Homework Set 1

Due Thursday, 06/30

Problem 1: A spring has a relaxed length of 15.0cm. When a 100g mass is hung from the spring, the spring is stretched to 17.5cm. When an unknown mass M is hung from the spring, the spring stretches to 18.0cm.

(a) What is the mass of M?

(b) What is the frequency of oscillations with the 100g mass and with mass M?

(c) If a 250g mass is attached to the spring while the spring is relaxed, and then dropped from that position, what will be the amplitude of the oscillations?

Problem 2: The position of an oscillator is given by

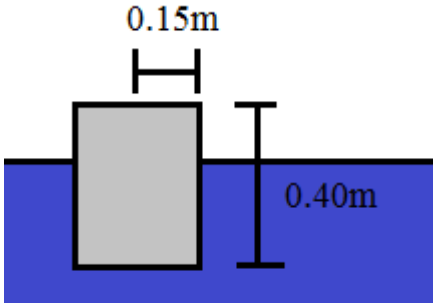
$$x(t) = 0.0135m \times cos\left(6.58s^{-1} \times t + 0.14\right)$$

(a) What is the amplitude, angular frequency, frequency and period of the oscillations?

(b) What is the speed of the oscillator at $t = 0$?

(c) If the oscillator has a mass of 0.05kg, what is the spring constant?

Problem 3: A cylindrical barrel with length L = 0.40m, radius R = 0.15m and density ρ = 600 kg/m^3 floats in water (density 1000 kg/m^3). The forces acting on the barrel are gravity, pointing down, and the buoyant force, equal to the weight of the displaced water according to Archimedes' law, pointing up.



(a) If the barrel is in equilibrium (no net force), how high is the top of the barrel above the surface of the water?

(b) Show that the barrel will obey Hooke's law if displaced slightly up or down from equilibrium. What is the effective "spring constant" and the period of oscillations?

Problem 4: A 6.0 kg block is attached to a spring with k = 120kg/s, and slides on a floor with a coefficient of kinetic friction $\mu_k = 0.25$. The block begins at the equilibrium position. It is struck by a bullet with a mass of 15 grams, moving at 650 m/s and coming from the right. The bullet becomes embedded in the block.

How far to the left does the block move?

Problem 5: A 70kg bungee jumper jumps off a very tall bridge using a bungee cord with a relaxed length of 30 meters.

Assume the bungee cord exerts no force until it reaches its relaxed length, and obeys Hooke's Law when stretched beyond the relaxed length. To avoid dislocating the jumper's leg, the deceleration of the bungee jumper should not exceed 30 m/s^2. What is the maximum allowable spring constant of the bungee cord?

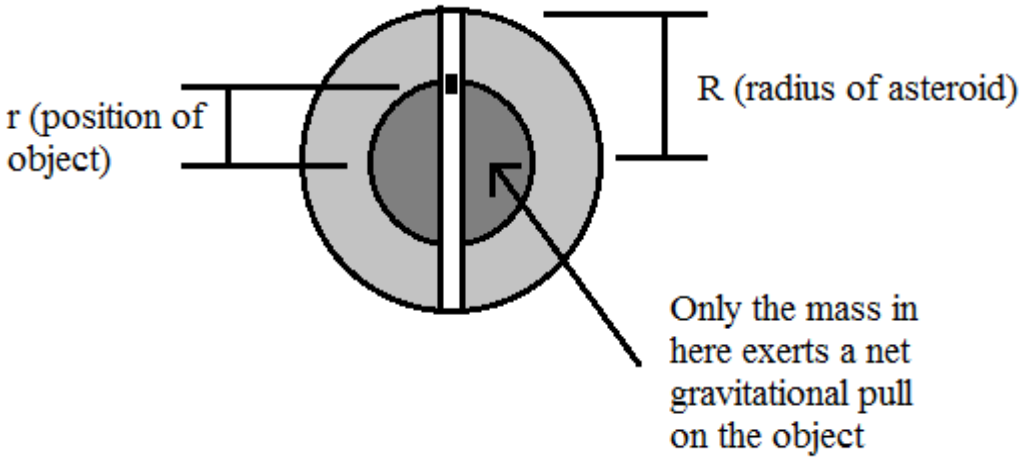
Problem 6: A simple pendulum is in an elevator that is accelerating upward at 1.5 m/s^2. The pendulum is swinging with a period of T = 0.8s.

(a) What is the length of the pendulum?

(b) When the elevator begins to decelerate at 1.5 m/s^2, what will be the period of this pendulum?

Problem 6: Darth Vader lands on an unknown planet. For some reason, he has a pendulum clock with him. On his spaceship, where the artificial gravity is kept at 8.0m/s^2, the clock keeps perfect time, but on the planet's surface, the clock's hour hand takes only 40 minutes to move through 1 hour. What is the acceleration due to gravity on the planet?

Problem 7: Suppose we have a spherical asteroid with radius R = 50km and density ρ = 3000 kg/m^3. We drill a straight tunnel from the surface of the asteroid to the center and out to the other side, and drop an object into the tunnel. There is a theorem, by Isaac Newton, that an object within a spherically symmetric distribution of mass (such as our asteroid) is attracted only by the gravity of the stuff that is closer to the center of the distribution than the object itself. So, if our object is 5km from the asteroid's center, it will only be attracted by the mass that is within 5km of the center of the asteroid. You will need to use this theorem to solve this problem.



(a) Show that if the object doesn't collide with the walls or otherwise experience friction, it will move back and forth through the tunnel with simple harmonic motion.

(b) How long will it take for the object to come back to you, after you throw it in?

(c) What is the object's maximum speed?

Problem 9 (Extra Credit): A 0.10 kg mass is oscillating on a weakly damped spring with a period of 0.5 seconds. It begins with an amplitude of A_0 = 16 cm, but after 30 seconds, the amplitude is reduced to 10 cm due to the damping force.

(a) What will the amplitude of the oscillations be after 1 minute?

(b) What is the magnitude of the frictional force when the mass passes through the equilibrium position for the first time, before the amplitude has decreased?

Problem 10 (Extra Credit): Suppose that a pendulum clock keeps perfect time on the equator. If this clock is moved to the South Pole, will it be too fast or too slow? How much time will it gain / lose over the course of 1 year? (Hint: assume that the difference in local gravity on Earth is primarily due to centrifugal acceleration due to the Earth's rotation.)