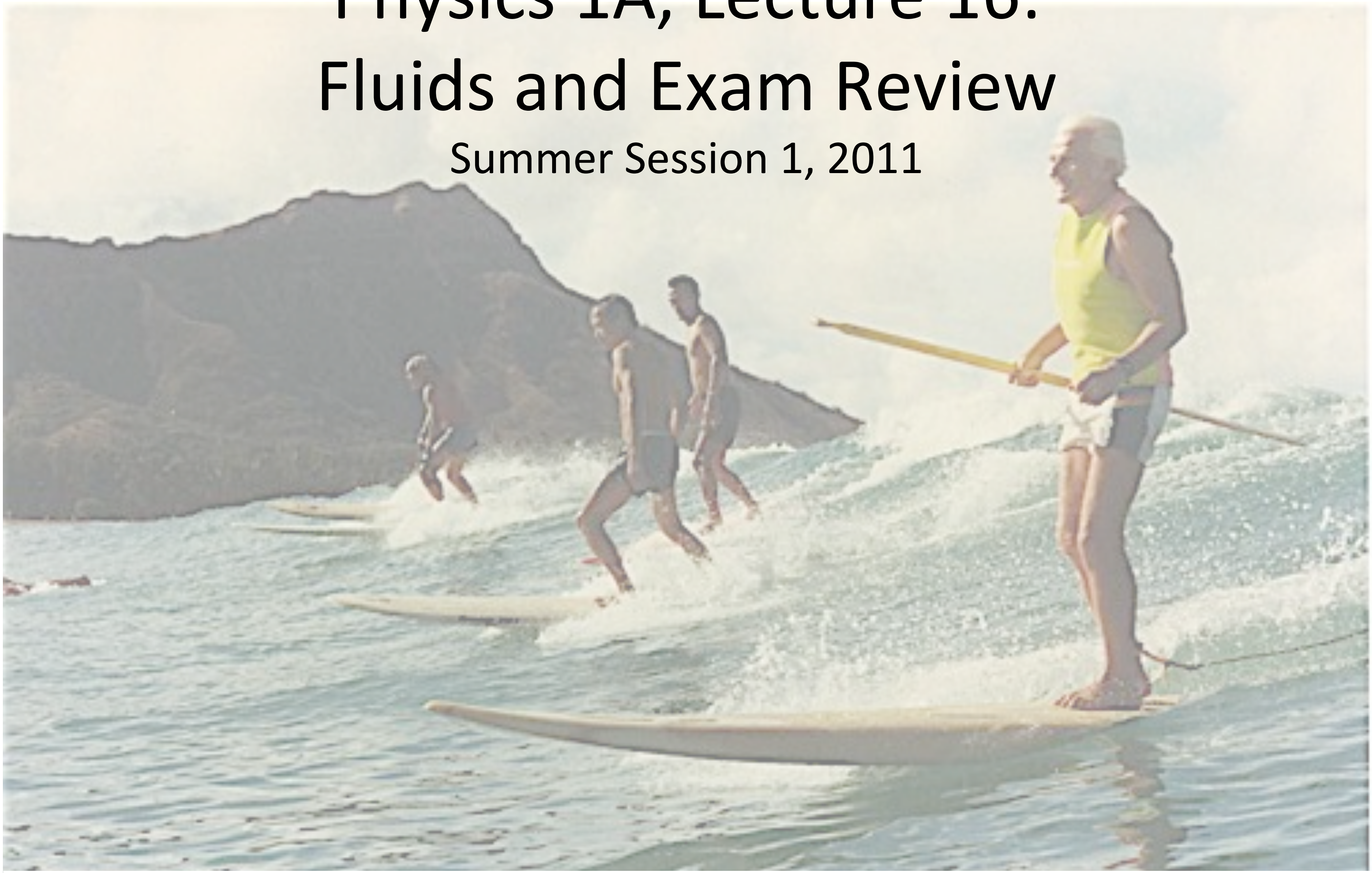
Physics 1A, Lecture 16: Fluids and Exam Review

Summer Session 1, 2011



No Reading Quiz today

Ladder problem

- Find the minimum angle that the ladder can make with the floor so that the ladder doesn't slip, given *m*, *L*, and μ_s between the ladder and the ground.

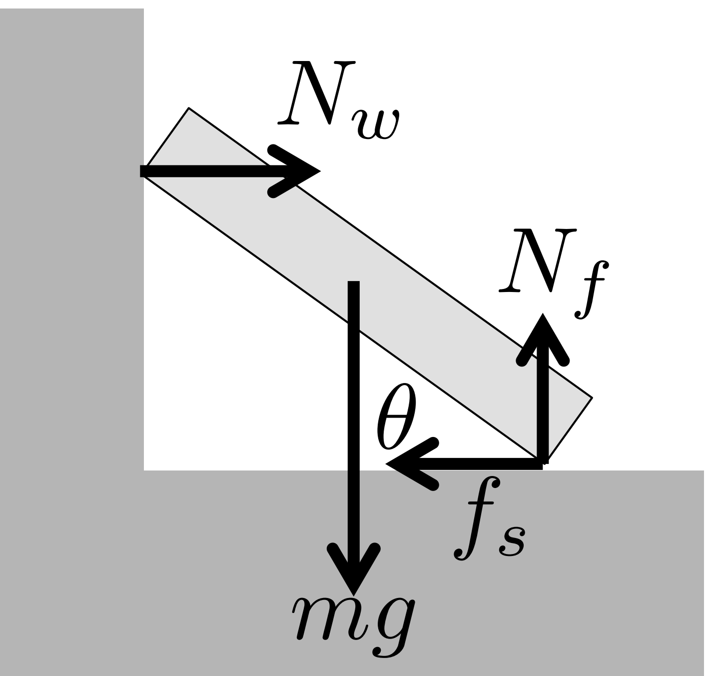
$$\sum F_x = N_w - f_s = 0$$

$$\sum F_y = N_f - mg = 0$$

$$\rightarrow N_w = \mu_s mg$$

$$\sum \tau = +mg\left(\frac{L}{2}\right)\cos\theta - \mu_s mgL\sin\theta = 0$$

$$\rightarrow \theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$$

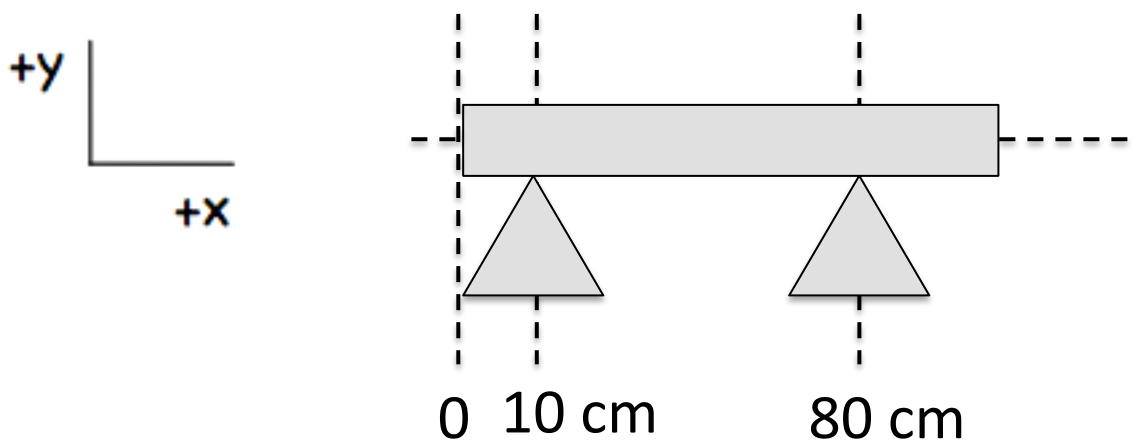


Rotational equilibrium problems

- Draw a picture with a coordinate system
- Add forces, being careful about where they are acting on the object
- Set net force equal to zero in x and y
- Pick a pivot point
- Set net torque equal to zero
- Solve algebra

Example

A 0.1 kg meter stick is held in place at the 10 cm mark and the 80 cm mark by two fulcrums. Find the upward force exerted by the fulcrums.



Clicker question

Given everything we've learned about the analogy between translational and rotational motion, what do you think is the correct equation for Rotational Work?

A) $W = \frac{1}{2} m (\Delta\theta)^2$

B) $W = \tau \Delta\theta$

C) $W = \tau \omega$

D) $W = \frac{1}{2} \tau \omega^2$

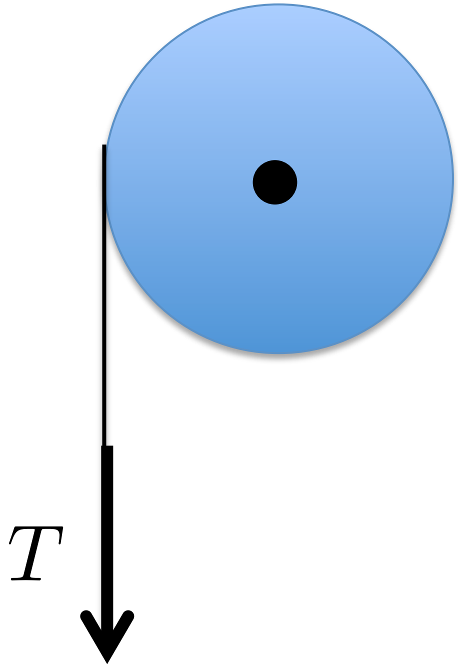
E) $W = I \alpha$

Work in rotational motion

- Pulling on a massive pulley.

$$I_{\text{cylinder}} = \frac{1}{2}MR^2$$

$$W = \tau \Delta\theta$$



Angular momentum

- Rotational analogy with the impulse equation:

$$\Delta L = \tau \Delta t$$

- Angular momentum

$$L = \int \tau dt = \int I \alpha dt \rightarrow \qquad L = I\omega$$

- If no external torques on a rotational "collision":

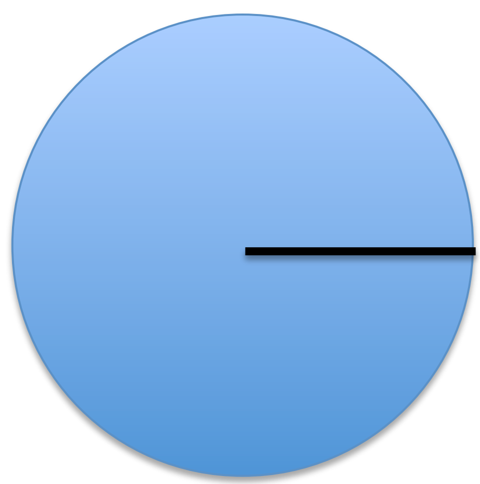
$$\Delta L = 0$$

Angular momentum conservation

$$L = I\omega$$

$$\Delta L = 0$$

If angular momentum is conserved and I changes → angular velocity must change!



Example

A merry-go-round (m = 100kg, r = 2.00m) spins with an angular velocity of 2.50(rad/s). A monkey (m = 25.0kg) hanging from a nearby tree, drops straight down onto the merry-go-round at a point 0.500m from the edge. What is the new angular velocity of the merry-go-round?

Matter

There are four basic states of matter:

- Solids - definite volume and definite shape.
- Liquids - definite volume but no definite shape.
- Gases - no definite volume and no definite shape.
- Plasmas - high temperature matter with electrons freed from their respective nuclei.

You can deform all of the above states of matter.

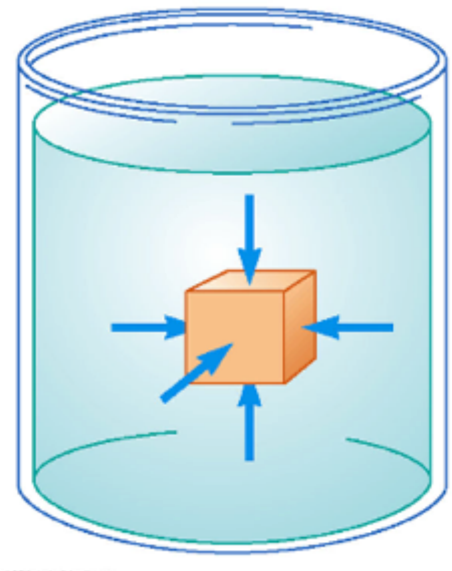
Solids are the hardest to deform, but it is possible through the application of external forces.

Fluids

- A fluid is a substance that can conform to the boundaries of any container.
- Liquids and gases fall into this category.
- Two important quantities for dealing with fluids are density and pressure.
- Density, ρ, is given by: $\rho = M/V$
- Density is measured in units of: kg/m^3.
- The specific gravity of a substance is the ratio of its density to the density of water at 4^oC.
- The density of water at 4^oC is $1000 kg/m^3$.

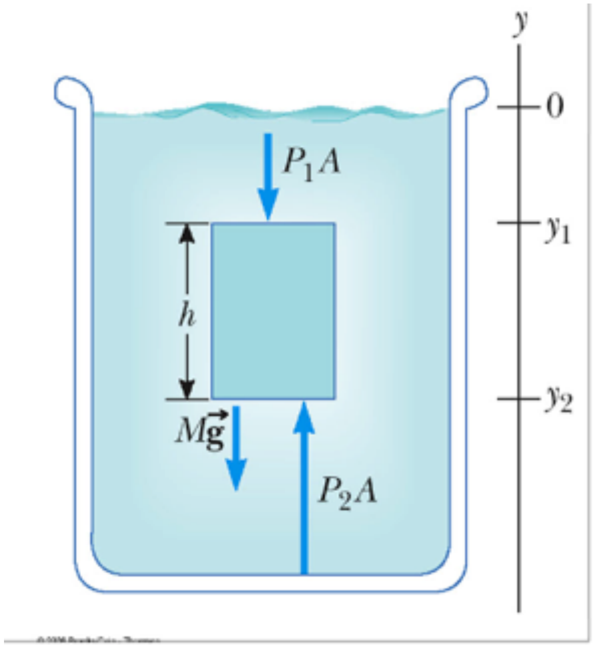
Fluids

- Pressure, P, is given by:
- Pressure is measured in SI units of: N/ m^2 = Pascal = Pa.
- There are plenty of units for pressure:
- atm, torr, psi, mm of Hg, kPa, bar...
- Because of this be aware of the basic conversions:
- 1 atm = 1.01×10^5Pa = 760torr = 14.7psi.
- Pressure is a scalar quantity, it doesn't have a direction.



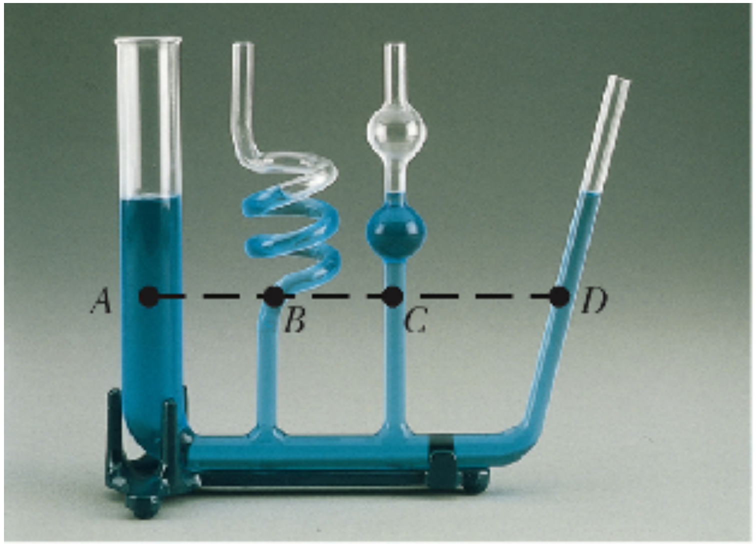
Fluids

- When discussing pressure in a fluid, it is important to measure with respect to a reference point.
- So, in reality you are measuring a change in pressure or ΔP.
- As you go deeper into the fluid, your pressure will increase (there is more mass on top of you pushing down on you).
- If the fluid is static (i.e. at rest) then we can say that:
- $\Delta P = P2 - P1 = \rho g(y2 - y1)$
- For a height h, $P2 = P1 + \rho gh$ $P2 > P1$



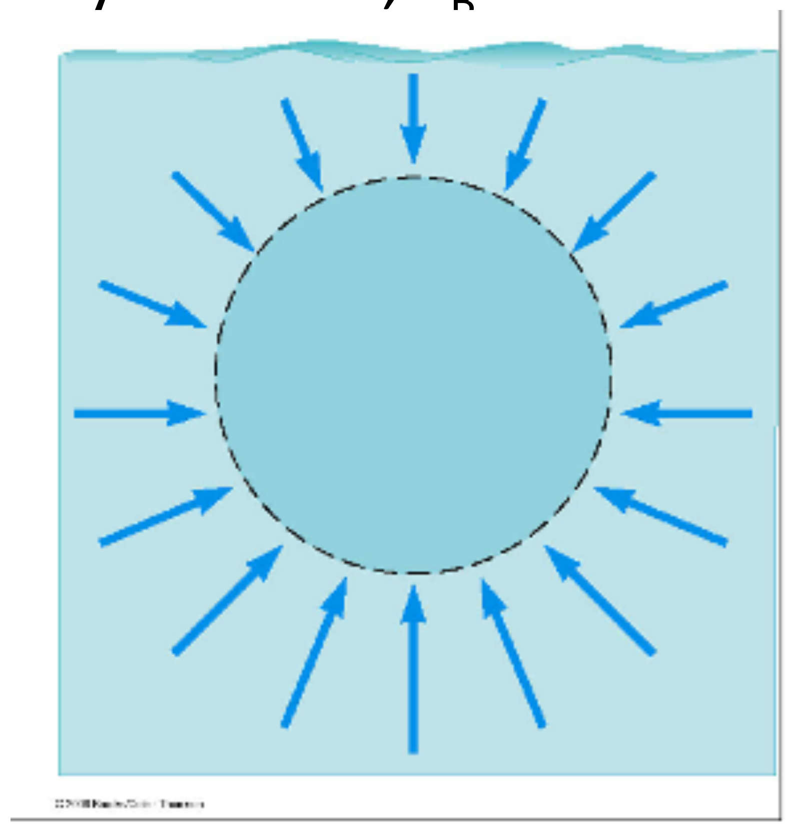
Fluids

- The pressure at any point in a fluid which is in static equilibrium depends on the depth of that point.
- The pressure does not depend upon the shape of the container.
- There are two types of pressure: Gauge and Absolute.
- Absolute Pressure is the total pressure at a location.
- Gauge Pressure is the measured pressure compared to a reference point (this reference is nearly always atmospheric pressure).



Fluids

- When an object is submerged in a fluid (even if only partially) it will be given an upward force by the fluid.
- This upward force is called the buoyant force, F_B or B.
- The physical cause of the buoyant force is the pressure difference between the top and the bottom of the object.
- Archimedes determined that this force depends upon the amount of fluid that the object has displaced.



Fluids

- Equationally this becomes: $F_B = \rho_{fluid} g V_{immersed} = \rho_f g V_i$
- where ρf is the density of the displaced fluid and Vi is the volume of the object that is immersed in the fluid.
- If the entire object is immersed in the fluid then you just use the volume of the object for V_i.
- But if the object is only partially immersed just use the volume that is immersed as V_i.
- Example: A ball is floating in a pool of water. 25% of its volume is immersed in the water. What is the density of the ball?

Clicker question

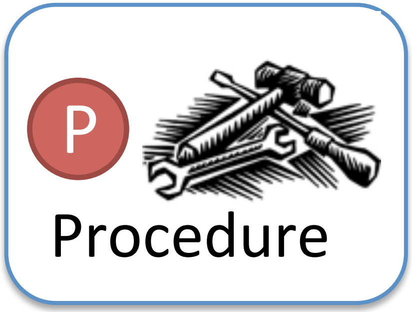
You are floating on a boat which is hauling iron on a small lake. You get disgruntled and decide to unload your iron into the water. What would happen to the overall water level of the lake (as measured from someone on the shore)?

A) The water level of the lake lowers slightly compared to when the iron was on the boat.

B) The water level of the lake rises slightly compared to when the iron was on the boat.

C) The water level of the lake will remain exactly the same as when the iron was on the boat.

Trig Review: Decomposing vectors

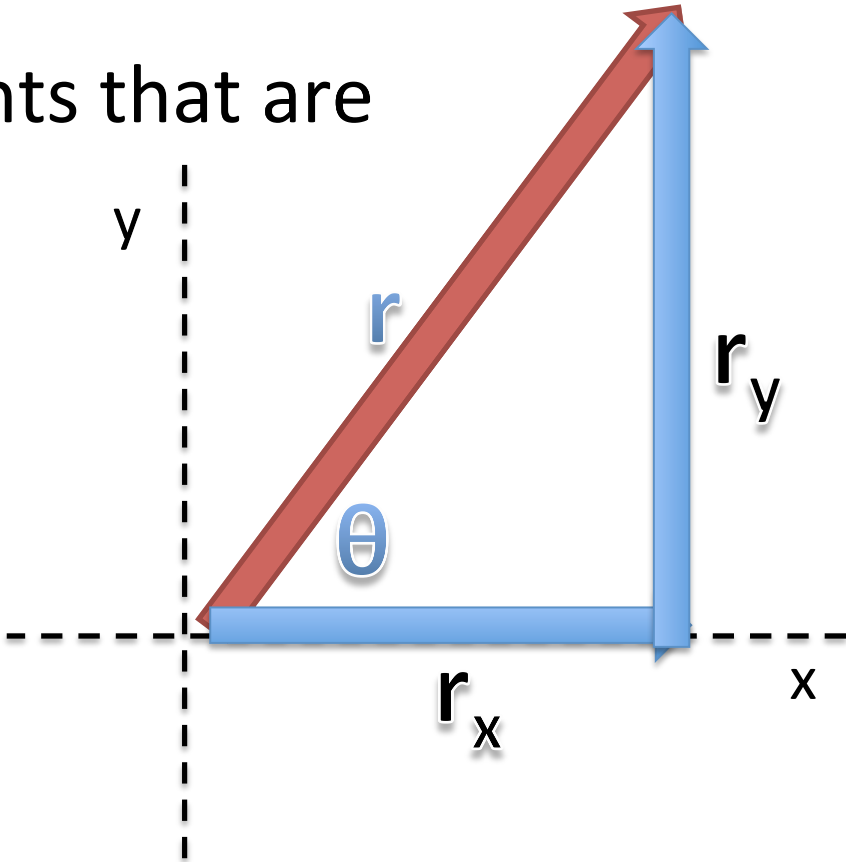


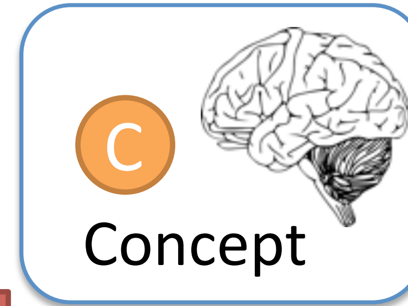
- Place vector r in a coordinate system of your choosing:
- Break up r into components that are

parallel to the axes:

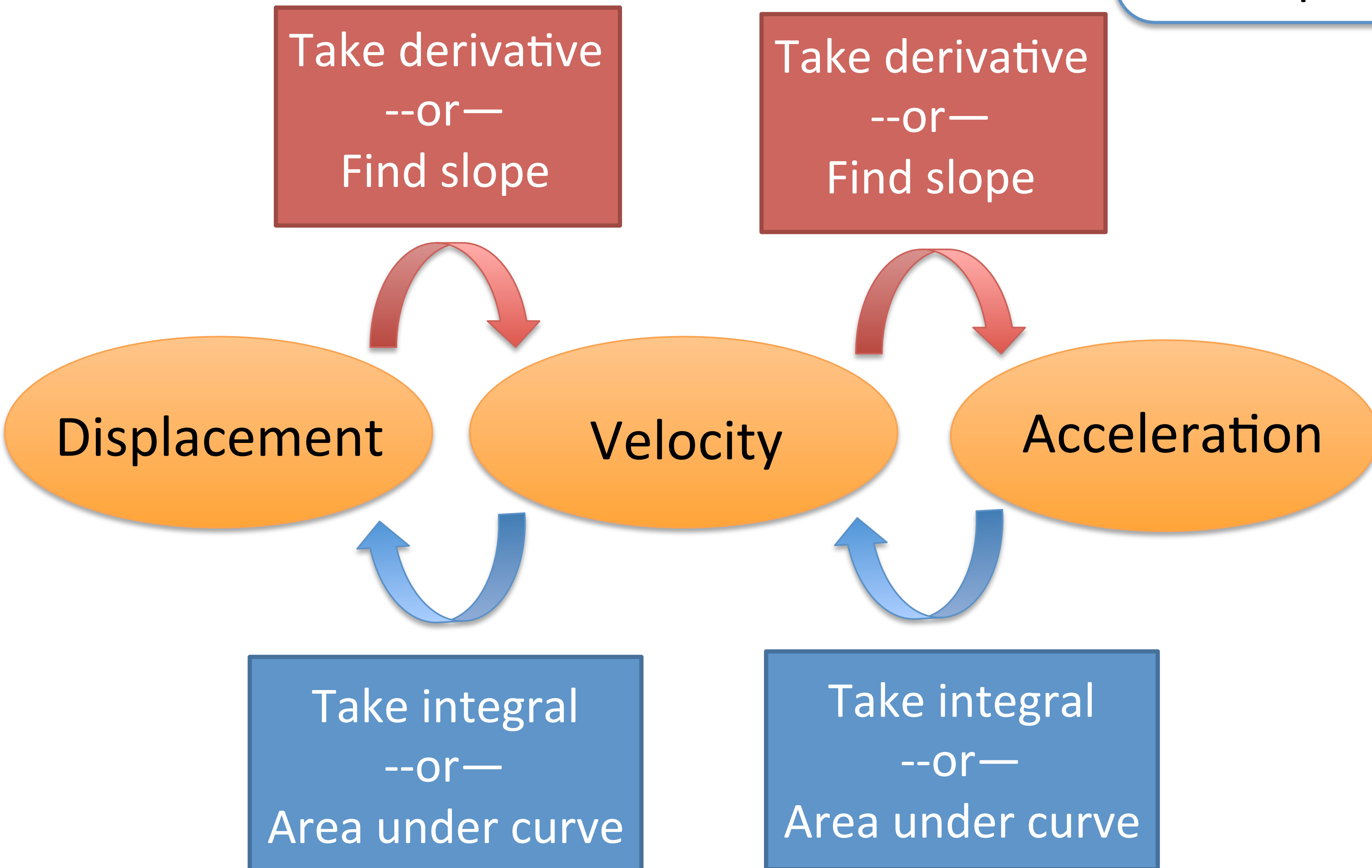
$$r_x = |r| \cdot \cos\theta$$

$$r_y = |r| \cdot \sin\theta$$

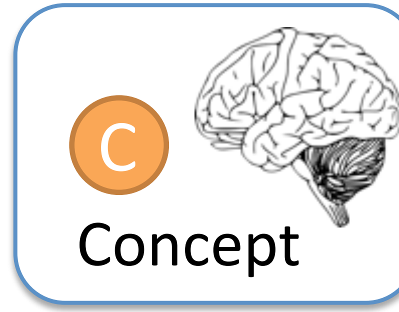




Major concept

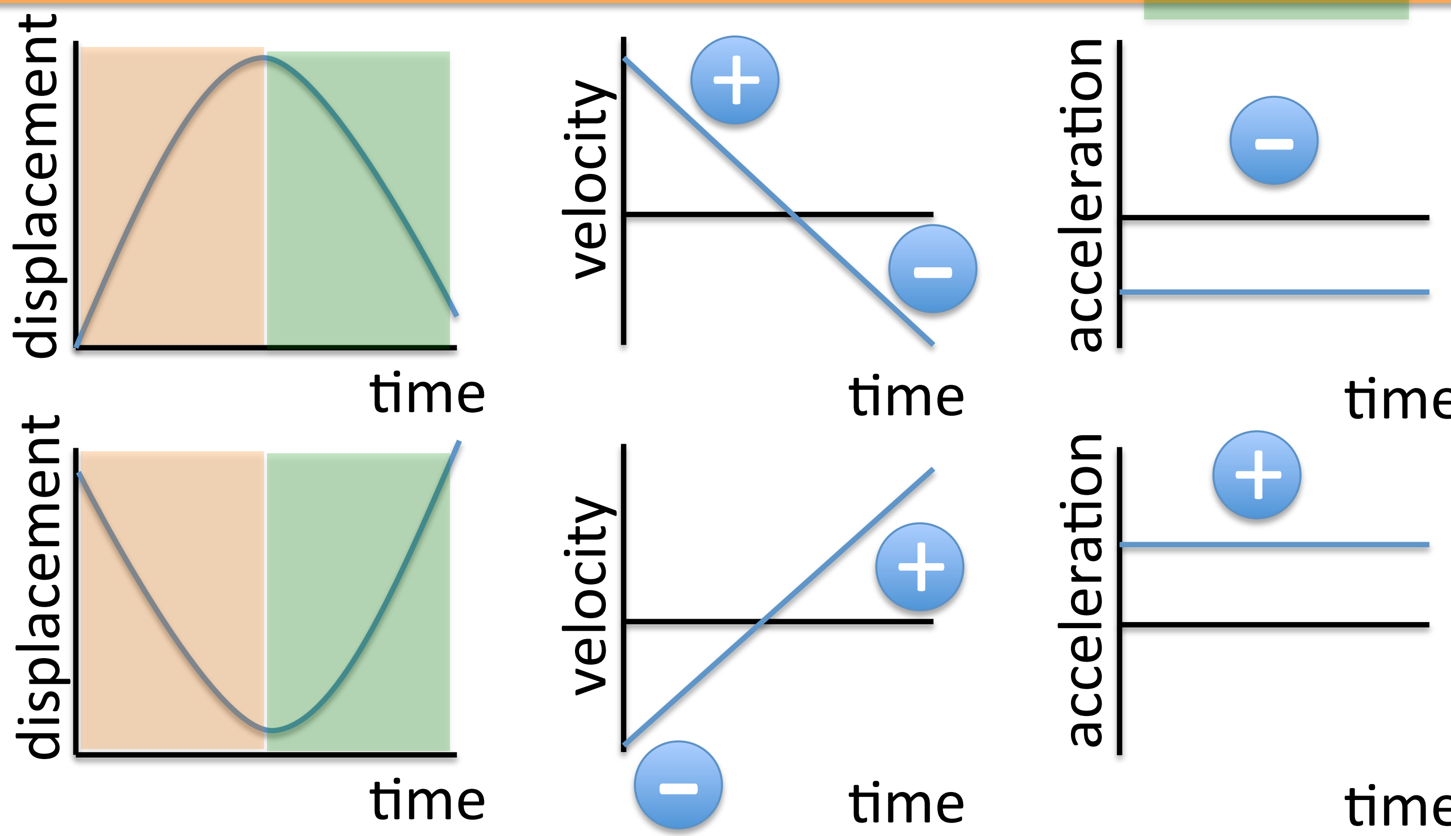


Graphical understanding of derivatives

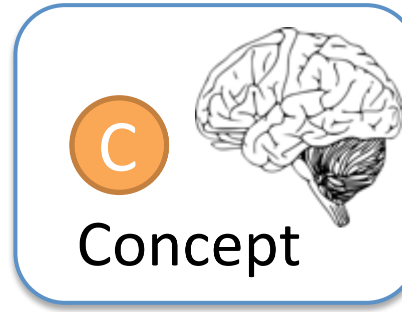


When acceleration and velocity are opposite signs → slow down

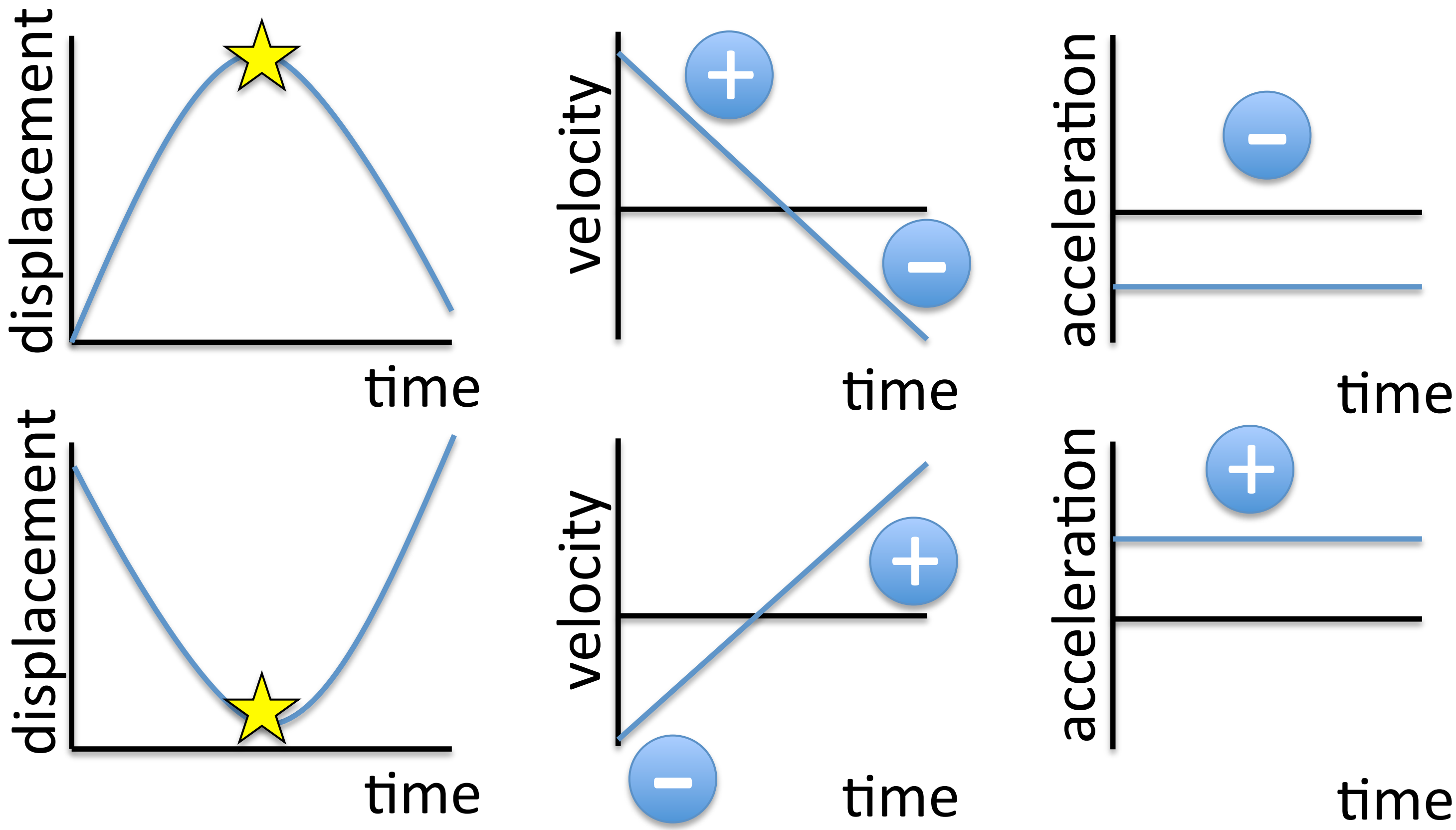
When acceleration and velocity are same sign → speed up



Graphical understanding of derivatives



- Velocity is zero but acceleration is nonzero!



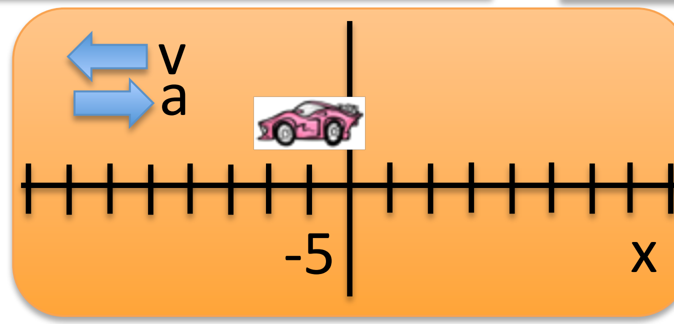
Procedure

How to solve physics problems

1) Collect all the knowns and unknowns

x_0 v_0 a t | x_f

2) Draw a picture with a coordinate system



3) Find the appropriate equations
   - Make sure you have enough equations!

$$x_f = x_0 + v_0 t + \frac{1}{2}at^2$$

4) Plug in the known parameters and solve

$$x_f = (-5m) + (-10\frac{m}{s})(2s) + \frac{1}{2}(1\frac{m}{s^2})(2s)^2 = -23m$$

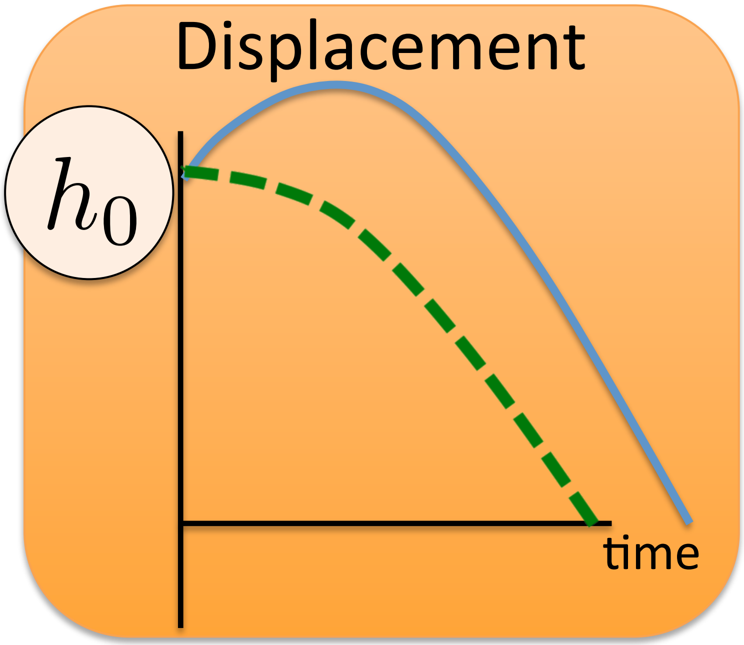
How to choose which kinematics equation to use?

Procedure

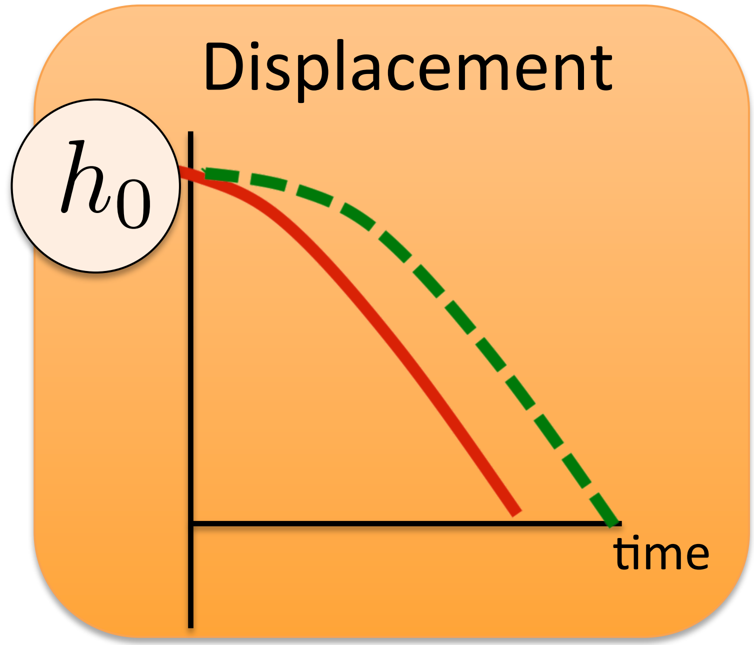
	Actors:	Missing:
$x_f = x_0 + v_0 t + \frac{1}{2} a t^2$	x_f x_0 v_0 t a	v_f
$v_f = v_0 + a t$	v_0 v_f t a	x_f x_0
$x_f = x_0 + \frac{1}{2}(v_f + v_0) t$	x_f x_0 v_0 v_f t	a
$v_f^2 = v_0^2 + 2a(x_f - x_0)$	v_f v_0 a x_f x_0	t

Air time depends on v_{0y}

Throwing up



Throwing down

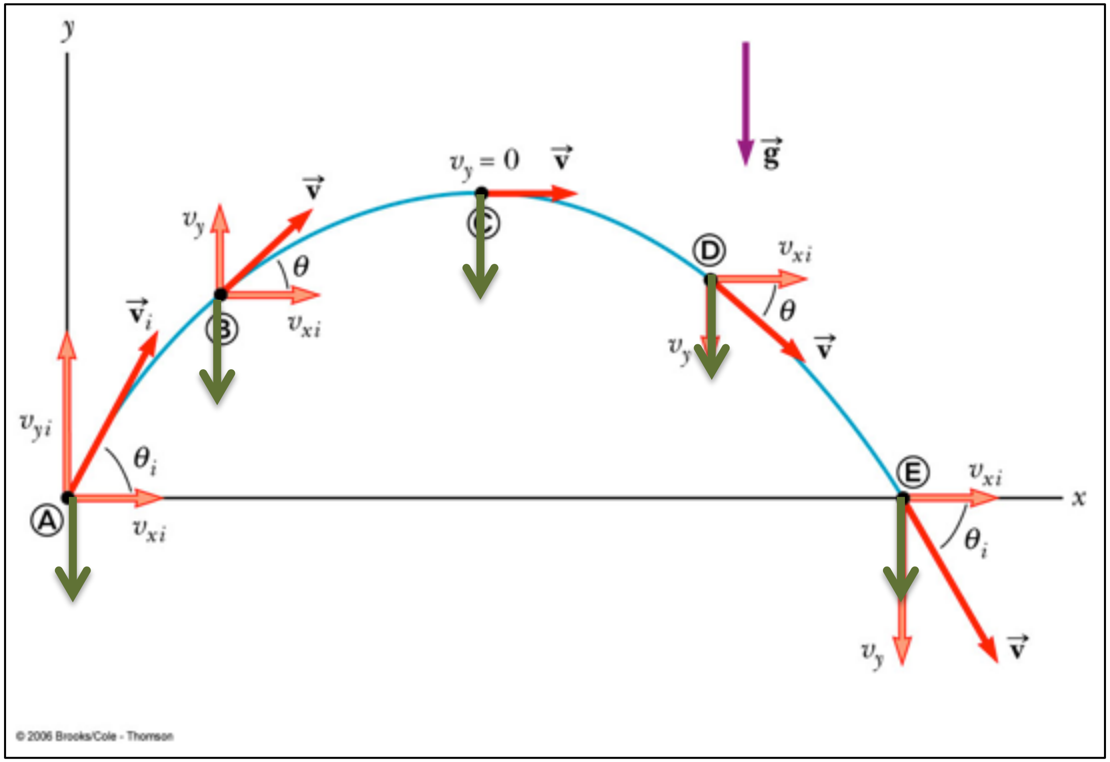


Dropping from rest

Projectile velocity and acceleration

- Constant v_x
- v_y decreases to zero then decreases to negative v_{0y}
- Constant a_y

$$a_y = -g$$



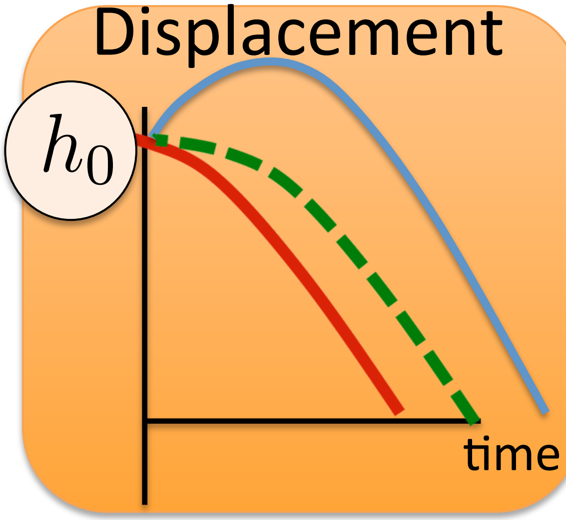
What determines air time and range?

C

Concept

- Air time is determined by h_0 and v_{0y}

$$h = h_0 + v_{0y}t + \frac{1}{2}a_y t^2$$



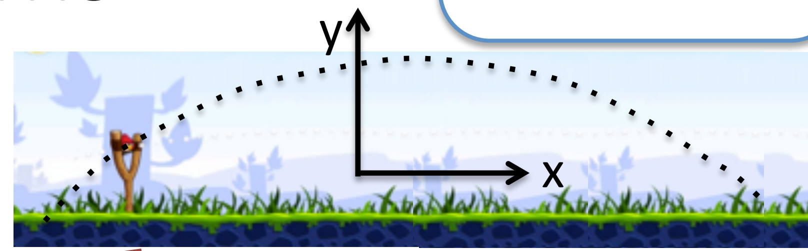
- Range is determined air time and v_{0x}

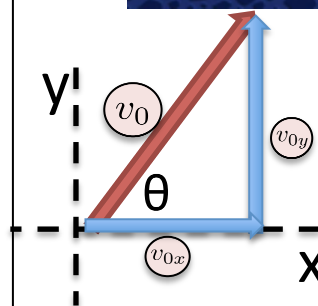
$$\Delta x = v_{0x}t$$

How to solve projectile problems

P 
Procedure

1) Draw a picture with a coordinate system
2) Collect all the knowns and unknowns, break up velocity into *x* and *y* components
3) Find the appropriate equations
4) Use *t* to connect between equations in *x* and *y*. Solve.





h_0 h v_{0y} a_y

x_0 v_x

x_f

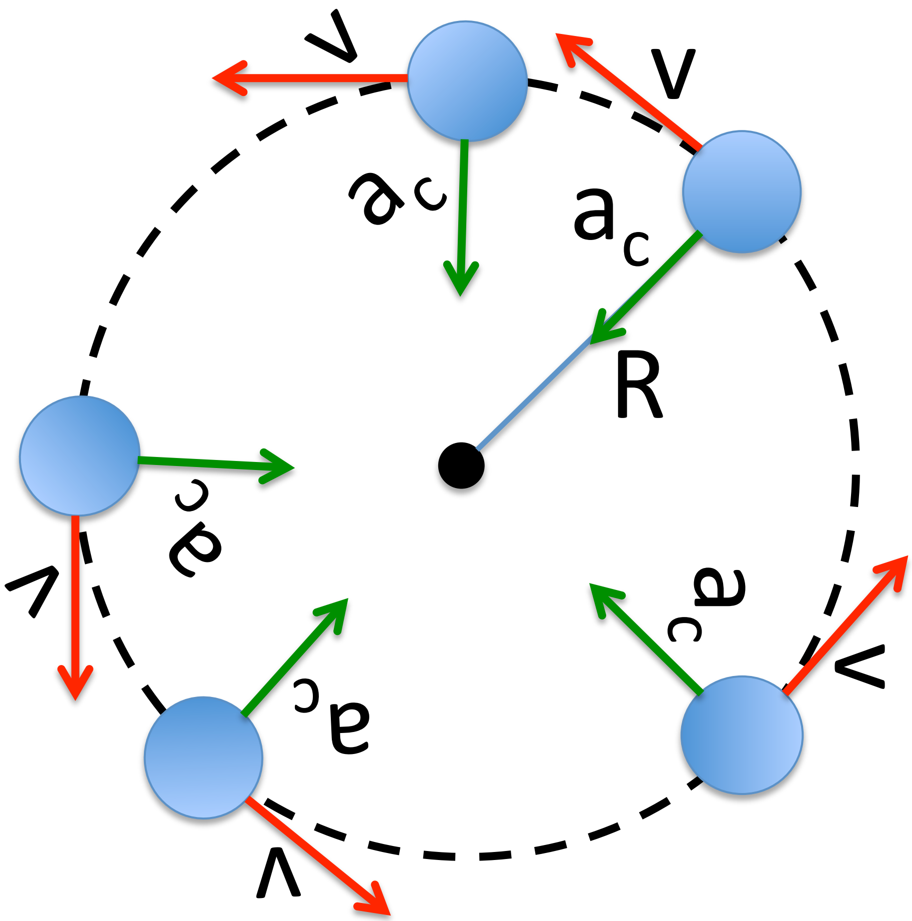
$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

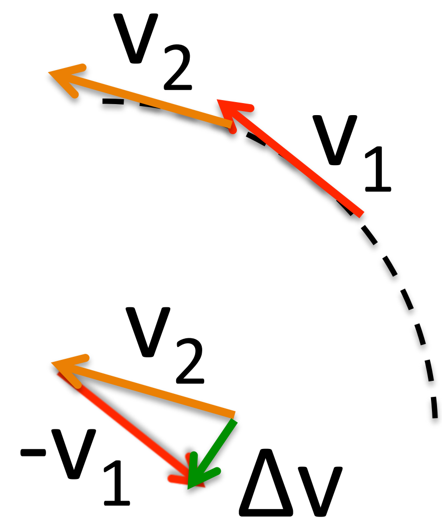
$$h = h_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$\Delta x = v_{0x}t$$

$$t = \frac{2v_{0y}}{g} \rightarrow \Delta x = \frac{2v_{0y}v_{0x}}{g}$$
$$= 866 \text{ m}$$

Uniform circular motion





$$a_c = \frac{v^2}{r}$$

$$T = \frac{2\pi r}{v}$$

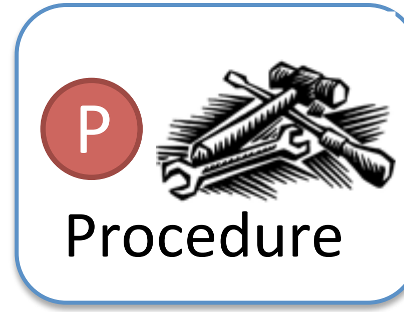
Force → acceleration

C

Concept

- Huge topic for next section:
  - Only way to have acceleration of an object is when there is a force acting on it
- Three things that cause acceleration:
  - Gravity (always constant in magnitude and direction!)
  - Gas or brake pedal (though cars can also coast)
  - Object in contact with that object applying a force
- Things that *do not* cause acceleration:
  - Throwing stuff (can only cause velocity)

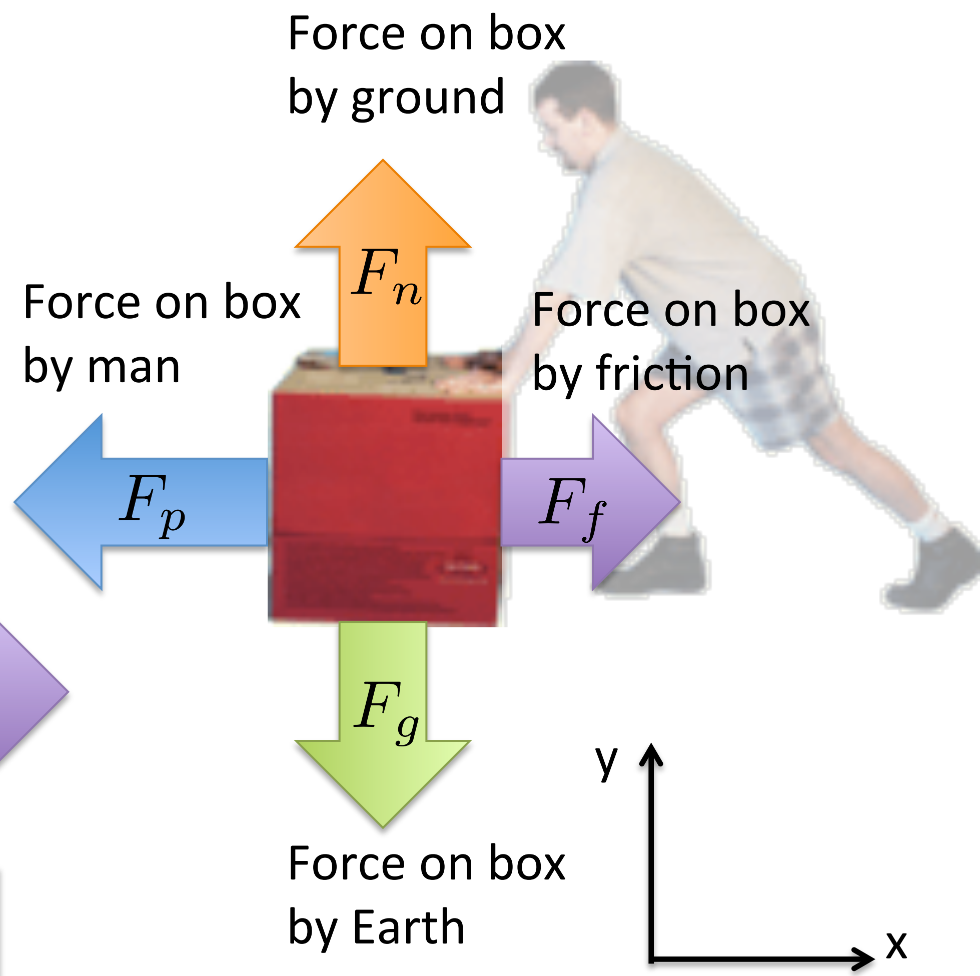
Free body diagrams and Net Force, ΣF



- To find the net force on an object:
  - Draw a picture with a coordinate system
  - Draw arrows going away from the object to represent the forces
  - Deal with each axis separately:

$$\sum \vec{F}_x = F_p + F_f$$

$$\sum \vec{F}_y = F_n + F_g$$



Newton's First Law

An object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force.

-Which means-

An object does not need need to be under the influence of a force to be in motion!

Newton's Second Law

A net force (ΣF) on an object of mass (m) results in an acceleration (a) according to the following formula:

$$\Sigma F = ma$$

Newton's Third Law

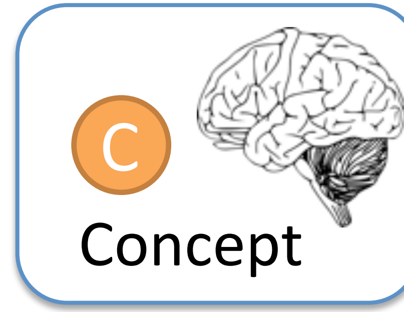
If two object interact, the force exerted by object 1 on 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on 1 :

$$\mathcal{F}_{12} = -\mathcal{F}_{21}$$

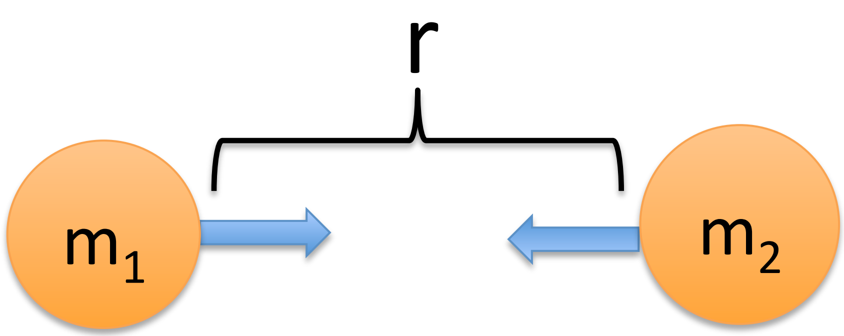
-In other words-

For every action there is an equal and opposite reaction.

Newton's Third Law



- Field forces obey Newton's Third Law:



Force on m_1 by m_2:

Force on m_2 by m_1:

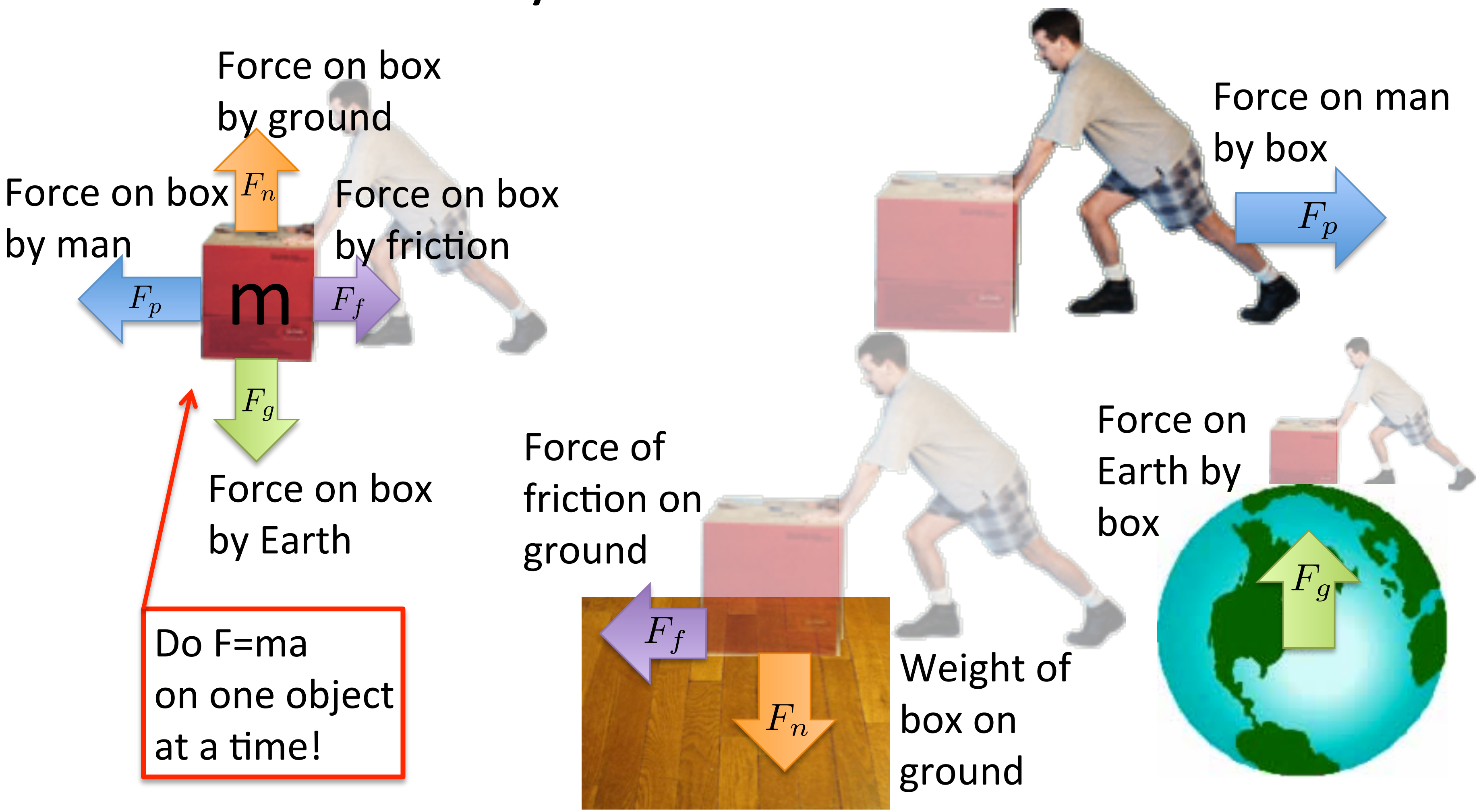
$$\left|\vec{F}_{12}\right| = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\,\frac{\mathrm{N\ m^2}}{\mathrm{kg^2}}$$

The gravitational constant

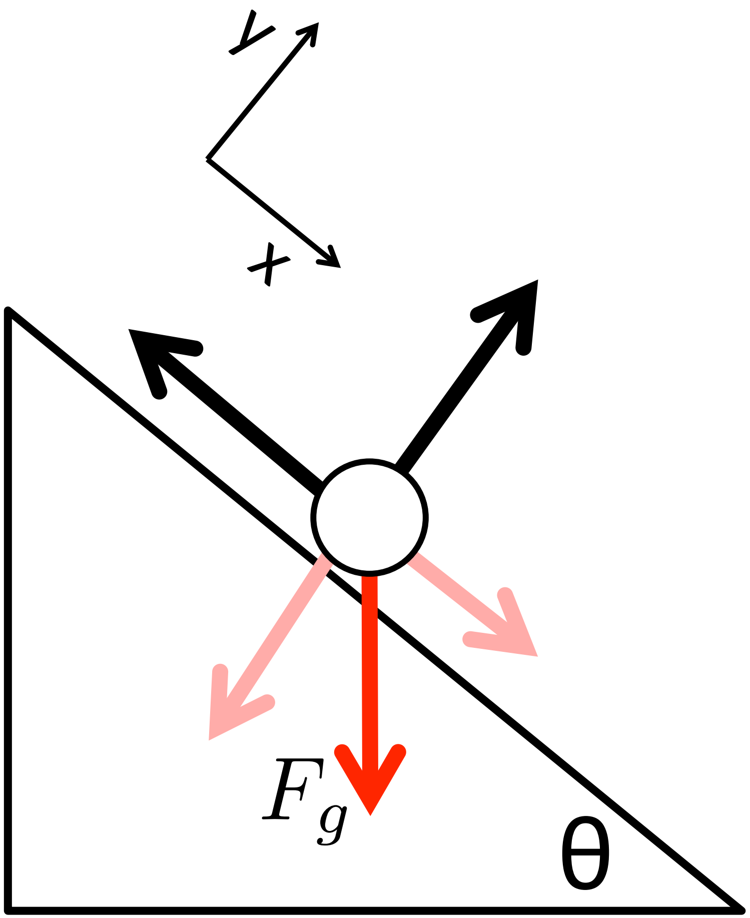
Newton's Third Law

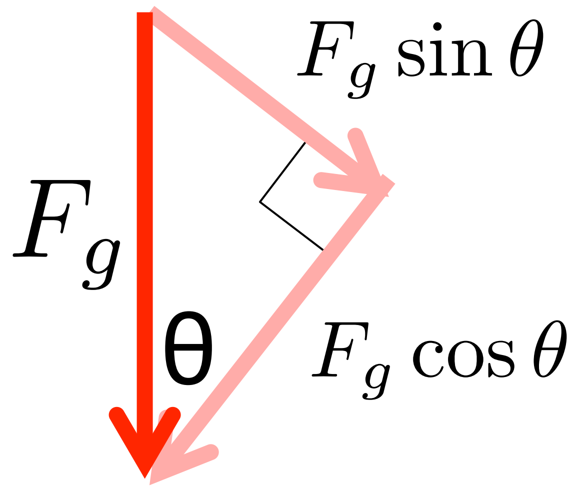
- All forces obey Newton's Third Law:



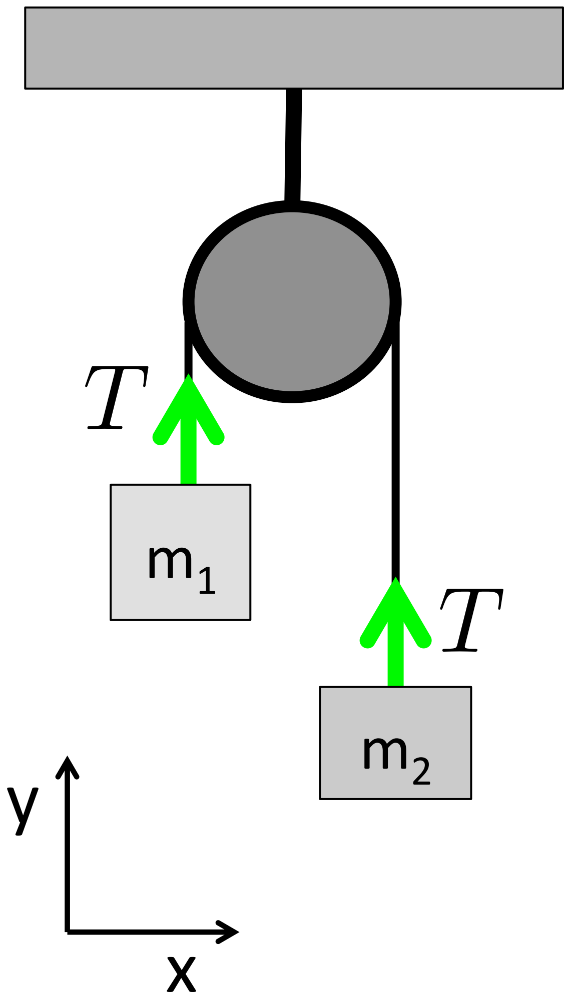
Breaking up forces into components on an incline

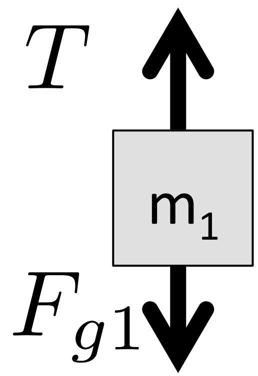
Procedure





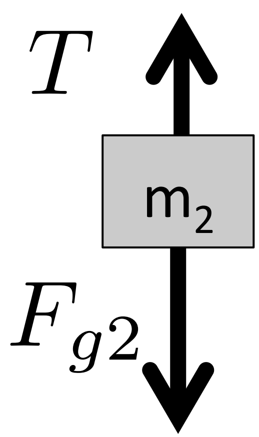
Attwood machine





$$\sum F_{y1} = T - F_{g1}$$

$$= m_1 a_1$$



$$\sum F_{y2} = T - F_{g2}$$

$$= m_2 a_2$$

C

Concept

Friction!

Sticking

(static friction)

Friction force balances forces in opposite direction → No motion!

$$f_s \leq \mu_s F_N$$

Slipping

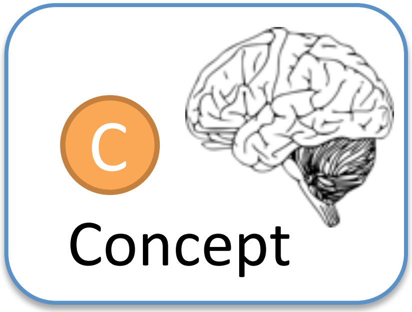
(kinetic friction)

Friction force Is less than the forces in opposite direction → Motion!

$$f_k = \mu_k F_N$$

$$0 \leq \mu \leq 1$$

Just a number, depends on the two materials that are in contact

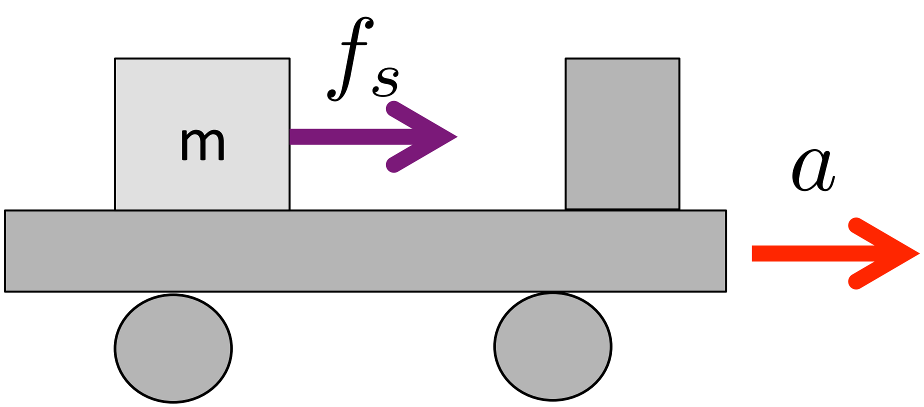


Friction!

Sticking

(static friction)

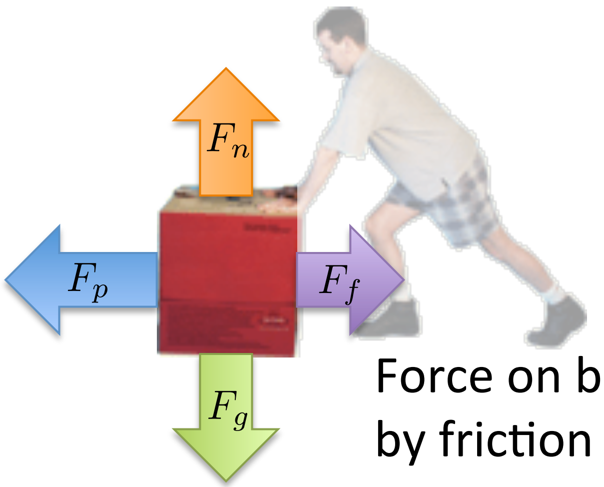
Points in the direction opposite the motion that would happen in friction was not there.



Slipping

(kinetic friction)

Points in the direction opposite the motion

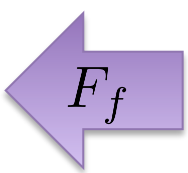


Force on box by friction

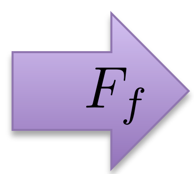
Clicker Question 8-3

What is the direction of friction when you take a step?

A)

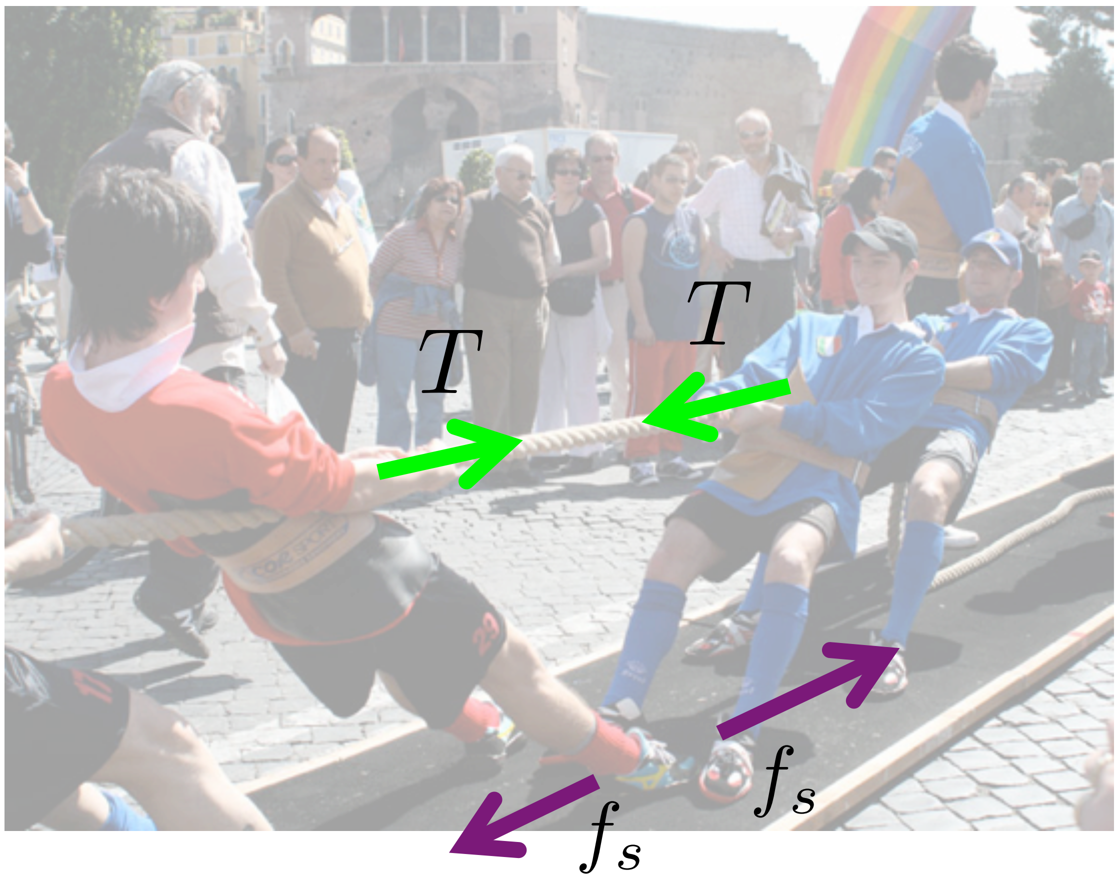


B)



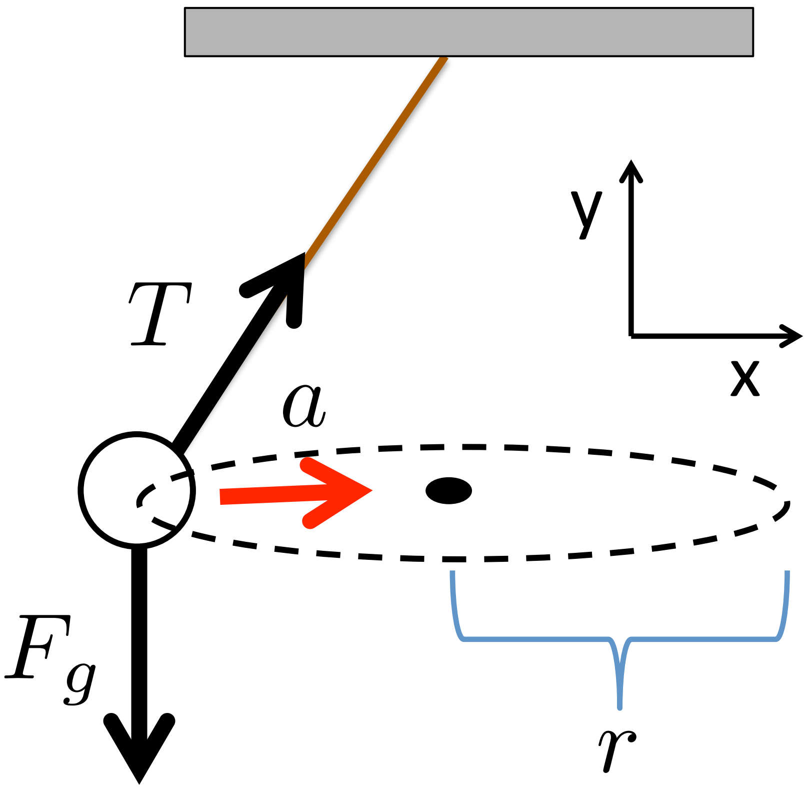
Practice with FBDs

If tension in a rope is always constant, how can anyone ever win a tug-of-war?



Force in circular motion

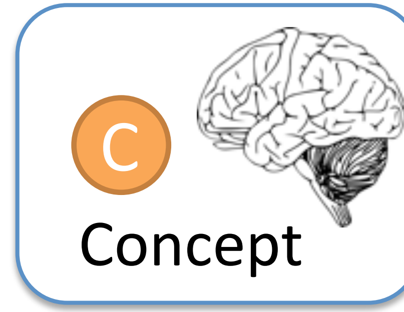
- Forces on a tether ball:



$$T_y - mg = 0$$

$$T_x = ma_x$$

Energy / Money analogy



<u>Types of Energy</u>

- Kinetic Energy
- Potential Energy
- Work

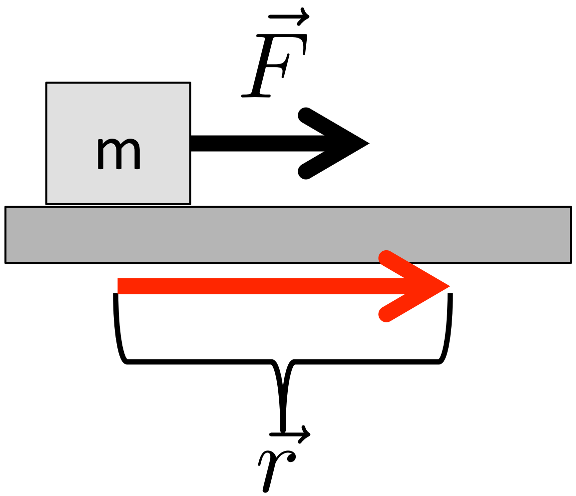
<u>Types of Money</u>

- Cash
- Money in bank account
- A paycheck or bill

C Concept

Work by a constant force

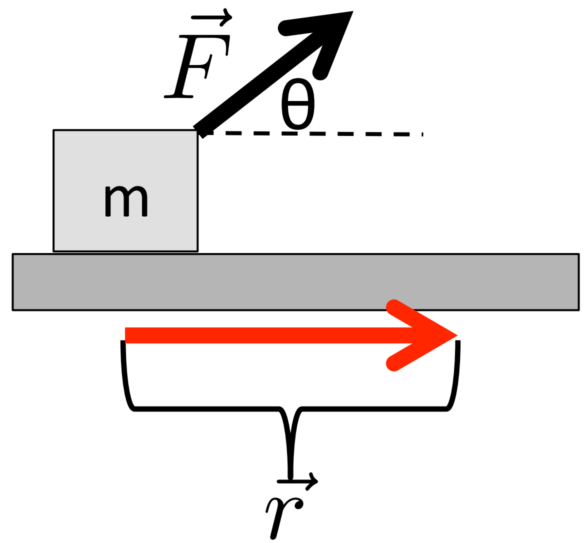
Force in direction of displacement:



$$W = \vec{F} \cdot \vec{r}$$

$$W = |F| \cdot |r|$$

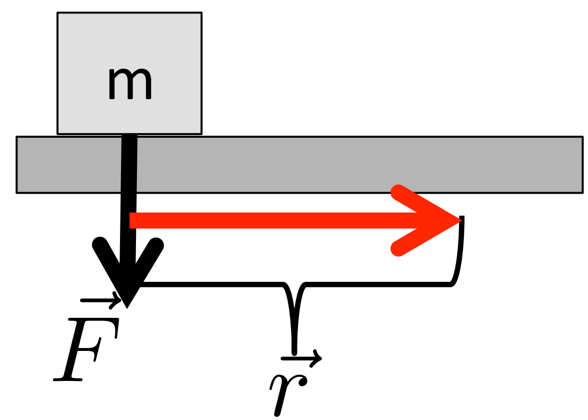
Force at an angle:



$$W = \vec{F} \cdot \vec{r}$$

$$W = |F| \cdot |r| \cos\theta$$

Force perpendicular to displacement:



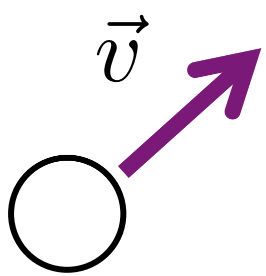
$$W = \vec{F} \cdot \vec{r}$$

$$W = 0$$

Kinetic and Potential Energy

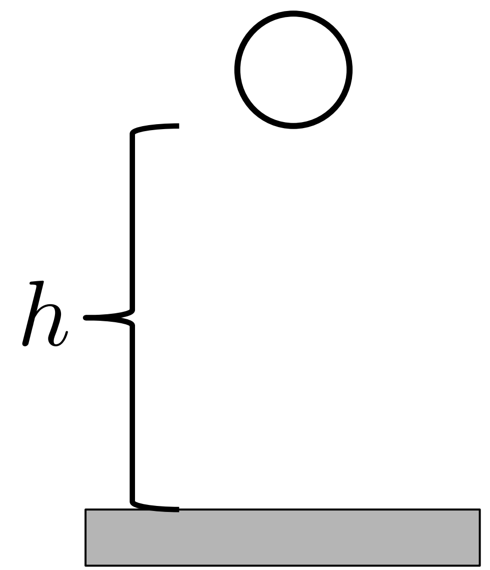
Translational Kinetic Energy

$$KE = \frac{1}{2}mv^2$$



Gravitational Potential Energy

$$PE_g = mgh$$



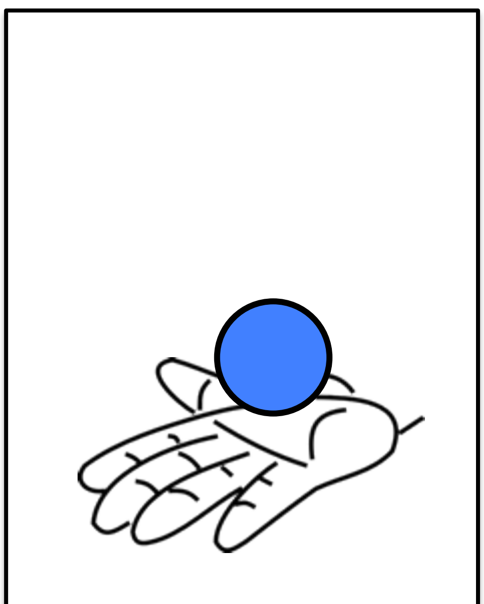
Isolated systems

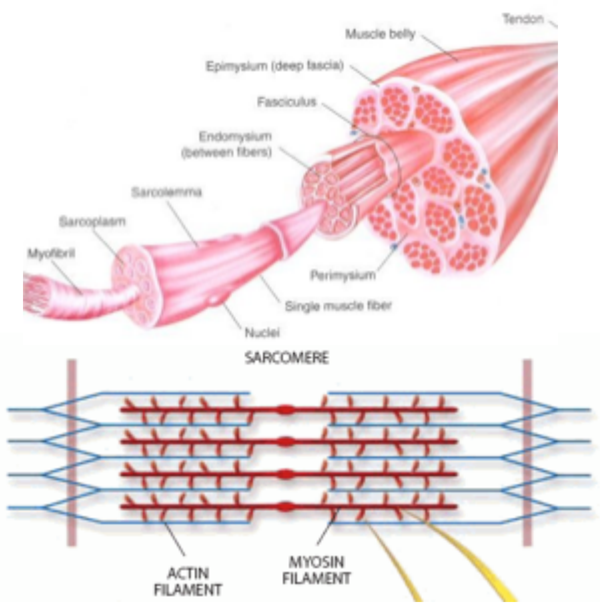
C

Concept

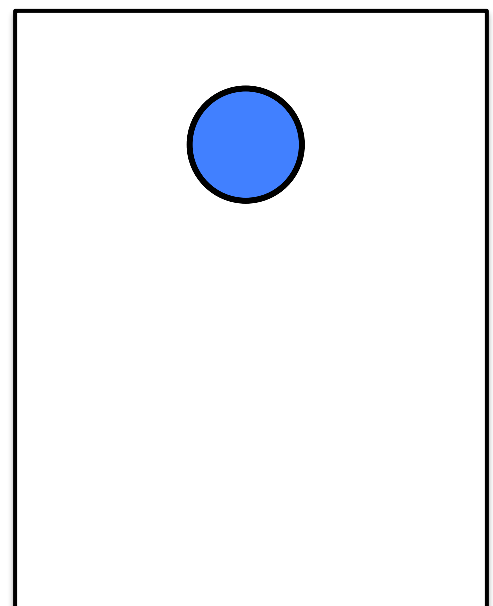
- System where total energy does not change over time (Energy is not added or dissipated away)
- Example:

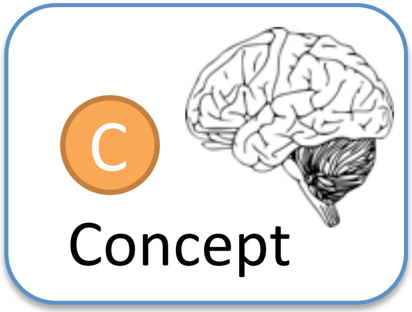
Not an isolated system





An isolated system

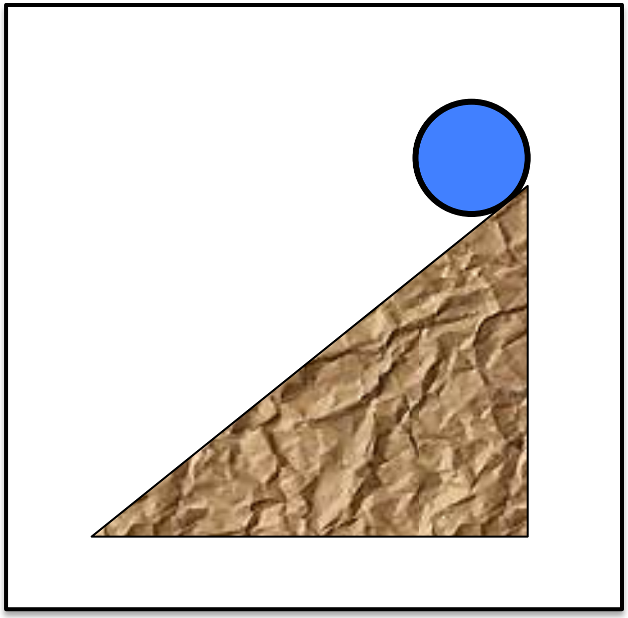




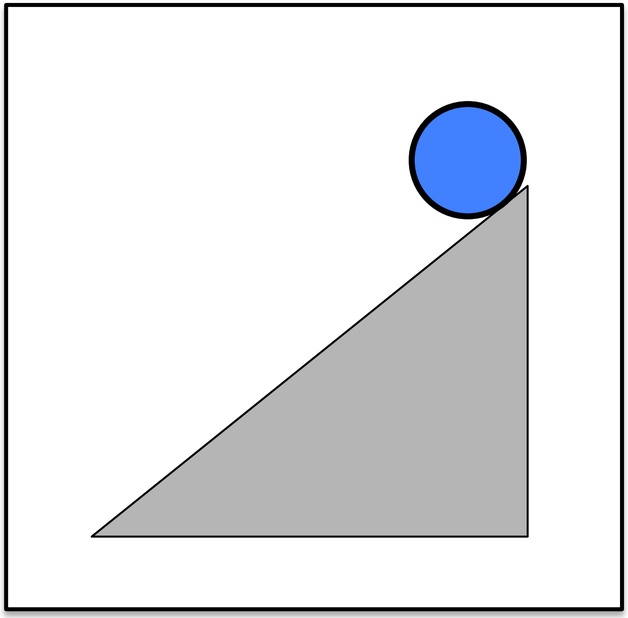
Isolated systems

- System where total energy does not change over time (Energy is not added or dissipated away)
- Example:

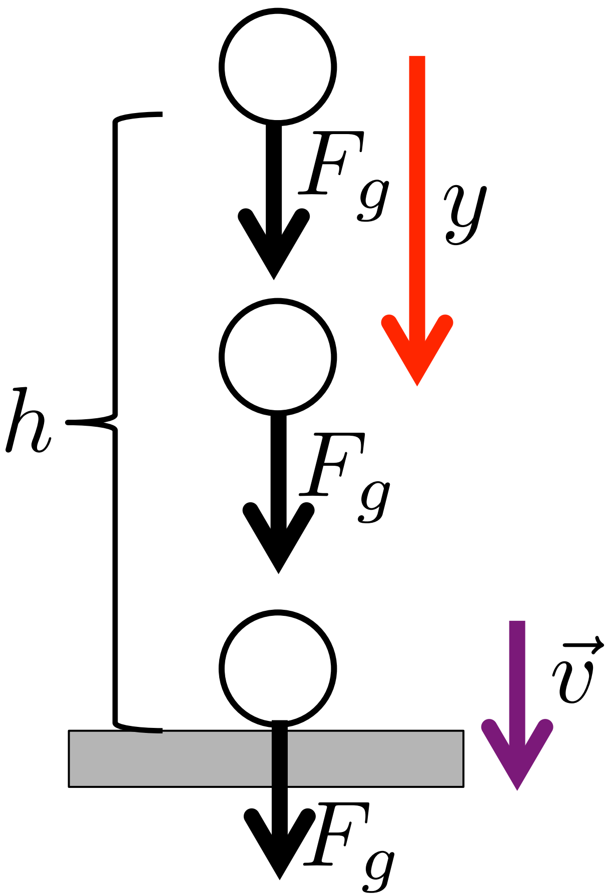
Not an isolated system



An isolated system



Potential Energy and Work

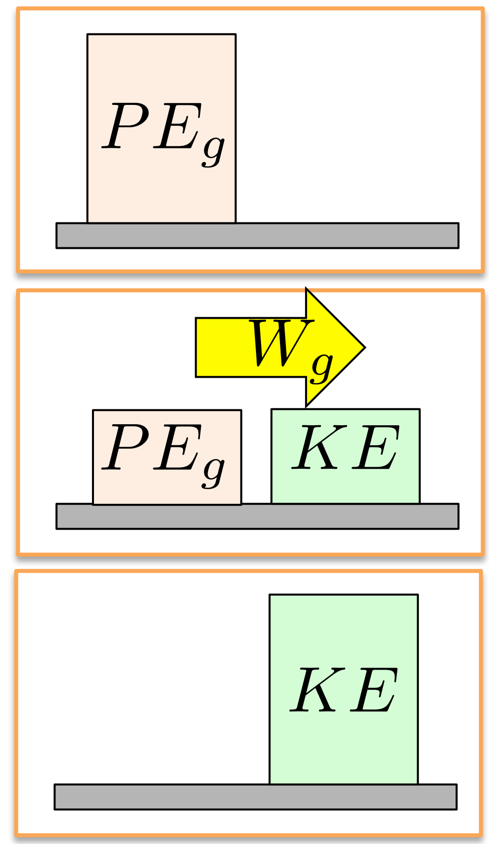


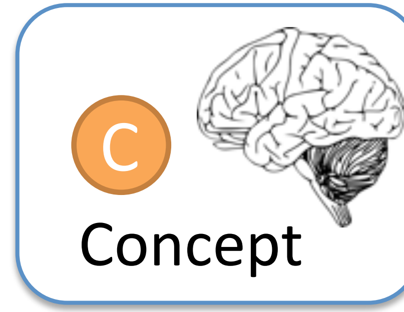
$$PE_g = mgh$$

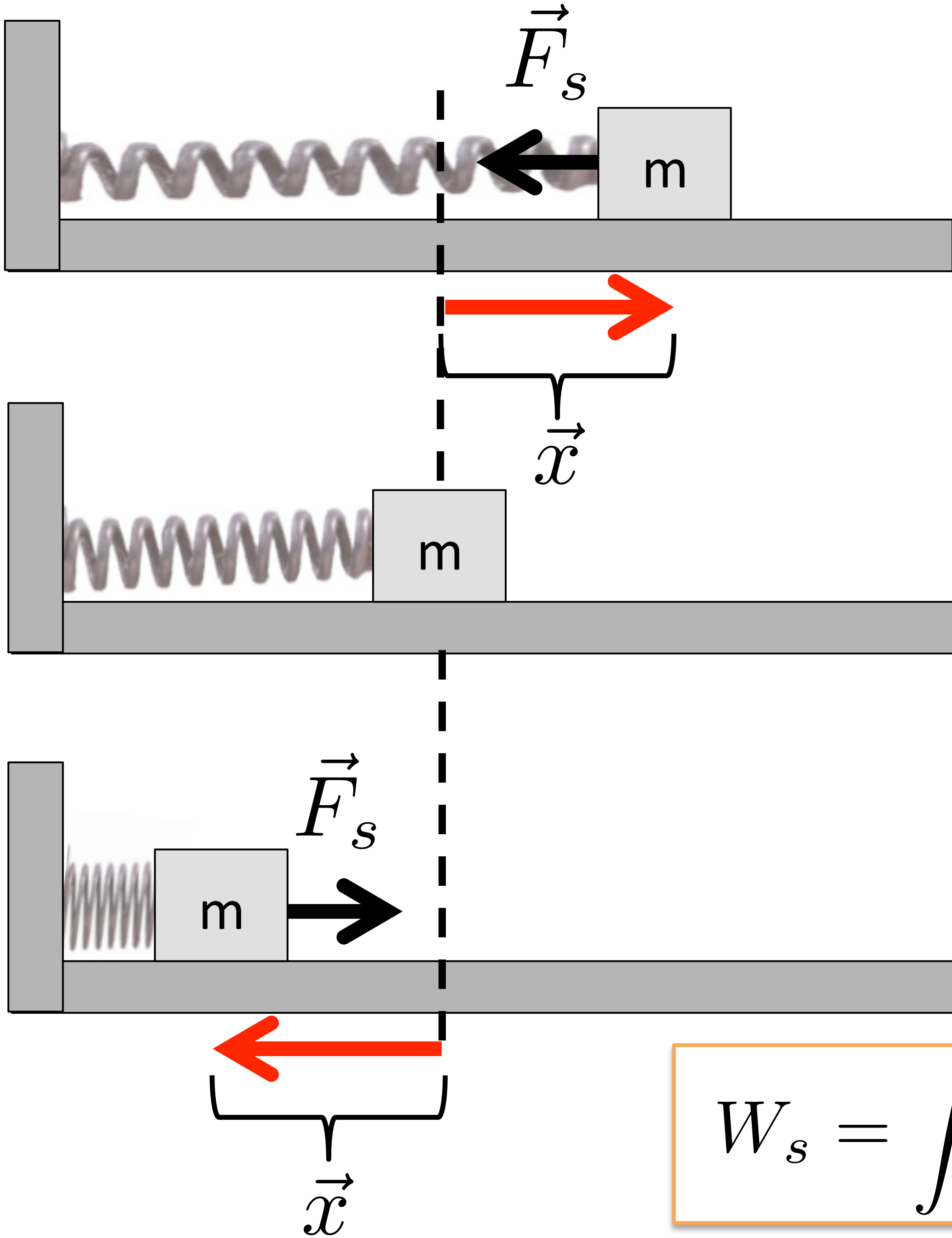
$$W = -\Delta PE$$

$$W = \Delta KE$$

$$KE = \frac{1}{2}mv^2$$







Hooke's Law

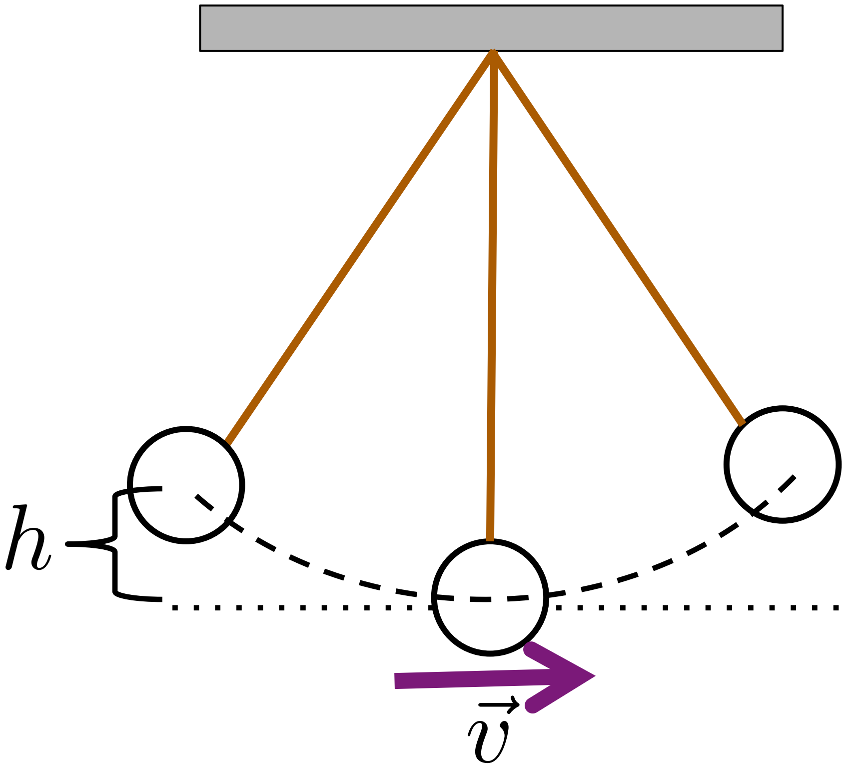
$$\vec{F}_s = -k\vec{x}$$

If force depends on displacement:

$$W = \int \vec{F} dx$$

$$W_s = \int (-kx)dx = -\frac{1}{2}kx^2$$

A pendulum isolated system



$$KE = \frac{1}{2}mv^2$$

$$PE_g = mgh$$

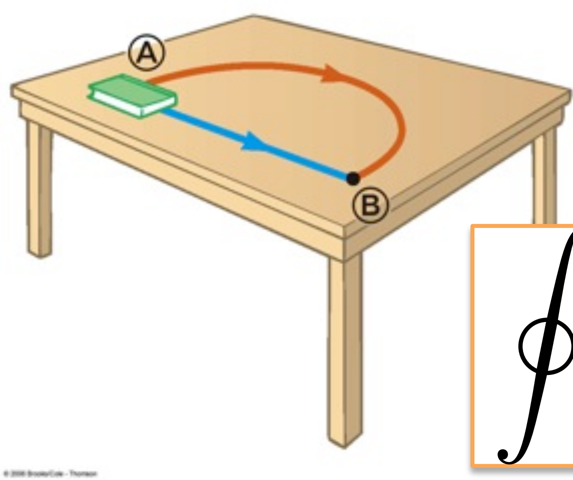
Conservative vs. Non conservative

Conservative Forces

- Work done is independent of the path taken
- Reversible

Non-conservative Forces

- Work done is not independent of the path taken
- Irreversible



$$\oint F_c \cdot dx = 0$$

Work and energy for nonconservative forces

- Modify Work-Energy equations:

$$W_c + W_{nc} = \Delta KE$$

$$W_c = -\Delta PE$$

- Energy conservation only when $W_{nc} = 0$.

$$\Delta E_{tot} = \Delta KE + \Delta PE = W_{nc}$$

$$E_f - E_i = W_{nc}$$

Obtaining Force from Potential energy

- Potential energy is related to force by the following:

$$PE = -W = -\int F \cdot dx$$

- But if you want to find force when given potential energy then take the derivative:

$$F = -\frac{d(PE)}{dx}$$

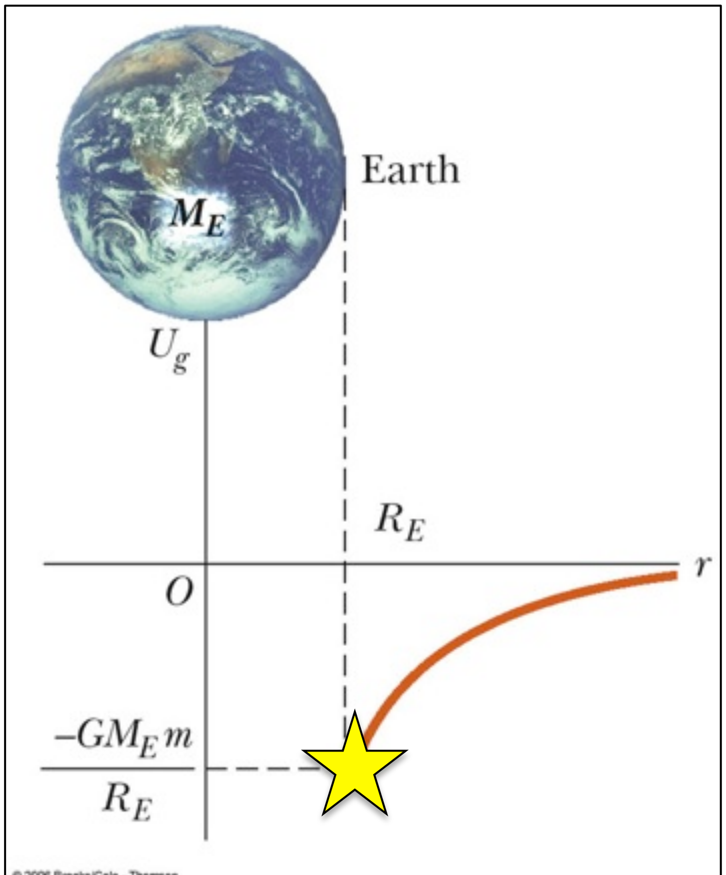
- Ex:

$$PE_s = \frac{1}{2}kx^2 \rightarrow \vec{F}_s = -kx$$

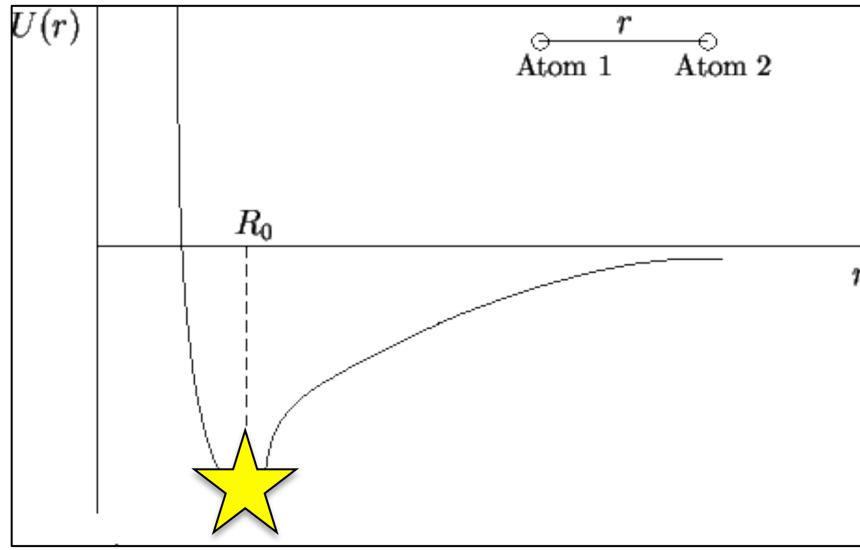
$$PE_g = mgh \rightarrow \vec{F}_g = -mg \cdot \hat{y}$$

Potential Energy for field forces

$$PE_g = -\frac{G m_1 m_2}{r}$$



Lennard-Jones potential



- Systems eventually settle to the minimum of the potential energy – stable equilibrium point

Momentum

- Linear momentum:

$$\vec{p} = m\vec{v}$$

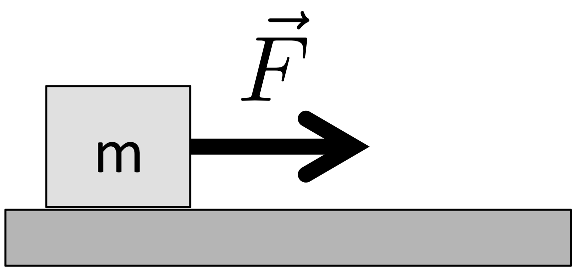
- Impulse, the change in momentum

$$\vec{I} = \Delta\vec{p} = \vec{F}\Delta t$$

- F is in the direction of the displacement

- Example:

You push on a 1 kg box that is at rest with a force of 1 N for 5 s. What is v_F?



Isolated systems

- The total momentum is conserved for *both*:

<u>Elastic collisions</u>

- Energy conserved!

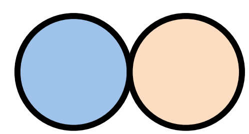
Bounce apart:

<u>Inelastic collisions</u>

- Energy *not* conserved!

Collide together and stick:

Throwing stuff / explosions:

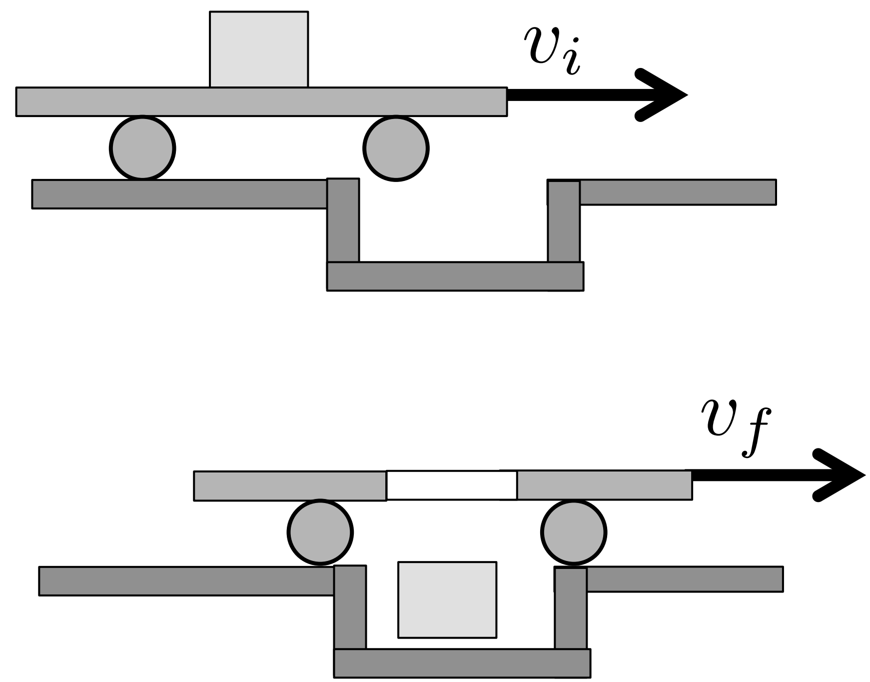


Example problem

- A bullet with mass *m* and velocity v_0 imbeds in a block of wood with mass *M*. If the coefficient of kinetic friction is μ, how far will the block of wood move across a horizontal floor before it comes to rest?

Clicker question 11-6

A cart with a mass on it is moving to the right with a constant velocity when it drives over a ditch. It releases the mass into the ditch through a trap door. What happens to the velocity of the cart?



A) It will slow down

B) It will speed up

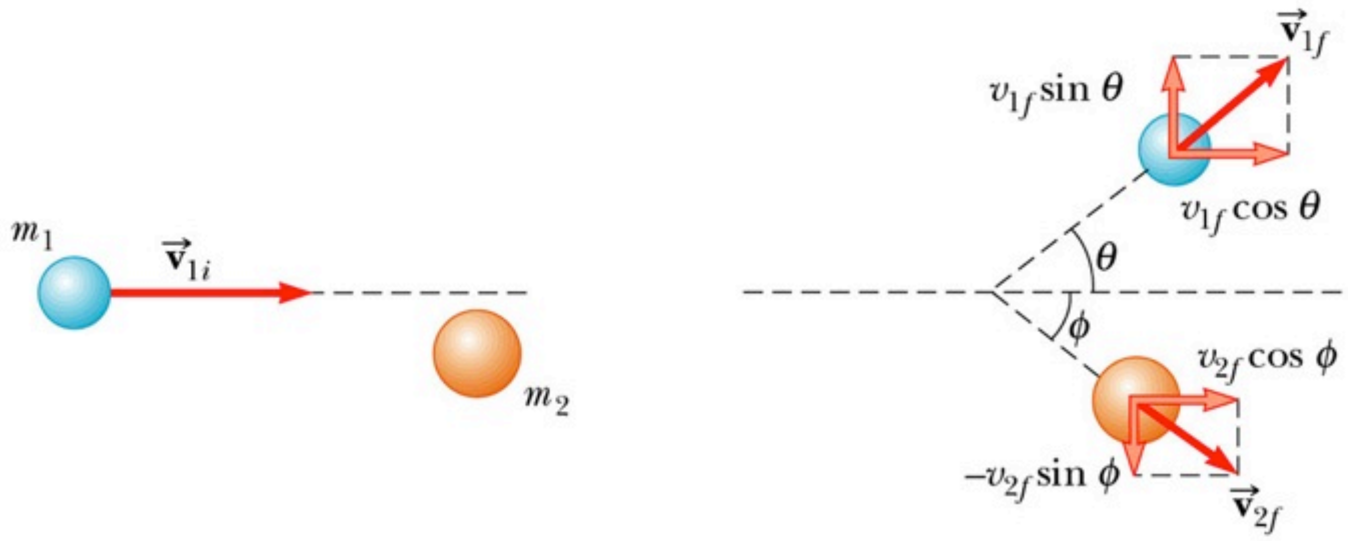
C) It will stay the same

2D collisions

- Conserve momentum in x and y separately.

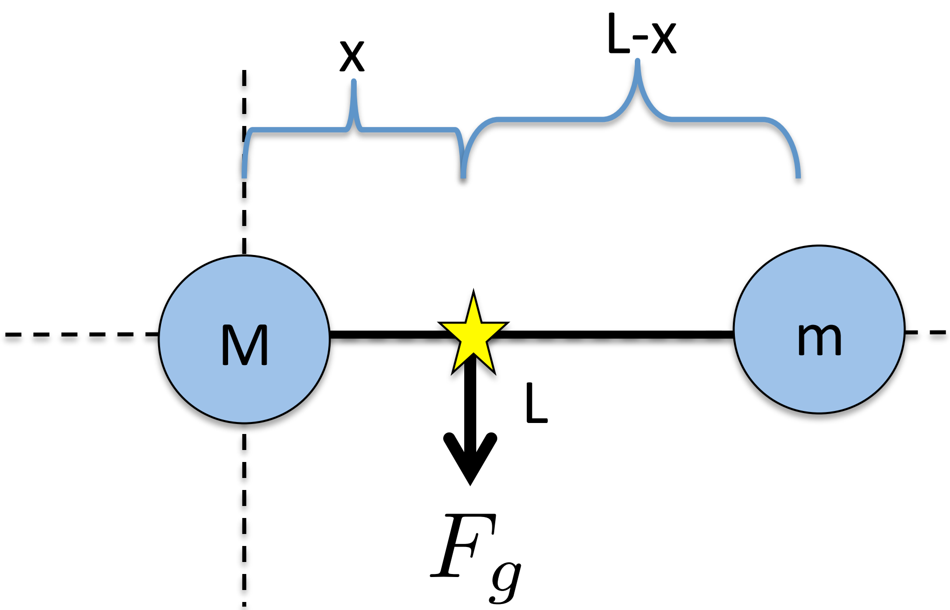
$$m_1\vec{v}_{(1xi)} + m_2\vec{v}_{(2xi)} = m_1\vec{v}_{(1xf)} + m_2\vec{v}_{(2xf)}$$

$$m_1\vec{v}_{(1yi)} + m_2\vec{v}_{(2yi)} = m_1\vec{v}_{(1yf)} + m_2\vec{v}_{(2yf)}$$



(a) Before the collision (b) After the collision

Center of mass



$$x_{CM} = \frac{Mx + m(L - x)}{M + m}$$

Rigid objects:

- Assume gravity acts only on the COM

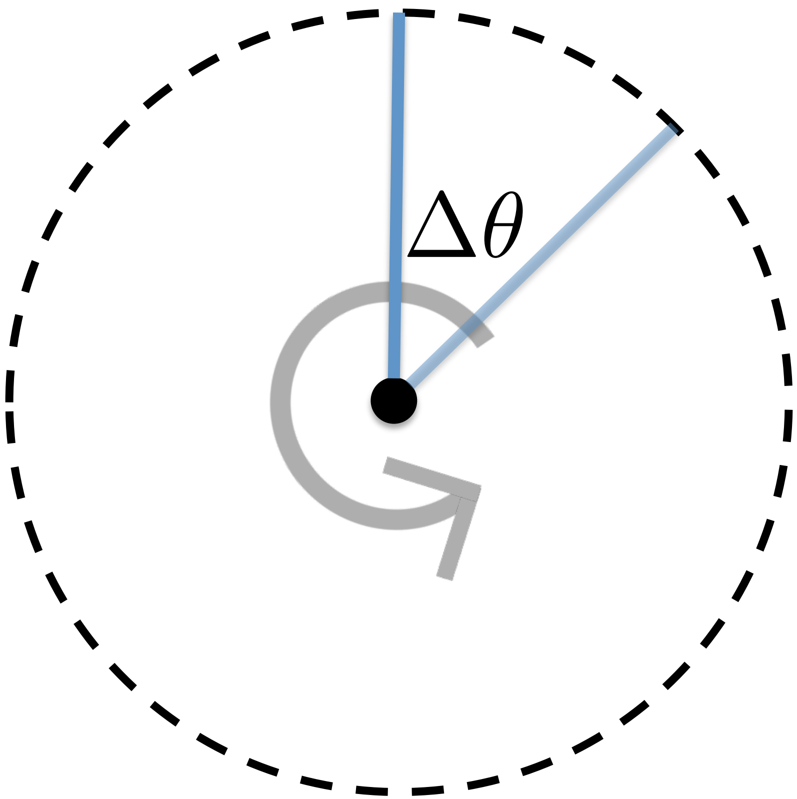
Collisions:

- Without external forces, velocity of center of mass is constant

Rotational motion
deja vu

Linear motion	Rotational motion
displacement (Δx)	angular displacement ($\Delta\theta$)
velocity (v)	angular velocity (ω)
acceleration (a)	angular acceleration (α)
mass (m)	moment of inertia (I)
force (F)	torque (τ)
$F=ma$	$\tau = I \alpha$
linear kinetic energy	rotational kinetic energy
linear momentum (p)	angular momentum (L)
$F=dp/dt$	$\tau = dL/dt$

Rotational Kinematics



- angular displacement
- angular velocity

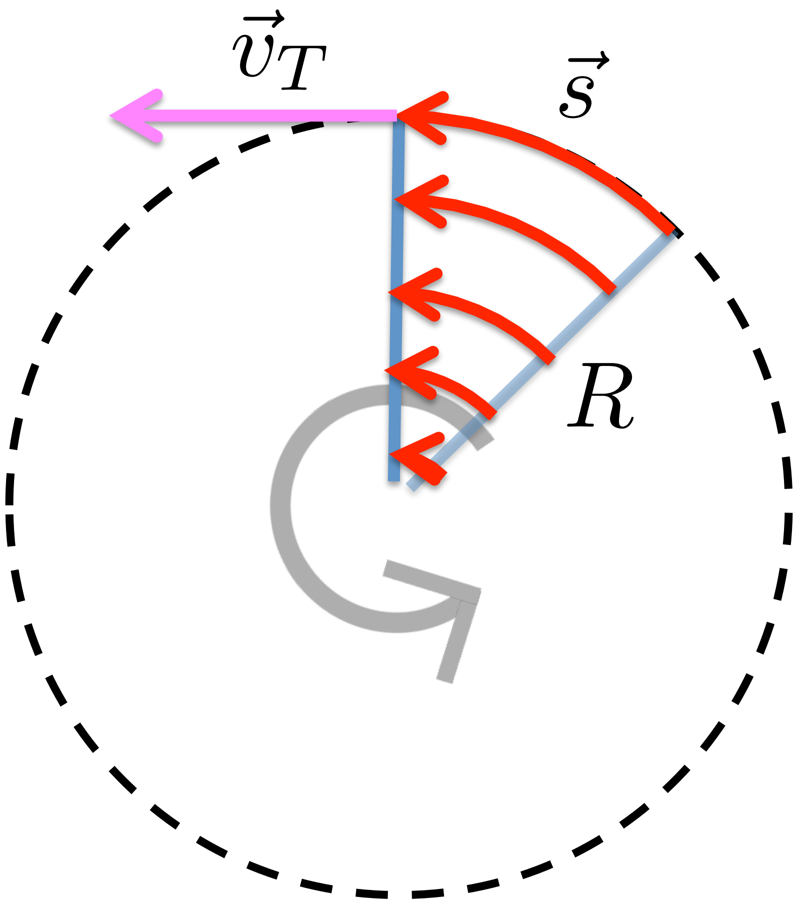
$$\omega = \frac{\Delta\theta}{\Delta t}$$

- angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$1 \text{ rotation} = 2\pi \text{ radians}$$

Rotational Kinematics



- arc length

$$s = R\theta$$

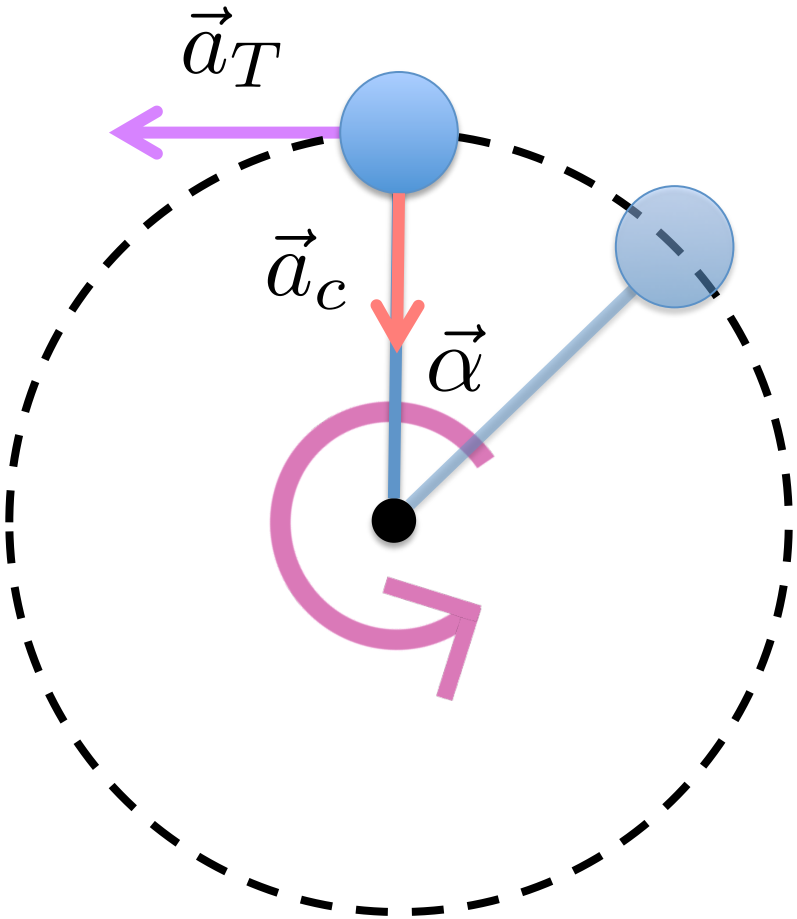
- tangential velocity

$$v_T = R\omega$$

- tangential acceleration

$$a_T = R\alpha$$

Three accelerations!



- centripetal acceleration

$$a_c = \frac{v^2}{r}$$

- tangential acceleration

$$a_T = R\alpha$$

- angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Rotational Kinematics

Take derivative
--or—
Find slope

$$\omega = \frac{d\theta}{dt}$$

Take derivative
--or—
Find slope

$$\alpha = \frac{d\omega}{dt}$$

angular displacement, θ(t)

angular velocity, ω(t)

constant angular acceleration, α

Take integral
--or—
Area under curve

$$\theta = \int \omega \cdot dt$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

Take integral
--or—
Area under curve

$$\omega = \int \alpha \cdot dt$$

$$\omega_f = \omega_i + \alpha t$$

Rotational Kinematics

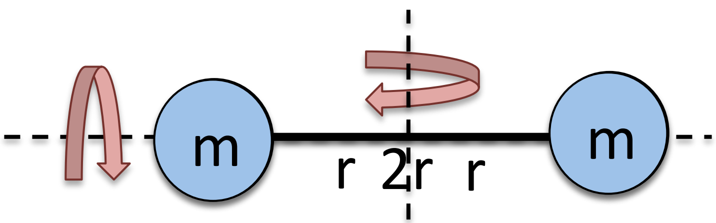
Linear motion	Rotational motion
$x_f = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
$v_x = v_{0x} + a_x t$	$\omega_f = \omega_i + \alpha t$
$x_f = x_0 + \frac{1}{2}(v_x + v_{0x})t$	$\theta_f = \theta_i + \frac{1}{2}(\omega_f + \omega_i)t$
$v_x^2 = v_{0x}^2 + 2a_x \Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Moments of Inertia

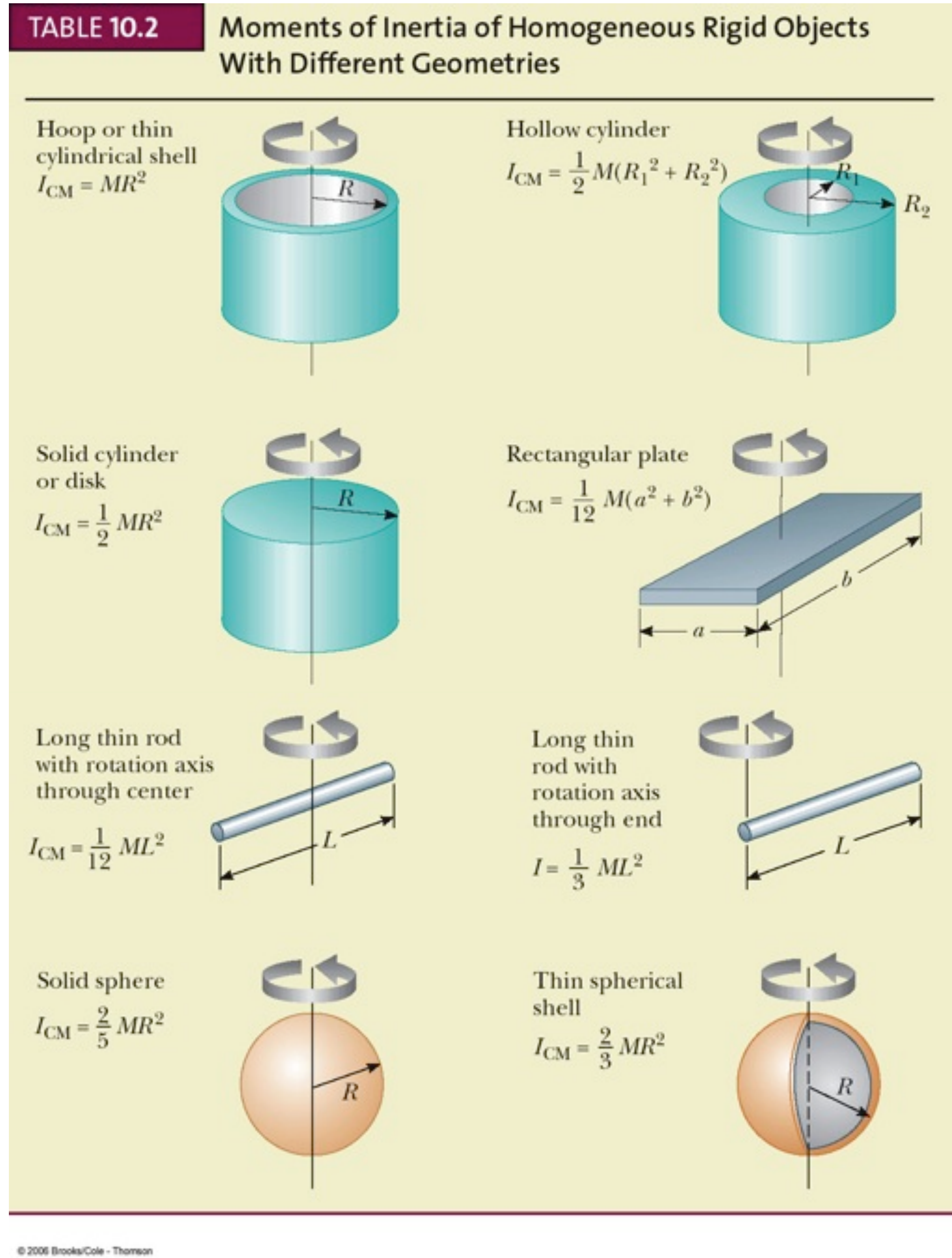
- How mass is distributed
- In general:

$$I = \sum_i m_i {r_i}^2$$

- Changes when you you consider different axes of rotation.



- Pre-calculated for common 3D objects→



Rotational Kinetic Energy

- Translational Kinetic Energy:

$$KE = \frac{1}{2}mv^2$$

- Rotational Kinetic Energy:

$$KE_R = \frac{1}{2}I\omega^2$$

Rolling without slipping

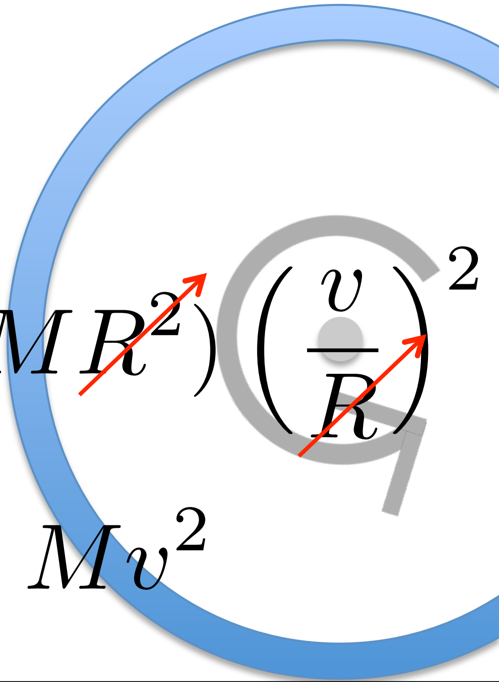
- Translational velocity is equal to tangential velocity!

$$I_{hoop} = MR^2$$

$$\omega = \frac{v_T}{R} = \frac{v}{R}$$

$$KE_R = \frac{1}{2} I_{hoop} \omega^2 = \frac{1}{2}(MR^2)\left(\frac{v}{R}\right)^2$$

$$KE_{tot} = KE + KE_R = Mv^2$$



Torque

Axis of rotation aka pivot point:

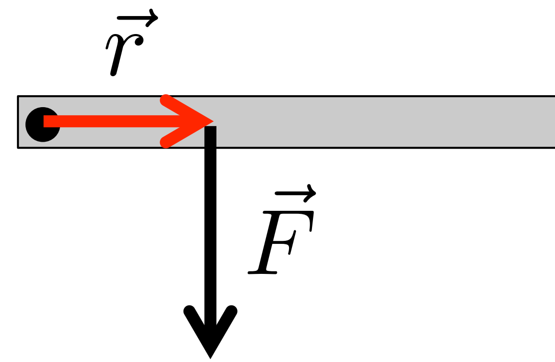
- A point that is not moving relative to the rotation

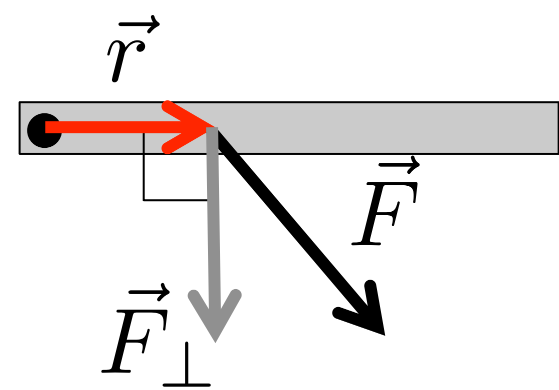
Moment arm:

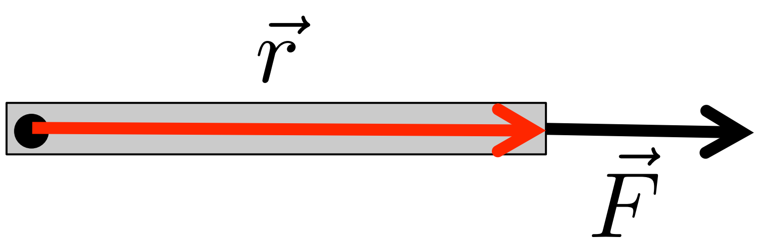
- A vector that extends from the pivot to the place where the force is being exerted.

In order for a force to exert a torque on an object:

- Force must have a component that is perpendicular to the moment arm.

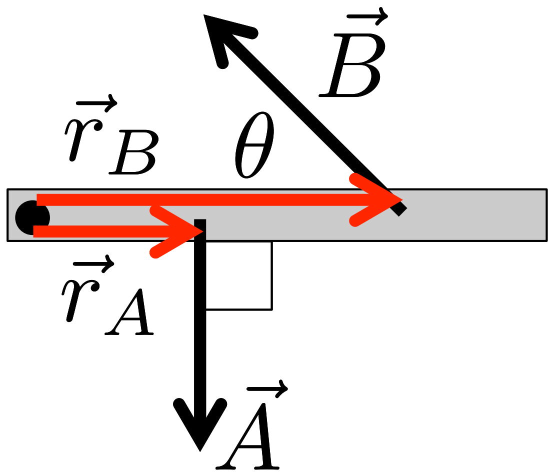






Newton's 2nd law for rotation

$$\sum \tau = I\alpha \quad = -Ar_A + Br_B \sin(180 - \theta)$$



Rotational equilibrium

- When pivot point is not fixed, even if net torque is zero, you can get translational motion when there is a net force. So you need to check both.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$