = 0) and used in such a way that the image forms at the eye's near point, p_{NP} . What is the angular magnification?

We want an image a distance of p_{NP} behind the lens, so $q = -p_{NP}$. From this, we can find the distance from the lens at which we should position the object, p:

$$p = \frac{1}{1/f - 1/q} = \frac{1}{1/f + 1/p_{NP}} = \frac{fp_{NP}}{f + p_{NP}}$$

Now we need the size of the image compared to the size of the object. If the image size is h' and the object size is h, then

$$h' = -\frac{q}{p}h = \frac{p_{NP}}{p}h = \left(1 + \frac{p_{NP}}{f}\right)h$$

The angle subtended by the object at the near point, with no lens, is (assuming the object is much smaller than the distance to the near point):

$$\theta_0 = \frac{h}{p_{NP}}$$

When using the lens, the angle subtended by the image is

$$\theta = \frac{h'}{-q} = \frac{h'}{p_{NP}} = \left(\frac{1}{p_{NP}} + \frac{1}{f}\right)h$$

The angular magnification is the ratio of the two angles:

$$m = \frac{\theta}{\theta_0} = \left(1 + \frac{p_{NP}}{f}\right)$$

Note that this is only true if the lens is held very close to the eye, and at such a distance from the object that the image forms at the eye's near point.

For example, suppose we have a magnifier with a focal length of 10cm and an observer with a near point of 25cm. Then, the angular magnification is m = 1 + 2.5 = 3.5.

The image subtends an angle that is 3.5 times larger than the object, and so details 3.5 times smaller can be resolved.

Example: An eye with normal distance vision is most relaxed when looking at an object at infinity. If we place the lens so that the image forms at infinity, what is the angular magnification?

In order to form an image at infinity, the object is placed at the focal point. The distance between the lens and the eye is now immaterial, since the image is infinitely far away. The ray diagram looks like this:



The angle subtended by the image is $\theta = h/f$, where *h* is the height of the object. Thus, the angular magnification is

$$m = \frac{\theta}{\theta_0} = \frac{h}{f} \frac{p_{NP}}{h} = \frac{p_{NP}}{f}$$

For the same eye and lens as above, with $p_{NP} = 25cm$ and f = 10cm, m = 2.5. Therefore if the same lens is used so that the image forms at infinity, the angular magnification is 2.5, rather than the 3.5 obtained by forming the image at the near point of the eye.

The telescope eyepiece

During last lecture, we discussed telescopes where the image was captured using a digital detector such as a charge-coupled device (CCD). We can instead magnify the image so that it can be viewed by the human eye. This is done by adding an additional lens, called the *eyepiece*. The eyepiece is basically a simple magnifier which uses the image from the telescope's lens or mirror as its object. It is usually adjusted so that the final image is at infinity. This is a lens diagram of a refracting telescope with an eyepiece:



From this diagram, $f_0 \tan \theta_0 = f_E \tan \theta$. For small angles, this becomes $f_O \theta_0 = f_E \theta$

$$m = \frac{\theta}{\theta_0} = \frac{f_O}{f_E}$$

Example: Suppose we have the 35*cm* refracting telescope with the objective focal length $f_0 = 60cm$. This telescope is diffraction-limited. We would like an observer looking through this telescope to just be able to make out the maximum detail that the diffraction limit allows. Suppose that the observer has a pupil diameter of 5*mm*, and the observer's eyes are also diffraction-limited. What should the focal length of the eyepiece be?

The minimum angle that can be resolved by this telescope is the angle through which parallel rays of light diffract after passing through the telescope's aperture:

$$\theta_{d,T} = \frac{\lambda}{a_T}$$

Here, a_T is the diameter of the telescope aperture (the objective lens). The minimum angle that can be resolved by the eye is similarly

$$\theta_{d,E} = \frac{\lambda}{a_E}$$

Here, a_E is the diameter of the eye's aperture (the pupil). The telescope can resolve smaller angles, since its aperture is bigger. We want to magnify the smallest angle that the telescope can resolve so that it is equal to the smallest angle that the eye can resolve, so that the observer can see it. Thus, the magnification we want is

$$m = \frac{\theta_{d,E}}{\theta_{d,T}} = \frac{a_T}{a_E} = \frac{f_O}{f_E}$$

Therefore the optimal focal length of the eyepiece is

$$f_E = \frac{a_E}{a_T} f_O = \frac{5mm}{350mm} \times 60cm = 0.86cm$$

This gives an angular magnification of 70X. At this magnification, the observer's eye can just make out details at the telescope's diffraction limit. Increasing the magnification further would result in a larger, but blurrier, image.

The compound microscope

For high magnifications, the simple magnifier suffers from the problem of lens aberration. The compound microscope is a system of two lenses that corrects the aberration. It looks superficially somewhat similar to a refracting telescope with an eyepiece, except that the object is placed close to the focal point rather than far away:



The focus of the microscope is typically adjusted so that the image at q falls on the focal point of the eyepiece. This creates a final image at infinity, allowing the observer's eye to be fully relaxed when viewing the image.

Example: A microscope has a length L = 15cm, an objective with a focal length of 1.2cm and an eyepiece with a focal length of 0.8cm. What is its magnification compared to a normal eye with a near point of 25cm?

First we will find the distance between the object and the objective lens. The distance should be such that the intermediate image falls on the focal point of the eyepiece: $q = L - f_E = 15cm - 0.8cm = 14.2cm$. This gives

$$p = \frac{1}{1/f - 1/q} = \frac{1}{1/1.2cm - 1/14.2cm} = 1.31cm$$

Referring to the ray diagram above, we see that the objective creates an inverted image of size h' = (q/p)h = (14.2/1.31)h = 10.8h. For a very small object, the angle subtended by this image on the eyepiece can be estimated as $\theta = h'/f_E = 10.8h/f_E$. This is the same as the angle subtended by the final image on the eye.

Now if the object was simply viewed with the eye at the near point, it would subtend an angle of $\theta_0 = h / p_{NP}$. Thus, the angular magnification of this microscope is

$$m = \frac{\theta}{\theta_0} = \frac{10.8h}{0.8cm} \frac{25cm}{h} = 338$$

Thus, when viewed through this microscope, objects appear about 340 times as big as when viewed with the eye at the near point.

Most microscopes have several objectives, labeled, for example, 300X, or 2000X. These objectives have different focal lengths, and thus give different magnifications. The nominal magnification of the objective is given as the magnification compared to an eye with a near point of 25cm, with the microscope adjusted so that the final image is at infinity.

The microscope's resolution is limited by the wavelength of light used. One cannot form an image of an object that is smaller than the wavelength of light used to illuminate it, since the light waves will diffract around such a small object, rather than passing through it or reflecting off it as shown in ray diagrams. So, for example, a microscope that operates with 500*nm* light will not be able to resolve objects smaller than 500*nm*. Magnifying the image in an attempt to get better resolution will simply result in a more blurry image.

The interferometer

The interferometer has many uses, most of which involve being able to measure a change in light path length or phase to a great degree of precision. The Michelson interferometer looks like this:



The light is emitted from a coherent source, so that initially all the EM waves are in phase with each other. The light reaches a beam splitter, which is a half-silvered mirror that reflects approximately half the light to Arm 1 and allows the other half to pass through to Arm 2. The light is reflected by mirrors at the end of each arm, and some of it is reflected or transmitted by the beam splitter towards the view port.

If the phase shift experienced by light along Arm 1 and Arm 2 is exactly the same or different by an even multiple of π , then the two beams reaching the view port will interfere constructively, resulting in a bright spot. If the phase shifts are different by an odd multiple of π , then we get destructive interference, resulting in a dark spot. The difference between the two cases corresponds to a change in distance of only half a wavelength, so any change in the length of one of the arms, or in any other factor that causes the light along one of the arms to accumulate more or less phase, can be detected to a great degree of precision.

Typically, one of the mirrors is slightly angled, so that the path length difference changes with position on the mirror. This results in a pattern of light and dark bands from light coming from different parts of the mirror. These bands are called *fringes*. If one of the mirrors is moved back slowly, so the path along that arm increases, the fringes move across the mirror. The observer can count the number of fringes that have gone by, and determine the distance by which the length of the arm has changed.

Example: A small interferometer's two arms each have a length of 0.25*m*. The light source has a wavelength of 500*nm*. Both arms are held in a vacuum. The chamber containing one of the arms is slowly filled up with helium, until the pressure in the chamber reaches 1 atmosphere. If the index of refraction of helium at 1 atmosphere is 1.000036, how many fringes move by the view port as helium is added to the chamber?

Suppose that, initially, light paths along both arms are identical, so light goes through the same number of wavelengths along each arm. The number of wavelengths along each path is $0.50m / 500nm = 10^6$.

After the helium is added, the wavelength of light in that chamber decreases by a factor of 1.000036. Therefore, the number of wavelengths along this path increases by a factor of 1.000036, and is now 1,000,036. The difference between this and the other path, which remains in vacuum, is 36 wavelengths. Thus, 36 fringes go by.

The Michelson-Morley experiment

Maxwell's equations predict a universal speed of light, independent of the observer. For a long time, people did not believe this conclusion, and assumed that Maxwell's equations were not entirely correct. By analogy with the theory of sound, where waves propagate through air and the wave equation is only valid in the frame where the air is at rest, 19th century physicists assumed that there was a medium known as the ether through which light waves propagated. Maxwell's equations would, then, be valid only in the frame where the ether is at rest.

The deviations from Maxwell's equations would allow an observer moving, say, in the x direction through the "ether", to see light coming from the x direction go faster, and light coming up from behind go slower, as expected from everyday experience. We now know this is not true, of course, but most 19th century physicists assumed it was true.

If such an observer-dependent effect did occur, it could be measured by using the Michelson interferometer. Suppose arm 1 was oriented in the observer's direction of travel through the ether. Due to the difference in the speed of light and the motion of the apparatus relative to the "ether", light traveling down arm 1 and back would accumulate a different phase than light traveling along arm 2. The apparatus could now be rotated until arm 2 was oriented in the direction of motion. This would cause the phase difference between the two paths to become flipped, and as a result, fringes would move as the apparatus was rotated.

Since the Earth moves around the Sun, the observer's motion would change depending on time of year. Thus, if the ether hypothesis was true, the results of the Michelson-Morley experiment would vary depending on the season.

Of course, no such variation was found, and in fact the fringes didn't move at all, no matter what time of year it was. This implied that the speed of light was in fact the same in all directions for all observers. Other experiments confirmed these findings. Eventually, Einstein began with the assumption that Maxwell's equations, rather than our everyday experience with relative speeds, were correct, and derived his theory of relativity.

Detailed treatment of the Michelson-Morley experiment is left as a homework assignment.