

Physics 1A, Lecture 14: Rotational Motion

Summer Session 1, 2011

The quiz will commence at 9:33 AM.

Key Questions: (Discuss with neighbors before quiz)

- 1) What are the units of angular velocity?
- 2) Is angular velocity positive or negative when rotation is counterclockwise?
- 3) How do you convert from angular velocity to tangential velocity?
- 4) What is rotational kinetic energy?
- 5) What measurements of an object determine its moment of inertia?

Reading Quiz 14-1

- What are the units of angular velocity?

A) m/s

B) radians/s²

C) 1/s²

D) radians/s

E) 1/s

Reading Quiz 12-2

- Is angular velocity positive or negative when rotation is counterclockwise?

A) positive

B) negative

Reading Quiz 12-3

- How do you convert from angular velocity to tangential velocity?

A) $v = \frac{1}{2} \omega R^2$

B) $v = \omega R$

C) $v = R/\omega$

D) $v = R/\omega^2$

E) $v = R\omega^2$

Reading Quiz 12-4

- What is rotational kinetic energy?

A) $\frac{1}{2} m v^2$

B) $\frac{1}{2} I \omega^2$

C) $\frac{1}{2} m \omega^2$

D) $\frac{1}{2} m v^2 r$

E) $\frac{1}{2} m v^2 / r$

Reading Quiz 12-5

- What measurements of an object usually determine its moment of inertia?

A) radius, mass

B) volume, mass

C) volume, density

D) center of mass

E) mass, velocity

Announcements

- Homework due tomorrow by 4pm
- Office hours today at 5pm
- Today is the deadline to drop with a W
- Fill out survey online
- No quiz this week! Just a Final on Friday.

Calculating your grade (as Anat rolls her eyes at you...)

- Points already earned:
 - $0.35 \times$ (Final quiz grade)
 - $0.2 \times$ (Homework grade) (all of them are out of 3)
 - $0.1 \times$ (Reading quiz grade)
 - $0.05 \times$ (Clicker grade)
- Final Grade cutoffs (that might be lowered):
 - A ... above 88%
 - B ... above 75%
 - C ... above 63%
 - D ... above 50%
- Take the cutoff that you want to reach, subtract points already earned, divide by 0.35 ... That is what you need to get on the Final to guarantee that grade.

Anonymous poll

- How awesome was Evan so far?
 - A) Who is Evan?
 - B) I only saw him at the quizzes.
 - C) Evan was ok, but I didn't enjoy getting help from him.
 - D) Evan was ok, but I didn't need much extra help.
 - E) Evan was super helpful.

Anonymous poll

- Do you think having the worksheet homework assignment was a good idea?
 - A) No.
 - B) Yes, but it shouldn't have been for credit.
 - C) Yes.

Example problem

- A bullet with mass m and velocity v_0 imbeds in a block of wood with mass M . If the coefficient of kinetic friction is μ , how far will the block of wood move across a horizontal floor before it comes to rest?

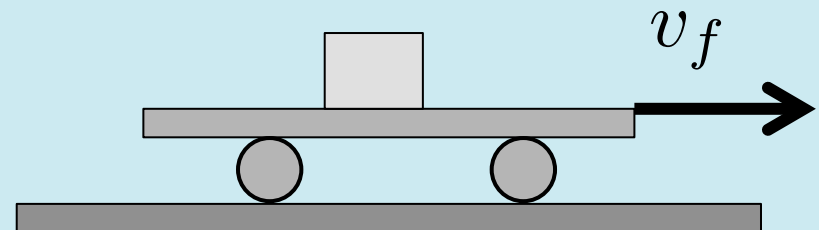
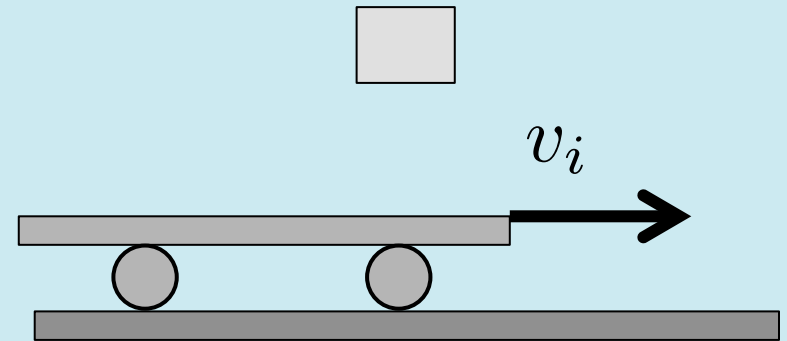
Clicker question 12-1

A cart is moving to the right with a constant velocity when a mass is dropped vertically onto it. What happens to the velocity of the cart?

A) It will slow down

B) It will speed up

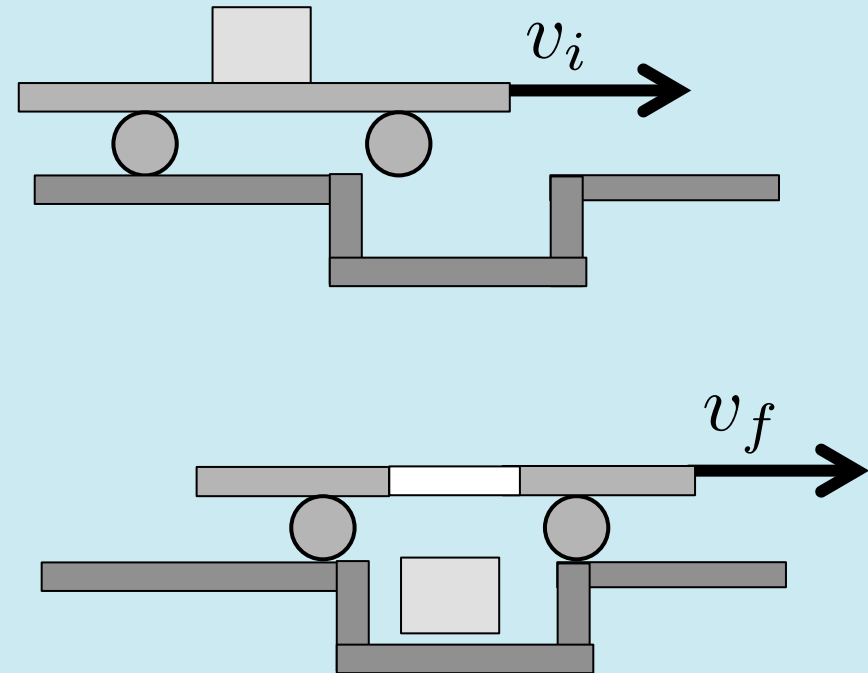
C) It will stay the same



Clicker question 12-2

A cart with a mass on it is moving to the right with a constant velocity when it drives over a ditch. It releases the mass into the ditch through a trap door. What happens to the velocity of the cart?

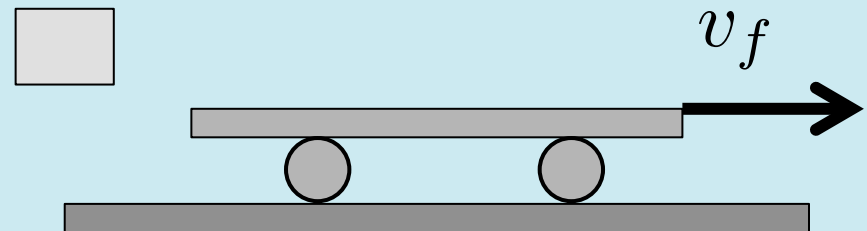
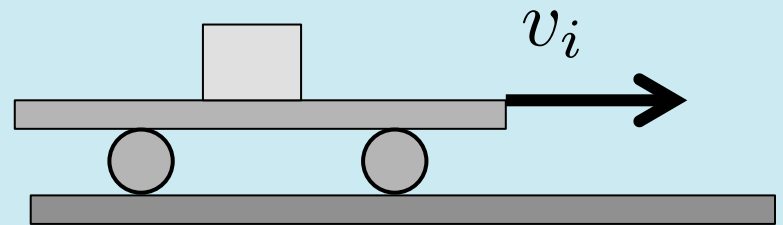
- A) It will slow down
- B) It will speed up
- C) It will stay the same



Clicker question 12-3

A cart is moving to the right with a constant velocity when a man throws the mass off the back of the cart. What happens to the velocity of the cart?

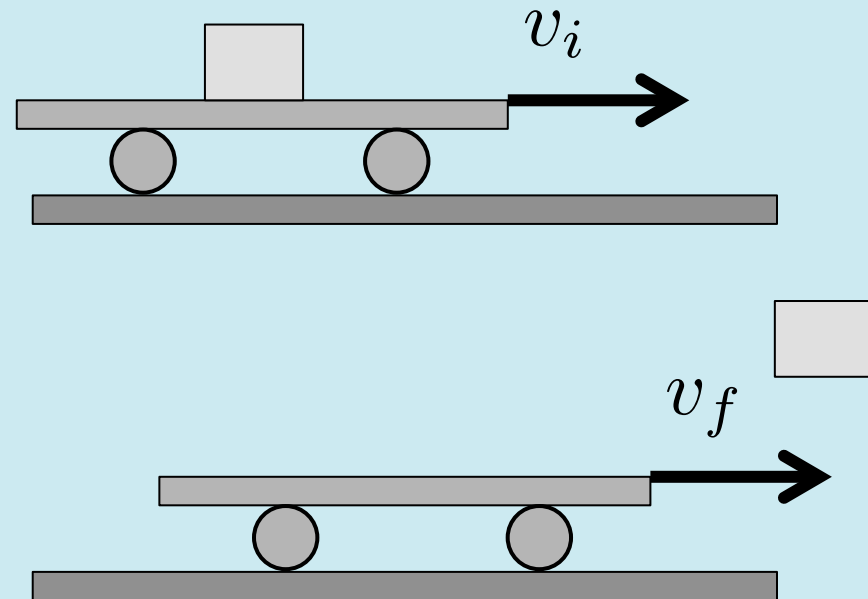
- A) It will slow down
- B) It will speed up
- C) It will stay the same



Clicker question 12-4

A cart is moving to the right with a constant velocity when a man throws the mass off the *front* of the cart. What happens to the velocity of the cart?

- A) It will slow down
- B) It will speed up
- C) It will stay the same
- D) It reverses direction
- E) A or D

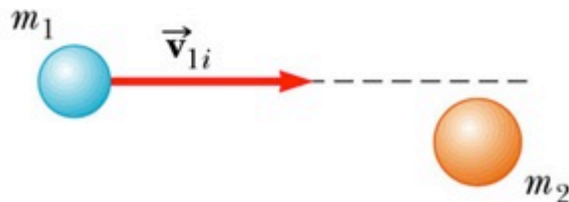


2D collisions

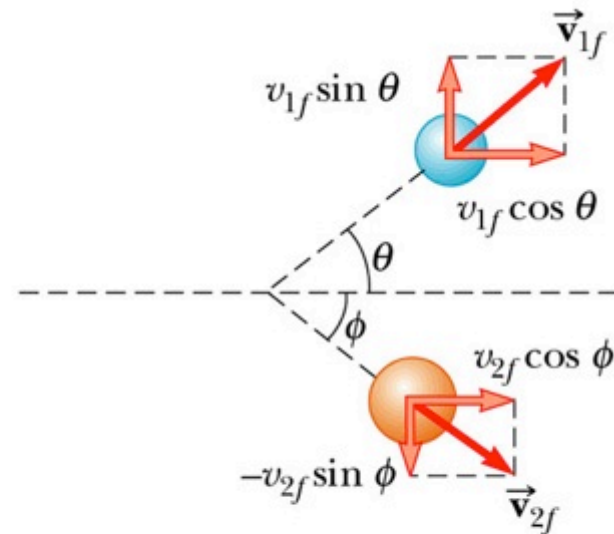
- Conserve momentum in x and y separately.

$$m_1 \vec{v}_{(1xi)} + m_2 \vec{v}_{(2xi)} = m_1 \vec{v}_{(1xf)} + m_2 \vec{v}_{(2xf)}$$

$$m_1 \vec{v}_{(1yi)} + m_2 \vec{v}_{(2yi)} = m_1 \vec{v}_{(1yf)} + m_2 \vec{v}_{(2yf)}$$



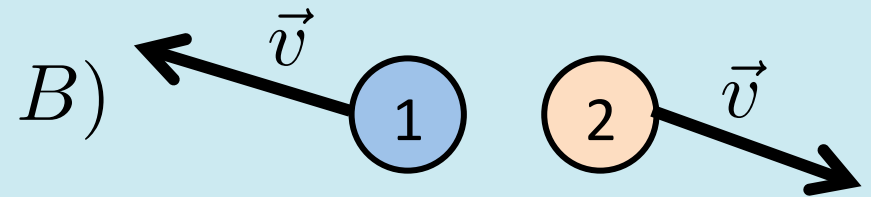
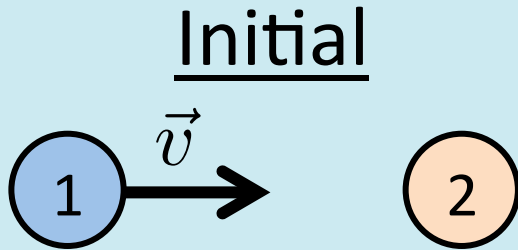
(a) Before the collision



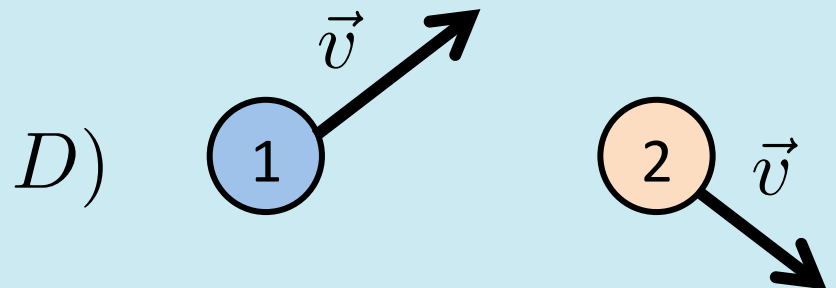
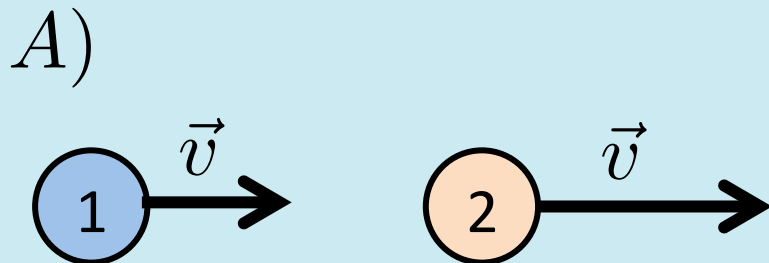
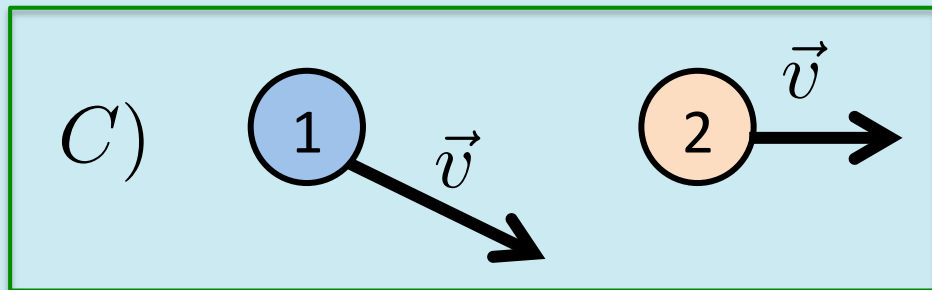
(b) After the collision

Clicker question 12-5

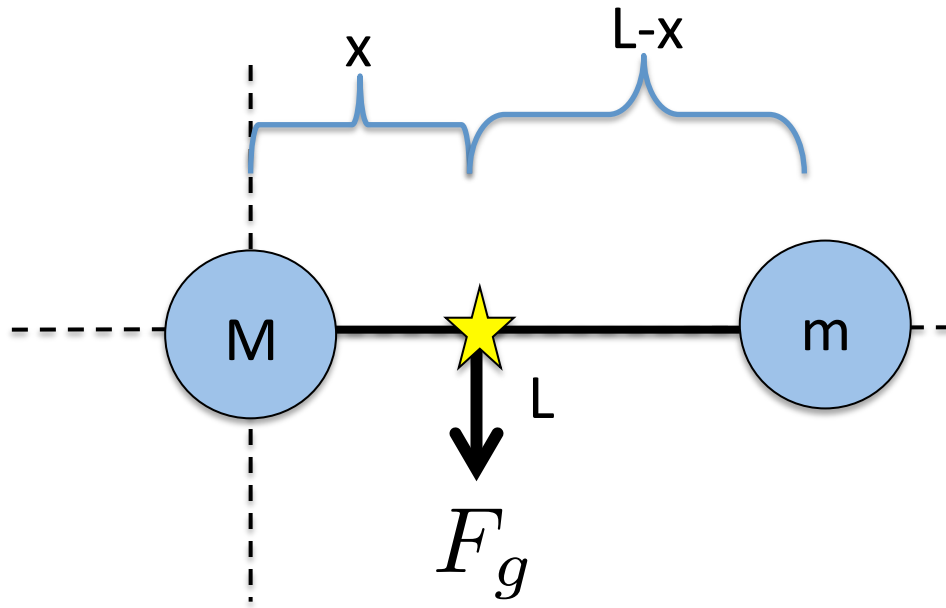
- Which of the following FINAL pictures of an elastic collision is not possible?



Final



Center of mass



$$x_{CM} = \frac{Mx + m(L - x)}{M + m}$$

Rigid objects:

- Assume gravity acts only on the COM

Collisions:

- Without external forces, velocity of center of mass is constant

Rotational motion

deja vu

Linear motion

displacement (Δx)

velocity (v)

acceleration (a)

mass (m)

force (F)

$$F=ma$$

linear kinetic energy

linear momentum (p)

$$F=dp/dt$$

Rotational motion

angular displacement ($\Delta\theta$)

angular velocity (ω)

angular acceleration (α)

moment of inertia (I)

torque (τ)

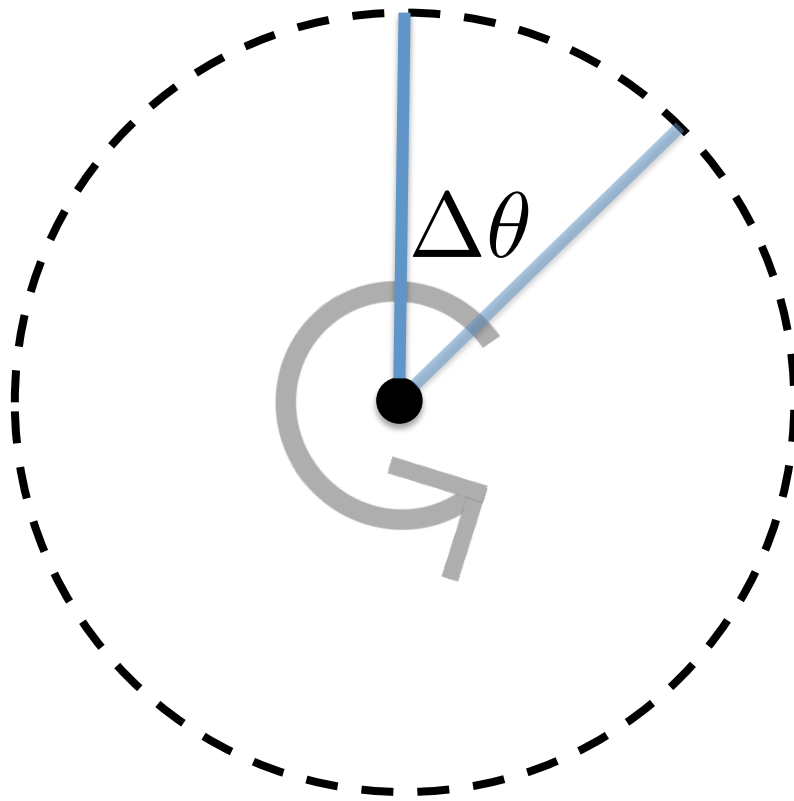
$$\tau = I \alpha$$

rotational kinetic energy

angular momentum (L)

$$\tau = dL/dt$$

Rotational Kinematics



- angular displacement
- angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- angular acceleration

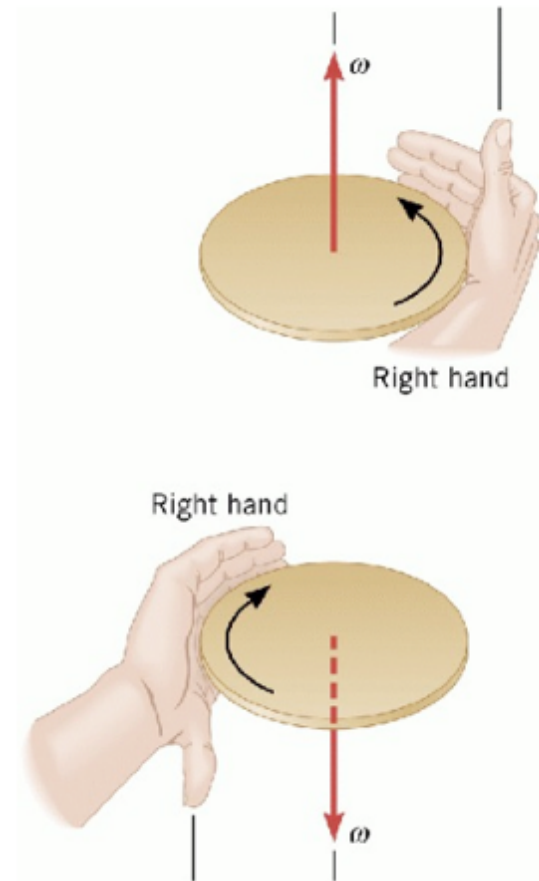
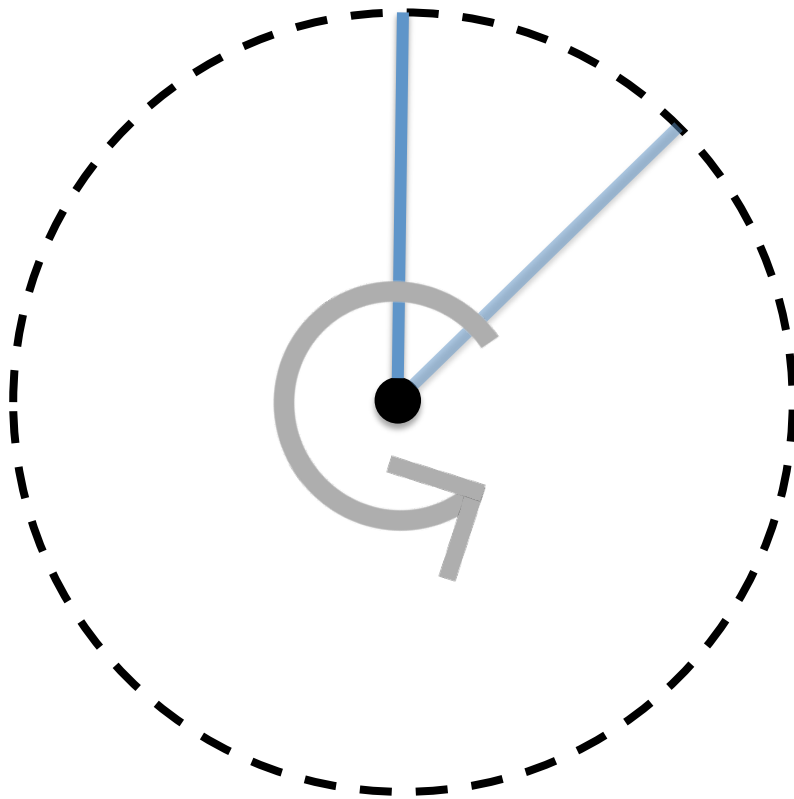
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

1 rotation = 2π radians

Rotational Kinematics

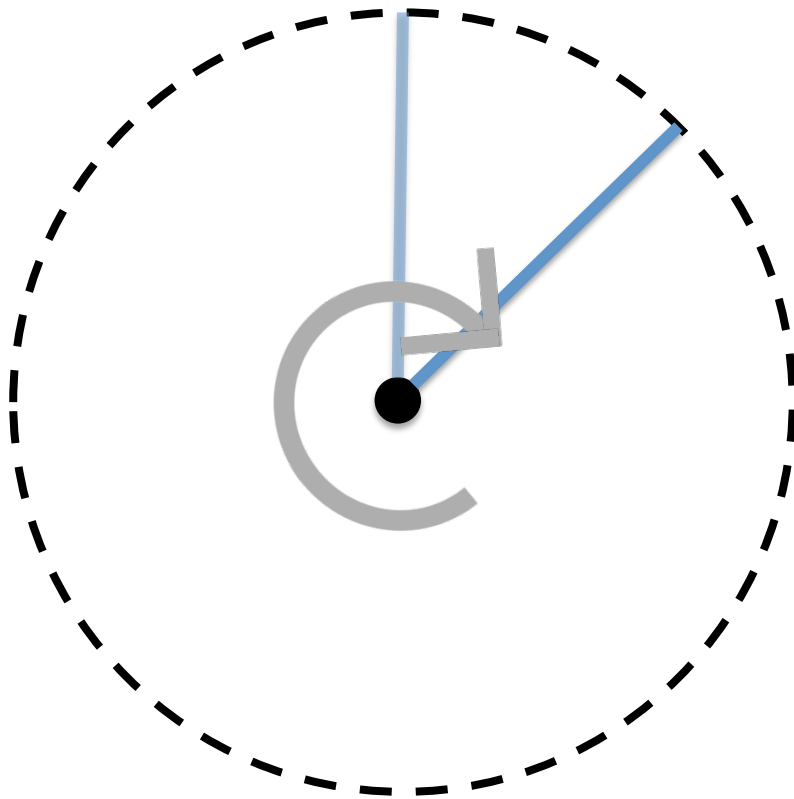
Positive rotation

- Define a pivot point
- Determine sign of ω

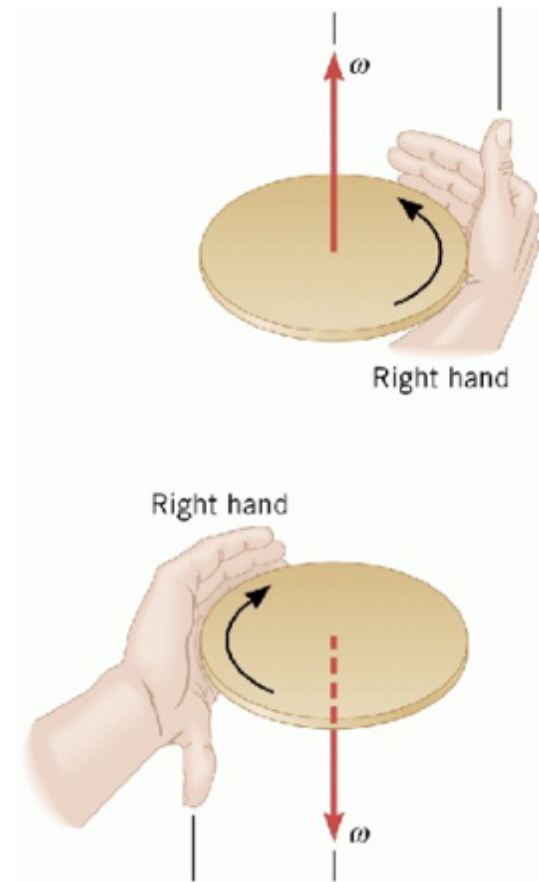


Rotational Kinematics

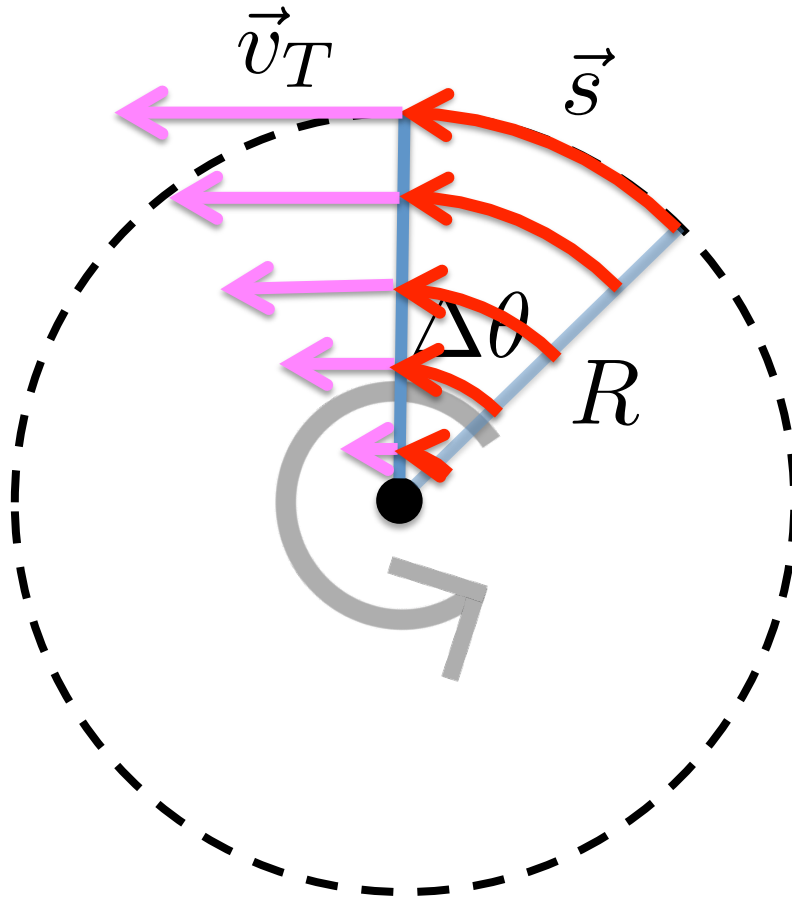
Negative rotation



- Define a pivot point
- Determine sign of ω



Rotational Kinematics



- arc length

$$s = R\theta$$

- tangential velocity

$$v_T = R\omega$$

- tangential acceleration

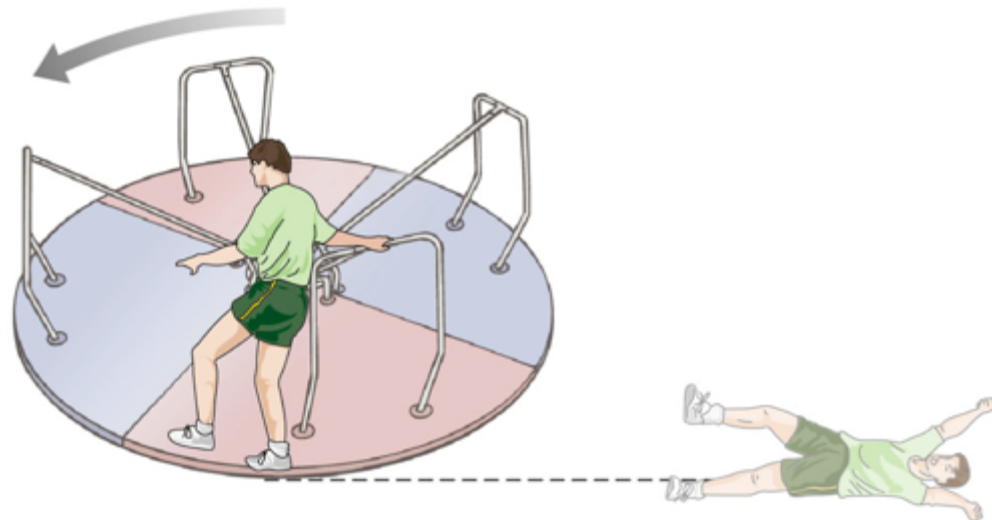
$$a_T = R\alpha$$

Playground accidents

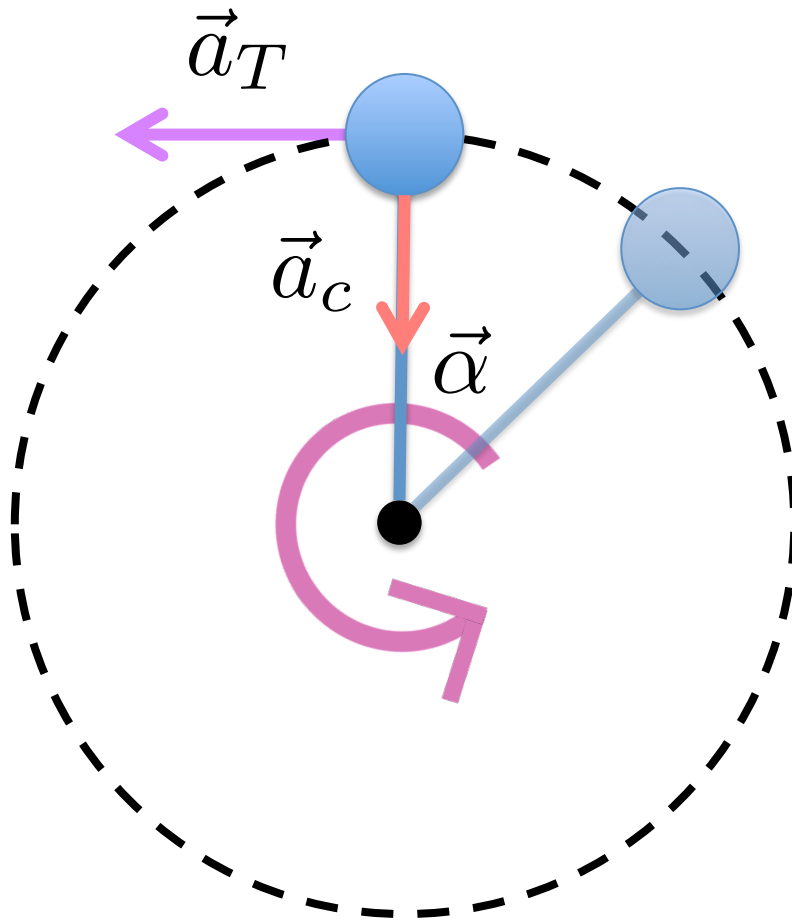
- Example: A boy is riding on a merry-go-round with a radius of 1 m at an angular speed of 12 rotations per minute. What is the speed that he goes flying off if he accidentally lets go?

$$12 \text{ rpm} \times \frac{2\pi}{1 \text{ rotation}} \times \frac{1 \text{ min}}{60 \text{ s}} = 1.25 \text{ radians/s}$$

$$v = R\omega = 1.25 \text{ m/s}$$



Three accelerations!



- centripetal acceleration

$$a_c = \frac{v^2}{r}$$

- tangential acceleration

$$a_T = R\alpha$$

- angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Rotational Kinematics

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

angular displacement, $\theta(t)$

angular velocity, $\omega(t)$

constant angular acceleration, α

$$\theta = \int \omega \cdot dt$$
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega = \int \alpha \cdot dt$$
$$\omega_f = \omega_i + \alpha t$$

Rotational Kinematics

Linear motion

$$x_f = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$x_f = x_0 + \frac{1}{2}(v_x + v_{0x})t$$

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

Rotational motion

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \frac{1}{2}(\omega_f + \omega_i)t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$$

Clicker Question 12-6

Which kinematics equation should you use?

A record initially spins at 78 revolutions per minute (rpm) in the counter-clockwise direction. This decreases to 45 rpm during a time period of 5.1 seconds (still in the counter-clockwise direction). What is the average angular acceleration (α) of the record during this time period?

A)
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

B)
$$\omega_f = \omega_i + \alpha t$$

C)
$$\theta_f = \theta_i + \frac{1}{2} (\omega_f + \omega_i) t$$

D)
$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

Clicker Question 12-7

Which kinematics equation should you use?

What is the angular velocity of the earth as it goes around the sun?

A)

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

B)

$$\omega_f = \omega_i + \alpha t$$

C)

$$\theta_f = \theta_i + \frac{1}{2} (\omega_f + \omega_i) t$$

D)

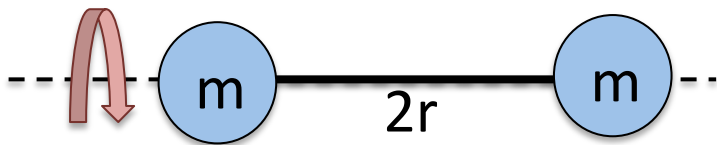
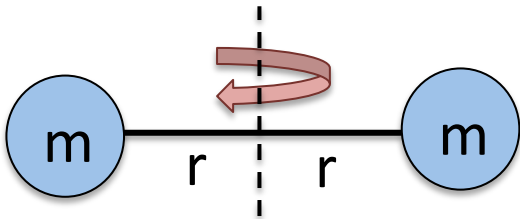
$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Moments of Inertia

- How mass is distributed
- In general:

$$I = \sum_i m_i r_i^2$$

- Changes when you consider different axes of rotation.

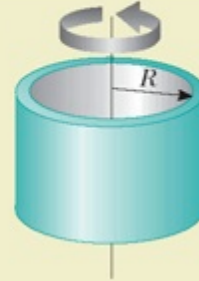


- Pre-calculated for common 3D objects →

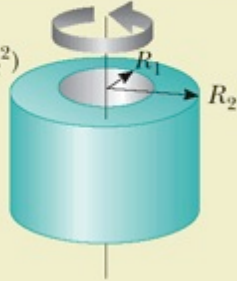
TABLE 10.2

Moments of Inertia of Homogeneous Rigid Objects With Different Geometries

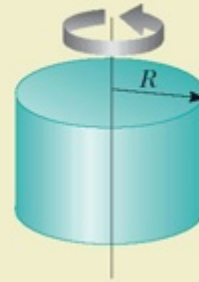
Hoop or thin cylindrical shell
 $I_{CM} = MR^2$



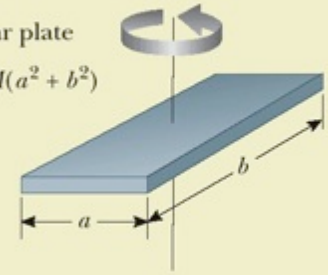
Hollow cylinder
 $I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$



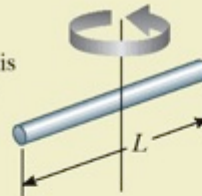
Solid cylinder or disk
 $I_{CM} = \frac{1}{2} MR^2$



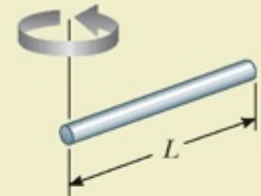
Rectangular plate
 $I_{CM} = \frac{1}{12} M(a^2 + b^2)$



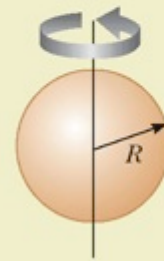
Long thin rod with rotation axis through center
 $I_{CM} = \frac{1}{12} ML^2$



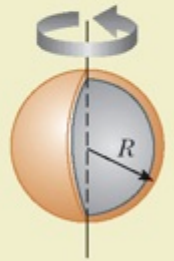
Long thin rod with rotation axis through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell
 $I_{CM} = \frac{2}{3} MR^2$



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