Physics 1C, Summer 2011 (Session 1)
Midterm 1 (50+5 points) Solutions

Problem 1 (4+4+6 = 14 points)

A particle is undergoing simple harmonic motion. Its velocity is given by the equation $v(t) = (230\text{ m/s})*\sin[(1000\text{s}^{-1})*t]$. Give the answers to the following questions in SI units:

a. What is the amplitude of the particle's motion?

b. What is the period of the oscillatory motion?

c. What is the first time t after t = 0 seconds that the particle will reach its equilibrium position?

Solution
a. The maximum speed is 230 m/s = ωA, where A is the amplitude and $\omega = (1000\text{s}^{-1})$ can be read off from the problem statement. This gives A = 0.23 meters.

b. The period $T = (2\pi)/\omega = 6.3$ ms.

c. This is the value of t for which $(1000\text{s}^{-1})*t = (\pi/2)$ radians: t = 1.57 ms.

Problem 2 (4+4 = 8 points)

Suppose you would like to create a pendulum with a 10 kg bob such that the period is exactly one second.

a. What length of string should you use to create your pendulum?

b. What length of string should you use if instead you use a 20 kg bob?

Solution
a. The period of a pendulum is (2π)Sqrt[L/g]. Setting this equal to 1sec, plugging in g = 9.8 m/s^2, and solving for L, we obtain L = 0.248 meters.

b. The length of string does not depend on the mass of the bob hanging on the pendulum, so you should still use a 0.248-meter piece of string for a 20 kg bob.

Problem 3 (4+4+4 = 12 points)

Transverse waves with a speed 50.0 m/s are to be produced in a string. The string is hanging from the ceiling with length 2 meters and total mass 5 grams.

a. What mass should be attached to the end of the string to achieve the required tension?

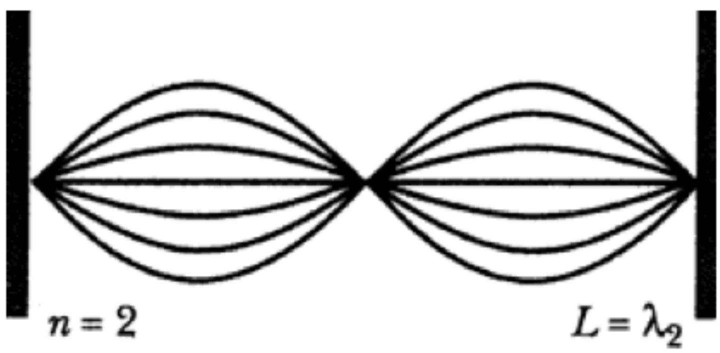
b. Draw a graph (displacement vs. position along the string) of the 2nd harmonic.

c. What is the wavelength of the 2nd harmonic?

Solution

a. The tension T required in the string satisfies v = 50.0 m/s = Sqrt[T/μ]. Mu (μ) is the mass per unit length of the string, or (0.005kg)/(2meters) = 0.0025 kg/m. Solving for T we obtain T = 6.25 Newtons. This is achieved by hanging a mass of 0.638 kg = 638 grams from the string.

b. The horizontal axis should read position along string:



c. As seen in the above graph, the wavelength is given by the length of the string: 2m.

Problem 4 (4+4 = 8 points)

A family ice show is held in an enclosed arena. The skaters perform to music with a sound level of 80.0 dB. This is too loud for your baby, who consequently yells at a level of 75.0 dB.

a. What total sound intensity engulfs you, assuming the intensities of the two waves add together with no interference?

b. What is the combined sound level in dB?

Solution

a. If β is the sound level of a sound wave, in decibels, then the intensity of the wave is given by $I = I_0 * 10^{\beta/10}$, where I_0 is 1.00×10^{-12} W/m^2. Thus, the loudspeaker has an intensity 10^{-4} W/m^2, and your baby has an intensity of 3.16×10^{-5} W/m^2. The two combine (add) to a total intensity of 1.32×10^{-4} W/m^2.

b. $\beta = 10 * \log_{10}[I/I_0] = 10 * \log_{10}[1.32 \times 10^{-4} / 10^{-12}] = 81.2$ dB.

Problem 5 (4+4 = 8 points)

a. Suppose a car is driving down the street at 20 m/s relative to the road. The driver of the car sees an ambulance approaching the car from behind, going 40 m/s relative to the road. The ambulance has a siren, producing sound at a frequency of 440 Hz. What is the frequency of the siren as heard by the driver of the car?

b. After the ambulance passes the car, what is the frequency of the siren as heard by the driver of the car?

Solution

a. Let v be the speed of sound in air (340 m/s), v_{obs} the speed of the car, v_{source} the speed of the ambulance, and f_0 the frequency of the siren as given off by the ambulance:

$$f_{obs} = [(v - v_{obs}) / (v - v_{source})] f_0 = [(340-20)/(340-40)] * 440\text{Hz} = 469 \text{ Hz}$$

b. Now, $f_{obs} = [(v + v_{obs}) / (v + v_{source})] f_0 = [(340+20)/(340+40)] * 440\text{Hz} = 417$ Hz

Extra Credit (5 points)

A uniform cord of length L and mass m is hung vertically from a support. What is the speed of transverse waves in this cord as a function of h, the height above the lower end?

Solution

Look at a small piece of the cord at a distance h above the lower end. This piece has a tension T pulling up on it from above, and a weight of cord μgh below it pulling down on it. The two forces cancel in equilibrium, and so $T = \mu gh$. Plugging this into $v = \text{Sqrt}[T/\mu]$, we obtain $v = \text{Sqrt}[gh]$.