Physics 1C, Summer 2011 (Session 1)

Practice Midterm 2 (50+4 points) Solutions

Problem 1 (5x2 = 10 points)

Label the following statements as True or False, with a one- or two-sentence explanation for why you chose your answer. Even if you get the answer correct, you will receive no credit unless your explanation is clear.

a. In order to have total internal reflection for light, you must be in a medium that has a smaller index of refraction than the surrounding medium.

b. When you walk away from a flat mirror, the size of your image does not change.

c. A lens can produce an upright, magnified, real image.

d. When looking at the intensity distribution for diffraction from a small single slit, projected onto a screen very far away from the slit, the width of the central bright spot is twice that of surrounding bright spots.

e. To resolve two very close objects, it is helpful to use light with a small wavelength.

Solution

a. False: The opposite is true according to Snell's Law. If you start in a medium with index of refraction $n_1 < n_2$, there is always an angle θ_2 such that $n_1\sin(\theta_1) = n_2\sin(\theta_2)$.

b. True: The size of the image is always equal to the size of the object for a flat mirror, since $1/p + 1/q = 1/f = 0 \rightarrow q = -p \rightarrow M = -q/p = 1 = h' / h$.

c. False: For a single lens, real images are only produced by converging lenses, which can only form inverted real images on the opposite side of the lens as the object.

d. True: This is why the formula for diffraction minima excludes m=0.

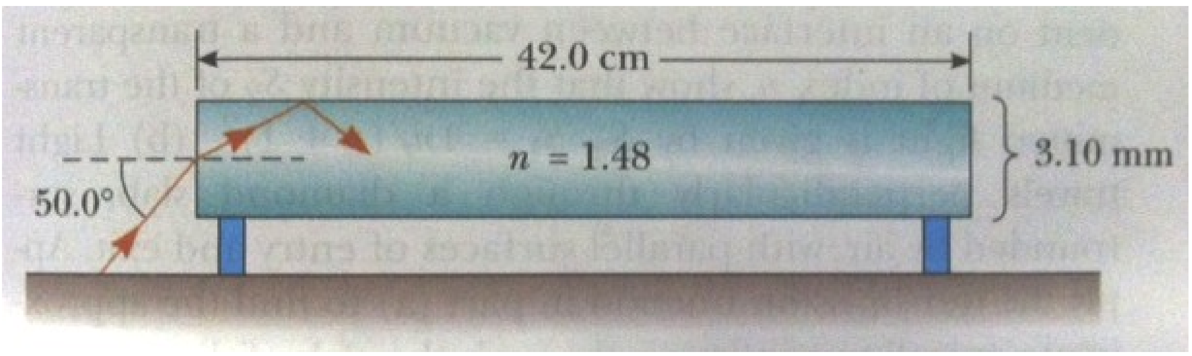
e. True: Rayleigh's criterion says that the limiting angle of resolution for a slit of width a is $\theta_{min} = \lambda / a$. The lower the wavelength, the smaller θ_{min}, and the better we are able to see two objects separated by a very small angle.

Problem 2 (5+5 = 10 points)

A laser beam strikes one end of a slab of material as shown in the figure below. The index of refraction of the slab is 1.48.

a. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.

b. Is there an incident angle ψ (instead of 50.0^0) such that the beam inside leaks out of the slab (that is, the beam is not undergoing total internal reflection)?



Solution

a. (Copied from the solution to P25.35 in the course textbook). As the beam enters the end of the slab, the angle of refraction is found from Snell's law:
$\sin(\theta) = (n_{air}/n_{slab}) \sin(50.0^0) = 0.518 \rightarrow \theta = 31.2$ degrees.

This is the angle of refraction of the beam just as it enters the slab. The beam then strikes the top of the slab, at an incident angle $\varphi = 90^0 - 31.2^0 = 58.8^0$. Note that this is greater than the critical angle of $\theta_c = \sin^{-1}(1/1.48) = 42.5^0$, and so the beam inside the slab undergoes total internal reflection.

Suppose we make a right triangle in the above picture by taking the dotted line (inside the slab) as the base, and the arrow pointing up and to the right as the hypotenuse. The distance the beam travels down the length of the slab for each reflection is 2d, where d is the base of that triangle. Then:
$d = h / \tan(\theta) = (3.10\text{mm}/2) / \tan(\theta) \rightarrow 2d = (3.10 \times 10^{-1}\text{ cm}) / \tan(31.2^0) = 5.12\times10^{-1}\text{cm}$.
The number of internal reflections made before reaching the opposite end of the slab is then $N = (\text{length of slab}) / (2d) = (42.0\text{cm}) / (5.12\times10^{-1}\text{cm}) = 82$ reflections.

b. The critical angle was found in part a to be 42.5^0, which means that light will leak out if the light hitting the top edge of the slab is less than 42.5^0. This means that the angle of the refractred beam that enters the slab is greater than $90^0 - 42.5^0 = 47.5^0$. Using Snell's law, we find $n_{air}\sin(\psi) = n_{slab}\sin(47.5^0) \rightarrow \sin(\psi) = 1.09 \rightarrow$ There is no solution for ψ. Therefore, there is no incoming angle for which light will leak out of the slab.

Problem 3 (5+5 = 10 points)

Light of wavelength 590 nm passes through two narrow slits 0.60 mm apart. The screen is 1.70 m away, perpendicular to the line connecting the slits to the screen.

a. How far away from the central maximum is the second-order fringe? The second-order fringe is not the bright spot next to the central maximum, but the bright spot that is two away from the central maximum.

b. A second source of unknown wavelength produces its second-order fringe 1.33 mm closer to the central maximum than the 590-nm light. What is the wavelength of the unknown light?

Solution
a. $d\sin(\theta) = d(y/L) = m\lambda \rightarrow y = (mL\lambda)/d = (2)(1.7)(590 \text{ x } 10\text{^-9})/(0.60 \text{ x } 10\text{^-3})$ meters.
Therefore, y = 3.34 mm is the distance from the central maximum to the 2^{nd}-order fringe.

b. The distance of the new light's second-order fringe is 3.34 mm – 1.33 mm = 2.01 mm.
$y_{new} = (mL\lambda_{new})/d \rightarrow \lambda_{new} = (y_{new}d)/(mL) = (2.01 \text{ x } 10\text{^-3})(0.60 \text{ x } 10\text{^-3}) / [2(1.7)]$ meters.
Thus, $\lambda_{new} = 355$ nm.

Problem 4 (6+3+3 = 12 points)

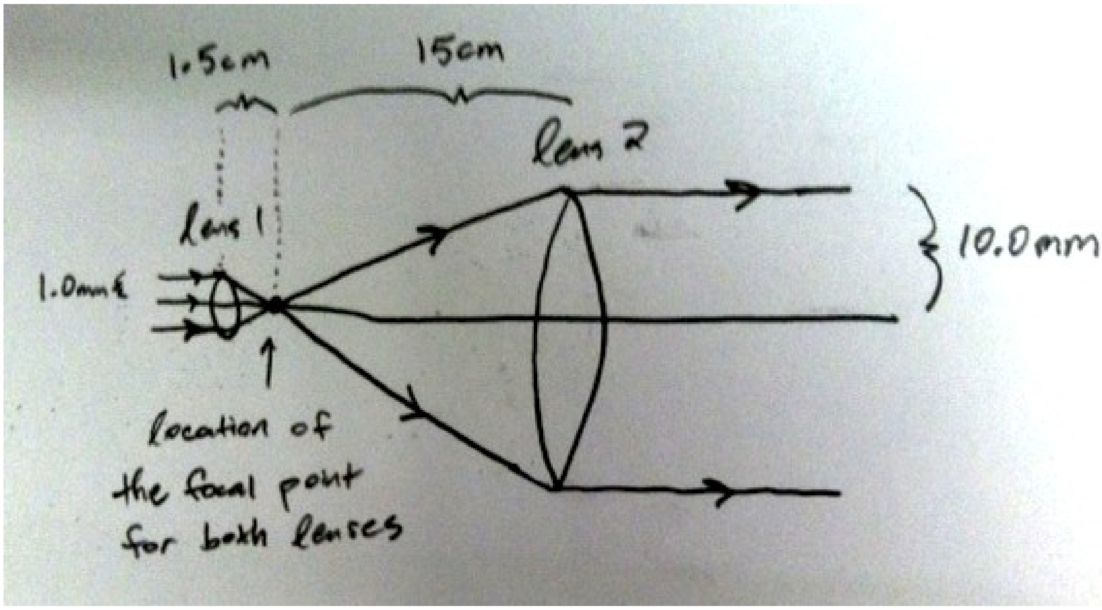
A collimated laser beam (meaning the light is coming out parallel) emerging from a commerial HeNe laser has a diameter of 1.00 mm. In order to convert this beam into a well-collimated beam of diameter 10.0 mm, two convex lenses are to be used. The first lens is of focal length 1.50 cm and is to be mounted at the output of the laser.

a. Draw a (neat) ray diagram to show the beam's width as it passes through the two lenses. Make sure you include the location of the focal points of the lenses.

b. What is the focal length of the second lens?

c. How far from the first lens should the second lens be placed?

Solution
a. See the following picture:



b. The focal length of the second lens needs to be 15 cm. This is evident from the above picture: in the region between both lenses, the beam grows 1.0 mm in width for every 1.5cm from the focal point.

c. The 2nd lens should be placed 16.5 cm from the 1st lens, also evident from the picture.

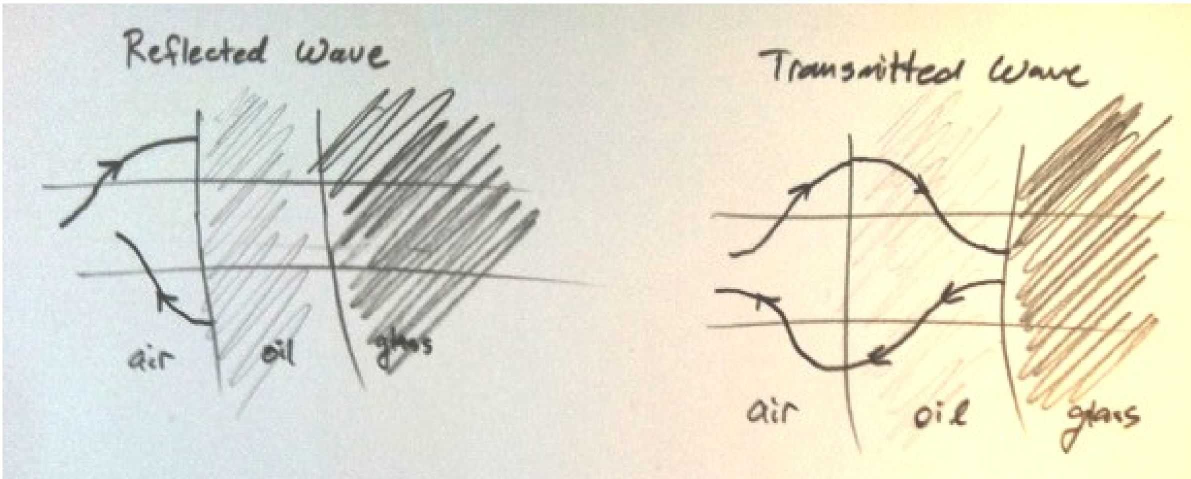
Problem 5 (8 points)

Blue light of wavelength 480 nanometers is most strongly reflected off a thin film of oil on a glass slide when viewed near normal incidence. The index of refraction of the oil is 1.2 and that of the glass is 1.6. What is the minimum thickness of the oil film (other than zero) ?

Solution
The following picture shows what's going on (the picture assumes the incident light has a maximum at the air-oil interface). Incident waves have two options:

1. to bounce off the oil at the air-oil interface, which comes with a phase difference of 180 degrees, or
2. to go though the oil-glass interface, bounce off the oil-glass interface (which also comes with a phase difference of 180 degrees), and then travel back through the air-oil interface.

The picture shows the minimum possible thickness for the two reflected waves to add together with constructive interference. It is evident from the picture the thickness should be half of the wavelength of blue light in the oil. Since $\lambda_n = \lambda / n$ for light in a material with index of refraction n, the wavelength of blue light in oil is (480nm)/1.2 = 400nm, and the thickness of the oil film is therefore 200nm.



Extra Credit (2+2 = 4 points)

Suppose we have light of initial intensity $2I_0$, and three polarizers. The first polarizer is fixed in the +x-direction, the third polarizer is fixed in the +y-direction, and the second polarizer (in between the two) can be rotated to lie at an angle θ with respect the +x-axis toward the +y-axis.

a. What is the intensity of the light passing through the entire system, as a function of θ?

b. Graph the intensity of light as a function of theta, for $0 <= \theta <= 90^0$.

Solution

a. After passing through the first polarizer, the intensity of light is I_0. After passing through the second, the intensity is $I_0*\cos^2(\theta)$. After passing through the third, the intensity is $I_0*\cos^2(\theta)*\cos^2(90^0-\theta)$.

b.