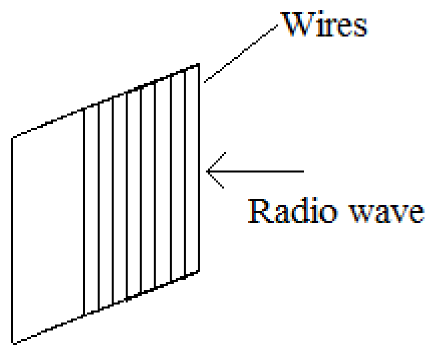
Lecture 11 Notes: 07 / 18

Polarization and polarizing filters

Light and other EM radiation is an oscillating EM wave. As we mentioned before, the electric field for an individual wave points in a particular direction perpendicular to the wave's motion, while the magnetic field is perpendicular both to the wave's motion and to the electric field. The direction that the electric field points is called ***polarization***.

Non-polarized light contains waves polarized in all directions, randomly mixed together. If every wave is is polarized in the same direction, then the light is said to be polarized.

Suppose we have a non-polarized radio wave. We can pass it through a contraption consisting of closely spaced conducting wires, all pointing in the same direction:



When a radio wave with random polarization passes through the wires, the vertical component of the electric field, which is aligned with the wires, will drive current up and down the wires, and will thus lose its energy to the resistance and dissipate. The horizontal component, however, will pass through unimpeded, as it cannot drive the electric charges side to side. The remaining wave will thus be polarized in the horizontal direction (we will call this the direction of the polarizing filter). Its amplitude will be the same as that of its original horizontal component. If the wave was originally polarized at an angle ϕ to the direction of the filter, and had amplitude E_0, its new amplitude will be the size of its original component that was aligned along the filter:

$$E = E_0 \cos \phi$$

The important thing is that the wires must be very close together compared to the wavelength, otherwise the waves will be able to go right through the gaps without causing current to flow, and we'll have a diffraction grating rather than a polarizing filter. We can construct a similar polarizing filter for light, but instead of parallel wires, we must use a material with a similar type of microscopic structure, that will admit only EM waves polarized in a particular direction. Such materials (various types of polymers or crystals) are quite cheap and easily available.

Note that an electromagnetic wave perpendicular to the filter cannot pass through at all, since in this case $\cos\phi = 0$, and thus the transmitted wave's amplitude is also 0. This is the principle behind 3D glasses. The image on the screen is a superposition of two polarized images: one for the left eye, polarized in the vertical direction, and another for the right, polarized in the horizontal direction. The left side of the glasses is a vertical polarizing filter, and so it only admits the image for the left eye, while the right side is a horizontal polarizing filter, and only admits the image for the right eye. Since depth perception is primarily based on the difference of images between the two eyes, the 3D image creates a perception of the distance of the objects on the screen from the viewer.

The ***intensity*** of an electromagnetic wave is proportional to the amplitude squared and is measured in watts per meter squared (just as for sound). Thus the intensity of the transmitted wave, for incident light polarized at an angle ϕ to the filter, is

$$I = I_0 \cos^2\phi$$

If the incident light is not polarized, then all waves have a random value of ϕ, varying from 0 to 2π. To get the intensity of the light after it passes through the filter, we should thus average this expression over ϕ. The average of $\cos^2\phi$ is

$$< \cos^2\phi >= \frac{\int_0^{2\pi} d\phi \cos^2\phi}{2\pi} = \frac{1}{2}$$

Thus, for unpolarized incident light, the transmitted light is polarized in the direction of the filter, and has an intensity of 1/2 that of the incident light:

$$I = \frac{1}{2} I_0$$

Example: A beam of light with $I = I_0$, polarized in the x direction, is passed first through an angle oriented at 30 degrees to the x direction, and then through a second filter oriented in the y direction, at 60 degrees to the first filter. What is the final intensity of the light? What if the first filter was removed?

We have polarized light, so the intensity will depend on the angle between the initial polarization and the filter. After the first filter, the intensity is

$$I_1 = I_0 \cos^2 30° = \frac{3}{4} I_0$$

After the second filter, the intensity is

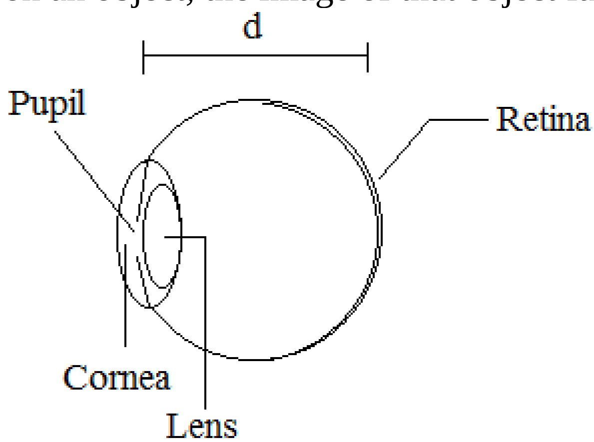
$$I_2 = I_1 \cos^2 60° = \frac{1}{4} I_1 = \frac{3}{16} I_0$$

If the first filter was removed, the second filter would be oriented at 90 degrees to the polarization of the light, and thus the final intensity would be zero. The intervening filter actually allows some of the light to pass through the second one.

Light can also be polarized by reflection from an interface between two transparent materials. This happens because the reflection coefficient is dependent on the orientation of the electric field, and the waves for which the electric field is perpendicular to the surface normal reflect more strongly. In addition, light scattered from its original path by particles such as air molecules is polarized depending on the angle from which it is viewed. You can read more about these phenomena in the book.

The human eye

For the purpose of optics, the eye consists basically of three parts: the cornea, the lens and the retina. The cornea and the lens together focus as a convergent lens. The lens is flexible and surrounded by muscles that can change its shape and adjust its power; thus, the focal length of the cornea-lens combination can change within a certain range. When the eye is focused on an object, the image of that object falls on the retina:



An opening (the pupil) admits incident light through the lens. The size of the pupil can adjust depending on the brightness of the light: in bright light, the pupil constricts, shielding the retina from excess light, while in low-light conditions, the pupil expands, allowing more light to reach the retina.

The retina consists of a dense array of photosensitive cells: the cones, which are sensitive to color but require a relatively high intensity of light to trigger, and the rods, which are sensitive only to the intensity but not the wavelength of light, but can detect even very faint light. In bright light, the rods are inactive, and the cones provide color vision; if the light is turned off, the rods come on over several minutes and provide monochrome night vision, while the cones do not see much as they require more light to operate.

Example: A person with good vision can perfectly focus on objects very far away, as well as on objects 30 centimeters away. This person has an eye diameter (lens to retina distance) of 1.9*cm*. What is the focal length of this person's lens-cornea combination when she is looking at the moon? What if she is reading a book 30 centimeters away from the eye?

The image distance should be the same as the diameter of the eye, so that the image falls on the retina. This means that

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad f = \frac{1}{1/p + 1/d}$$

For an object at infinity, this means that $f = d = 1.9cm$. For an object at 30*cm*, this gives a focal length of $f = 1 / (1/30cm + 1/1.9cm) = 1.79cm$. The slight change in focal length is accomplished by flexing and relaxing the muscles of the eye that change the shape of the lens.

Myopia and presbyopia

We see that for perfect vision, the focal length of the lens-cornea combination must be able to vary between a value somewhat less than the diameter of the eye, and the diameter of the eye. If the maximum focal length is less than the diameter of the eye, this is a condition known as ***myopia*** or ***nearsightedness***. This makes it so that the eye cannot focus on distant objects. Myopia can be caused by an abnormal eye shape, where the lens is too far away from the retina, or by a lens and cornea that is too curved (and thus has too short a focal length) for the normal eye.

A deficiency on the other side of the range, when the lens has a minimum focal length such that nearby objects cannot be seen clearly, is known as ***presbyopia***. This is most frequently a result of the loss of the eye's accommodation ability with age. To reduce the focal length and focus on nearby objects, the lens must be stretched. With age, the lens becomes more rigid, and the focal length cannot be reduced enough to allow nearby objects to be seen clearly.

Since myopia and presbyopia have different mechanisms and impact different sides of the eye's range of accomodation, they are not mutually exclusive. One could have both at the same time.

Example:

The far point of a myopic patient's eye is 60 *cm*. What is the focal length of a contact lens that this patient should wear to correct the vision? What is the power of this lens? What is the focal length and power of glasses, which will be separated by a distance of 2*cm* from the patient's eye?

The contact lens should bring an object at infinity to the patient's far point. This is 60*cm* from the eye, or 60*cm* behind the lens. Thus, by the lens equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad f=\frac{1}{1/\infty-1/60cm}=-60cm$$

The power is $1/f$, or 1 / 0.60*m* = 1.67 diopters.

For glasses, the object at infinity should also form an image 60*cm* from the eye, but this is now only 58*cm* behind the glasses. Thus the focal length should be -58*cm*. The power is 1 / 0.58*m* = 1.72 diopters.

If the patient's near point was 15*cm* before vision correction, what will it be with the contact lenses? With the glasses?

The near point will occur when the image from the lens or the glasses forms at 15*cm* from the eye. With the lenses, this means the image is 15*cm* behind the lens, and we obtain

$$p=\frac{1}{1/f-1/q}=\frac{1}{-1/60+1/15}=20cm$$

With the glasses, the near point is 13*cm* behind the lens, and we get

$$p=\frac{1}{1/f-1/q}=\frac{1}{-1/58+1/13}=16.8cm$$

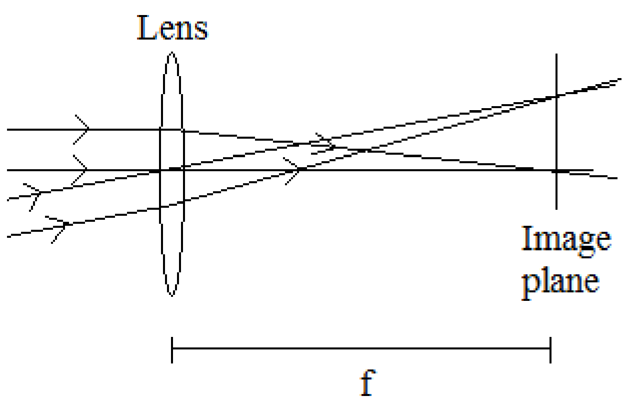
Since the glasses are 2*cm* in front of the eye, this means that the near point is now 18.8*cm* away from the eye.

The telescope

The telescope works much the same way as the eye, except it is intended to look at distant objects only, so there is no need to adjust the focal length of the main lens or mirror. Light is focused at the focal point, forming an inverted image of distant objects. The image can then be magnified by using an eyepiece (essentially a magnifying glass) or be directly photographed by letting it fall on a charge-coupled device (CCD) or photographic plate (before CCDs were put into wide use). The CCD is the same kind of detector as in digital cameras: it is a chip consisting of a bunch of closely-spaced pixels, each of which detects any light that falls on it, thus recording an image. The CCD is exactly analogous to the retina. We will discuss telescopes with CCDs first, since they are very similar to the eye and are most often used by professional astronomers, and discuss eyepieces later.

The purpose of the telescope is not to magnify the image. The CCDs have very high resolution, with a pixel every few microns, so even a tiny image will suffice, and an eyepiece can magnify the image to an arbitrary size for human inspection. Instead, the telescope is primarily a light amplification device. The bigger the mirror or lens of the telescope, the more light can be collected, and the fainter the objects that can be seen. Increasing the aperture size also helps make the image more clear by reducing diffraction; we will discuss this below.

Smaller telescopes used by amateurs can use a large convergent lens to collect the light. This kind of telescope is called a ***refracting telescope***.



Rays coming in from infinity are focused at the focal point, where one can place the CCD or inspect the image with an eyepiece. Rays coming from different directions are focused to different points on the image plane; as you can see in the image above, the angle by which the points on the image plane are separated is the same as the angle by which the distant sources are separated.

Example: An amateur astronomer photographs the Moon using a refracting telescope with a 35*cm* lens and a focal length of 60*cm*.

What is the size of the Moon's image on the image plane? If a CCD with a pixel size of 20*mm* is used, how many pixels across is the image of the Moon? How much brighter is the image of the Moon compared with the Moon as viewed with the eye, assuming the human pupil has a dark-adapted diameter of 8*mm*?

The Moon's image subtends the same angle with the lens as the Moon does. The Moon is 3476*km* across and, on average 384,000*km* away from the Earth. The angle subtended by the Moon is thus 9.05 x 10^{-3} radians.

At a distance of 60*cm* from the lens, this gives an image size of 0.543*cm*. With a 20-micron pixel size, the image will be 0.543 / 0.002 = 272 pixels across. The resolution of the image can be increased by using a CCD with a smaller pixel size, or by using a telescope lens with a larger focal length.

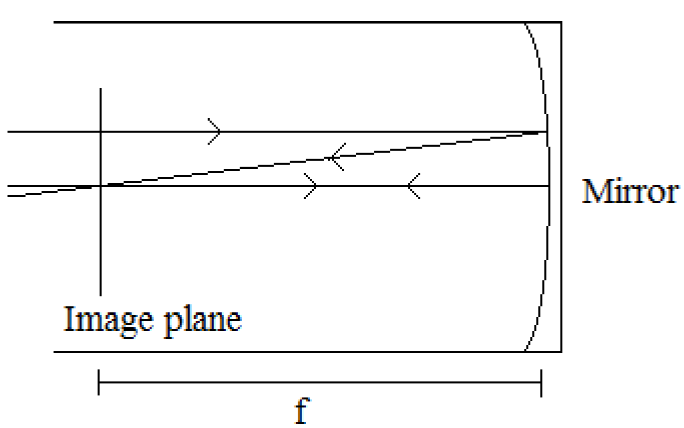
The amount of light that gets through is proportional to the area of the aperture, and thus its diameter squared. The telescope lens is 350*mm* / 8*mm* = 43.75 times bigger than the pupil. Thus, the brightness is increased by a factor of 43.75^2 = 1900.

Note that the image size, compared to the object size, can be calculated as follows:

$$h' = f\theta = f\frac{h}{R}$$

Here, *h'* is the image size, *h* is the object size, *R* is the distance to the object, and *f* is the focal length of the telescope lens.

It is very difficult to manufacture large lenses without introducing defects that will reduce the quality of the image. Large parabolic mirrors are much easier to make. Thus, large telescopes are ***reflecting telescopes***, using a parabolic mirror to focus the light:



Reflecting telescopes work the same way as refracting telescopes; the image size is related to the object size and the focal length in the same way, and the amplification of the light works the same way as well. However, now the mirror can be made quite large. Telescopes with single mirrors have aperture sizes of up to 10 meters; compound mirrors can be constructed from multiple segments and make this size even larger. Right now, a 30-meter reflecting telescope is being built using a compound mirrors, and the size is limited only by the light-gathering power desired and by the available funding.

The diffraction limit

Parallel light entering through a telescope aperture spreads through an angle θ due to diffraction. In the previous lecture, this angle was found to be

$$\sin\theta \approx \theta = \frac{\lambda}{a}$$

Light from sources separated by an angle less than θ will thus be mixed together due to diffraction. These sources therefore cannot be distinguished from each other. The diffraction limit thus limits the telescope's resolution. The only way to improve the resolution beyond the diffraction limit is to use a telescope with a larger aperture.

Example: Recall the 35*cm* reflecting telescope with the focal length of 60*cm* from the example above. When taking the photograph of the Moon, as in the previous example, what is the smallest size of the features that can be distinguished due to the diffraction limit? What CCD pixel size would be required in order to achieve image resolution at the diffraction limit? Is this reasonable? Assume the observations are done in the visible light, with an average wavelength of 520*nm*.

The smallest angle between two sources that this telescope can distinguish is

$$\theta = \frac{\lambda}{a} = \frac{5.20 \times 10^{-7} m}{0.35 m} = 1.49 \times 10^{-6} rad$$

The distance on the Moon that this corresponds to is

$$h = \theta R = (1.45 \times 10^{-6})(3.84 \times 10^{5} km) = 556 m$$

With a focal length of 60*cm*, this angle would give the following distance on the image plane:

$$h' = \theta f = (1.49 \times 10^{-6})(0.60 m) = 891 nm$$

This is actually about 6 times smaller than the smallest CCD pixel size available. Thus, to achieve resolution at the diffraction limit, the image should first be magnified optically. This can be done by photographing through an eyepiece. If the telescope had a longer focal length, the image would be larger and the eyepiece unnecessary.

Example: A particular individual's eye has a pupil diameter of 4*mm* under well-lit conditions, an eye diameter of 19*mm*, and perfect vision. The cones in the center of the field of vision are spaced 2 microns apart. What is this individual's angular resolution due to the diffraction limit? Due to the resolution limit of the retina? The term "20 / 20 vision" describes visual acuity, and refers to the ability to see, at a distance of 20 feet, a letter that has lines separated by 1 arc minute (1/60 of a degree); 20 / 40 is worse than normal, and 20 / 10 is better. What is this person's visual acuity in bright light?

The diffraction limit of this eye is

$$\theta = \frac{\lambda}{a} = \frac{5.20 \times 10^{-7} m}{4 \times 10^{-3} m} = 1.30 \times 10^{-4} rad = 0.44'$$

The resolution limit of the retina is

$$\theta = \frac{\Delta x}{d} = \frac{2 \times 10^{-6} m}{1.9 \times 10^{-2} m} = 1.05 \times 10^{-4} rad = 0.36'$$

The eye is therefore diffraction-limited, and can see parts of letters separated by 0.44', rather than the nominal 1' for 20 / 20 vision. Thus, this person has 20 / 20 x 0.44 = 20 / 9 vision. This is very good, but not uncommonly so.

To compare this example to the telescope above, this person would be able to see features on the Moon separated by a distance of 50*km* or so (in fact, a bit less so since the pupil dilates in low illumination, reducing the diffraction limit). The 35*cm* telescope allows up to 100 times higher resolution, due to the larger aperture, provided appropriate optics are used to magnify the image.

We will talk more about magnifying images next lecture, when we discuss magnifying lenses and microscopes.