Physics 1C, Summer 2011 (Session 1)

Practice Midterm 1 (50+5 points) Solutions

Problem 1 (5+5 = 10 points)

A mass m at the end of a spring vibrates with a frequency of 0.88 Hz; when an additional 1.25 kg mass is added to m, the frequency is 0.48 Hz.

a. What is the value of m?

b. What is the spring constant k?

Solution

a. Frequency f is proportional to Sqrt[1/m], and so f_1 / f_2 = Sqrt[m_2 / m_1]. Plugging in f_1 = 0.88 Hz, f_2 = 0.48 Hz, and $m_2 = m_1$+1.25 kg, we obtain an equation involving only m_1:

$$0.88 / 0.48 = \text{Sqrt}[(m_1+1.25\text{kg}) / m_1] \rightarrow m_1 = 0.53 \text{ kg}$$

b. $f = [1/(2\pi)]$Sqrt[k/m]. Plugging in m = 0.53 kg and f = 0.88 Hz, k = 16 N/m

Problem 2 (4+4+4 = 12 points)

A 0.650-kg mass vibrates according to the equation x = 0.25 sin (5.50t), where x is in meters and t is in seconds.

a. What is the period of the oscillations?

b. What is the kinetic energy when x is 10cm?

c. What is the potential energy when x is 10cm?

Solution

a. The angular frequency can be read off from the problem statement: ω = 5.50 rad/s. Therefore, the period is T = $(2\pi)/\omega$ = 1.14 seconds.

For parts b and c, it is useful to find a time t where x is 10cm:
x(t) = 0.10m = 0.25 sin (5.50t) → t = 0.0748 seconds

b. v(t = 0.0748 sec) = (0.25)*(5.50)*cos[5.50(0.0748)] = 1.26 m/s, where all quantities are in SI-units. KE = $(1/2)mv^2 = (1/2)(0.650kg)(1.26\ m/s)^2 = 0.516$ Joules

c. PE = $(1/2)kx^2$, where $k = m\omega^2 = (0.650kg)(5.50s^{-1})^2 = 19.66$ N/m.
PE = $(1/2)(19.66N/m)(0.10m)^2 = 0.098$ Joules

Problem 3 (5+5 = 10 points)

Suppose you produce two sound waves from the same source: the first wave has a period of 0.500 ms, and the second wave has a period of 0.520 ms. When you stand 1 meter away from the source, the intensity of each wave by itself is 50 dB.

a. What is the beat frequency caused by the two waves?

b. Assuming each source emits sound isotropically (in all directions equally), what is the intensity of each wave in dB if you stand 2 meters away from the source?

Solution
a. The frequency of the first wave is $1/(0.50 \times 10^{-3}$ sec) = 2000 Hz, and the frequency of the second wave is $1/(0.52 \times 10^{-3}$ sec) = 1920 Hz. The beat frequency is the difference, which is equal to 2000 Hz – 1920 Hz = 80 Hz.

b. The intensity of a wave is proportional to $1/r^2$, so the intensity of each wave drops by a factor of 4. The initial intensity I_i is given by

$$\beta = 50\text{ dB} = 10*\log_{10}[I_i / I_0] = 10*\log_{10}[I_i / (10^{-12}\text{ W/m}^2)] \rightarrow I_i = 10^{-7}\text{ W/m}^2$$

The final intensity is 4 times lower: $I_f = 0.25 \times 10^{-7}$ W/m^2

This translates to $\beta = 10*\log_{10}[I_f / I_0] = 44.0$ dB

Problem 4 (4+4 = 8 points)

Echolocation is a form of sensory perception used by animals such as bats, toothed whales and porpoises. The animal emits a pulse of sound (a longitudinal wave) which is reflected from objects; the reflected pulse is detected by the animal. Echolocation waves emitted by whales have frequencies of about 200,000 Hz.

a. What is the wavelength of the whale's echolocation wave, given that the bulk modulus of water is 2.0×10^9 N/m^2?

b. If an obstacle is 100 m from the whale, how long after the whale emits a wave will the reflected wave return to him?

Solution
a. v = Sqrt[B/ρ] = Sqrt[($2x10^9$ N/m^2) / (1000 kg/m^3)] = 1.41 x 10^3 m/s.
Wavelength $\lambda = v/f$ = (1.41 x 10^3 m/s) / (2.0 x 10^5 Hz) = 7.1 mm.

b. time = distance / speed = 2(100m) / (1.40 x 10^3 m/s) = 0.14 seconds.

Problem 5 (5+5 = 10 points)

A bat flies toward a wall at a speed of 5.0 m/s. As it flies, the bat emits an ultrasonic sound wave with frequency 30.0 kHz. The speed of sound in air is 343 m/s.

a. What is the frequency of the sound wave as received by the wall?

b. What frequency does the bat hear in the reflected wave?

Solution
a. Let v be the speed of sound in air (343 m/s), v_{source} the speed of the bat, and f_0 the frequency of the sound wave as given off by the bat:

$$f_{wall} = [(v) / (v-v_{source})] f_0 = [(343)/(343-5)]*30\text{kHz} = 30.44 \text{ kHz}$$

b. Now, the bat is the observer, and the source can be taken as the wall giving off the sound wave at a frequency of 30.44 kHz:

$$f_{obs} = [(v+v_{obs}) / (v)] f_{wall} = [(343+5)/(343)]*30.44\text{kHz} = 30.9 \text{ kHz}$$

Extra Credit (3+2 = 5 points)

A damped harmonic oscillator loses 5.0 percent of its mechanical energy per cycle.

a. By what percentage does its frequency differ from the natural frequency, ω_0 = Sqrt[k/m]?

b. After how many periods will the amplitude have decreased to 1/*e* of its original value?

Solution
a. If an oscillator loses 5% of its energy, it has lost ~2.5% of its amplitude.

$0.975 = \exp[-\gamma(\textit{time for 1 cycle})] \sim \exp[-\gamma(2\pi/\omega_0)] \rightarrow \gamma \sim [0.025/(2\pi)]\omega_0 \sim 0.004\omega_0$. The actual frequency is $\omega = \mathrm{Sqrt}[\omega_0^2 - \gamma^2] \sim \omega_0[1\text{-}0.5*(\gamma^2/\omega_0^2)] \sim 0.999992\omega_0$. Therefore, the frequency differs from the natural frequency by 0.0008%

b. We want the number of periods for the energy to decrease to $(1/e)^2 = 0.1353$ of its original value. Since the oscillator loses 5% of its energy each cycle, the total number of oscillations for this to occur (N) is given by $(0.95)^N = 0.1353$. Solving this for N we obtain N = 39 cycles.