Homework Set 5

Due Thursday, 07/28

1. Time dilation and length contraction

Problem 1: A spaceship travels from Earth to a star 100 light-years away. The ship is moving at a constant speed of $0.85c$.

(a) How long does the trip take according to an observer on Earth?

(b) How long does the trip take according to an observer on board the spaceship?

(c) What is the distance from Earth to the star, according to the observer on the spaceship?

Problem 2: Unstable particles are produced in a nuclear reaction in a laboratory. The particles have an average lifetime of 1.0×10^{-10} seconds when at rest. The particles travel an average distance of $5\,cm$ from the point of production before decaying, according to an observer at rest in the laboratory. What is the speed of the particles relative to the stationary observer?

Problem 3: Suppose that at $t = 0$, Bob synchronizes his watch with Alice, and gets on a high-speed train that travels around and around the Earth at a speed of $500\,km/h$. If the train never stops, how long would Bob need to stay on it before his watch was off from Alice's by one second, due to time dilation? Express the answer in years.

2. Velocity addition

Problem 4: Alice observes a particle moving with a speed of $0.8c$ in the x direction, while Bob observes the same particle moving with a speed of $0.5c$ in the x direction. How fast is Bob moving relative to Alice?

Problem 5: An observer sees two spaceships, A and B, coming towards him from opposite directions with an equal speed. If the crew of ship A sees ship B approaching ship A at a speed of $0.9c$, what is the speed of the two spaceships relative to the observer?

3. Lorentz transformation and 4-vectors

Problem 6: Bob is moving in the x direction at a speed of $0.8c$ relative to Alice. He detects two simultaneous explosions at $t = 0$. One explosion happens at position $(x, y, z) = (10^6\text{km}, 0, 0)$, while the other happens at $(x, y, z) = (2 \text{ x } 10^6\text{km}, 0, 10^6 \text{ km})$.

(a) According to Alice, the explosions are not simultaneous. How much time passes between them? Use the Lorentz transformation to find the result.

(b) What is the distance between the explosions according to Bob? According to Alice?

Problem 7: Bob is moving in the x direction, at a speed of $0.5c$ relative to Alice. He observers a particle moving in the z direction with a speed of $0.3c$.

(a) What is the particle's 4-velocity relative to Bob?

(b) What is the particle's 4-velocity relative to Alice?

(c) How fast is the particle moving relative to Alice?

The Doppler effect

Problem 8: A driver gets a ticket for running a red light. The driver claims that since he was driving towards the light, the light appeared green to him due to the Doppler effect. How fast would the driver have to be going if his story was true? Assume that red light has a typical wavelength of 650*nm* while green light has a wavelength of 520*nm*. Express the result as a fraction of the speed of light and in miles per hour.

Problem 9:
A radar emits microwave radiation with a wavelength of 20*cm*. The microwaves reflect off an aircraft, which is moving towards the radar with a speed of 400 *m/s*. In the radar device, the reflected waves are made to interfere with the outgoing waves; the slight difference in frequency causes beats, just as for sound waves. What is the beat frequency?

Wave-particle duality

Problem 10:
A 650*nm* laser operates at a power of 1.0W. The laser beam is approximately cylindrical, and has a diameter of 1*cm*.

(a) How many photons per second does the laser emit?

(b) How many photons are in a cubic centimeter of the laser beam?

Problem 11:
When ultraviolet light with a wavelength of 300*nm* is incident on a metal surface, electrons are emitted via the photoelectric effect. The electrons are stopped by a retarding potential of 1.2 volts.

(a) What is the work function of this metal?

(b) What is the longest wavelength of light that can eject electrons from this metal?

Problem 12:
An electron and a proton both have a kinetic energy of 10^4 eV.

(a) What are the speeds of the electron and the proton?

(b) What are the wavelengths of the electron and the proton?

Problem 13:
An electron microscope is to resolve features as small as 10*nm* across. What is the potential through which the electrons in the microscope must be accelerated?

Problem 14:
A beam of electrons is accelerated through a potential of 5000V. The electrons are incident on a crystal with interatomic spacing of 0.15*nm*. What is the angular separation between adjacent diffraction peaks?

Mass, energy and momentum

Problem 15:
How much energy would it take to accelerate an electron to a speed of 0.99c? What about 0.999c? Express the answers in electron-volts.

Problem 16:
An electron has a kinetic energy of 1MeV relative to Alice. It moves in the positive x direction.

(a) Bob is moving in the negative x direction (opposite to the direction of the electron's motion) at a speed of 0.5c. What is the kinetic energy of the electron relative to Bob?

(b) How fast is the electron moving relative to Alice? Relative to Bob? Obtain both result from the electron's kinetic energy.

(c) Now obtain the speed relative to Bob from the speed relative to Alice using relativistic velocity addition. You should get the same result as in part (b).

Problem 17:
A photon with an energy of E_γ= 25eV collides with an electron at rest, and is reflected back in the direction it came from.

(a) What is the 4-momentum of the electron and of the photon before the collision, in terms of the photon energy E_γ and the electron mass m? Assume the photon is initially traveling in the x-direction.

(b) What is the 4-momentum of the electron and the photon after the collision? What is the kinetic energy of the electron after the collision, in eV? Use energy-momentum conservation, and the mass-energy-momentum relations for the photon and electron.

Nuclear and particle physics

Problem 18:
Carbon 14 decays via beta decay.

(a) What isotope does it turn into?

(b) The carbon 14 nucleus has a mass of 14.003241 u, and the nucleus it turns into has a mass of 14.003074 u. Suppose that the electron and the antineutrino are emitted in such a way that the nucleus remains at rest. What are the energies of the electron and the antineutrino? How fast is the electron moving (as a fraction of the speed of light)?

Problem 19:
Plutonium-239 has a half-life of approximately 24,000 years. It decays to uranium-235 via alpha decay. The isotope mass of Pu-239 is 239.052156 u, the isotope mass of uranium-235 is 235.0439299 u, and that of the alpha particle is 4.002602 u.

(a) How many decays per second spontaneously occur in 1 kilogram of Pu-239?

(b) How much energy per second is released by all these decays?

(c) The heat radiated away by an object with temperature T is given by $P = \sigma A(T^4 - T_0^4)$, where T is the temperature of the object in kelvins, T_0 is the ambient temperature, σ is Stefan-Boltzmann constant ($\sigma = 5.67 \times 10^{-8} \mathrm{Wm^{-2}K^{-4}}$) and A is the surface area of the object. Plutonium has a density of 19.82g $\mathrm{cm^{-3}}$. If the 1kg piece of plutonium is shaped like a sphere and all the heat from the decays is radiated away, what is the temperature of the surface of the sphere? Assume the ambient temperature is 273K.

Problem 20:
Suppose that a collision causes a quark to begin accelerating away from a proton. Two particles are formed as a result, one of which is a neutron. What is the other particle?

Problem 21:
A neutral pion consists of a quark-aniquark pair, both of the same type. A charged pion consists of a pair of different types (for example, up quark and down antiquark). Pions are lighter than any particles apart from photons, electrons and neutrinos.

(a) How does a neutral pion decay? How does a charged pion decay? Draw Feynman diagrams for the decay processes.

(b) Which would you predict would have a longer lifetime, a charged pion or a neutral pion? Why?

Problem 22:
In some grand unified theories, the proton can decay into a pair of particles, with a very long lifetime.

(a) Based on conservation of charge, energy, momentum and spin, what kind of particles could a proton decay into?

(b) Pick a possible pair of particles. If the proton is at rest before it decays, what will be the energy of each of these particles after the decay?