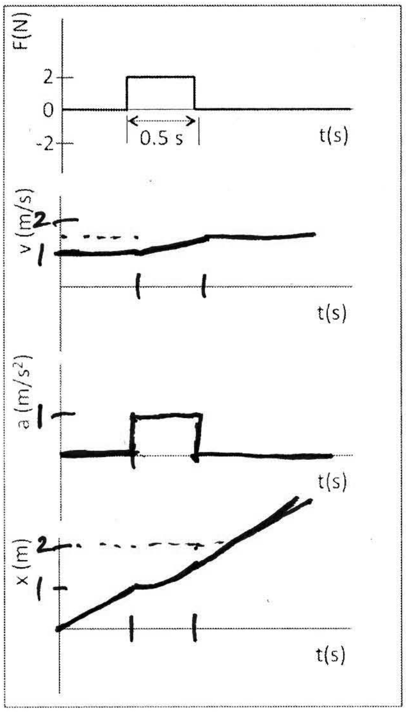
SOLUTIONS

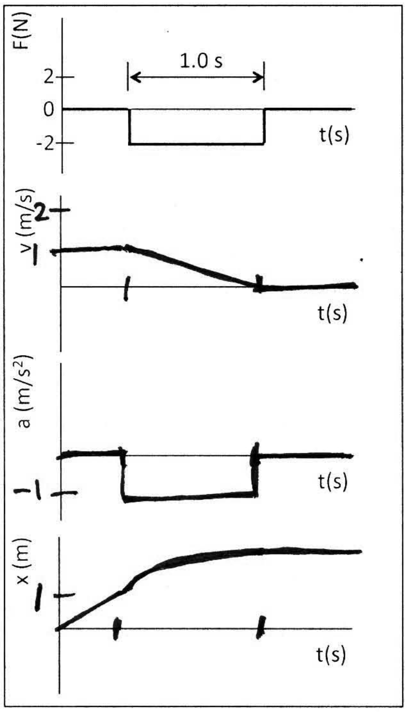
Name: ______________________ 3 Digit Code: __________

Worksheet 5: Momentum and Rot. Motion

1 Impulse

At t=0 a 2 kg object starts moves away from the origin to the right with a constant speed of 1 m/s when at t=0.5 s it experiences an impulse due the the force shown in the graph. Do your best to plot the object's velocity, acceleration, and position. Use the equations, $I = \Delta p = F\Delta t$.





2 Races

A) Blocks A and B, both initially at rest, are pushed to the right continuously by identical constant forces. Block B is more massive than Block A. Which block crosses the finish line with more momentum?

~~$\Delta P_A \geq \Delta P_B$ .. Since $m_B > m_A$, $\Delta v_A > \Delta v_B$~~ $\boxed{\Delta t_A < \Delta t_B, \text{ so } \Delta P_A < \Delta P_B}$

Block A will win because ~~they will have the same final momentum~~

$a_A > a_B$

B) Same situation as in A) except now A and B have equal mass, but A already has velocity when it crosses the starting line. Which block undergoes a larger *change* in momentum?

$a_A = a_B$ but now A will win because $V_{0A} > V_{0B}$. $\boxed{\Delta t_A < \Delta t_B, \text{ so } \Delta P_A < \Delta P_B}$

C) Same situation as in A) except now there isn't a finish line but the force is only applied for 1.0 s. Which block has more momentum after 1.0 s?

$\boxed{\Delta t_A = \Delta t_B \quad \text{So} \quad \Delta P_A = \Delta P_B}$

Elastic collisions

ne formulas for perfectly elastic collisions are the following:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

A 12.0 g bouncy ball is used to knock over a 100 g wood post in a carnival game. The ball hits e post with 20 m/s and the collision is perfectly elastic. What is the final velocity of the wood st?

$$v_{wood} = \frac{2\, m_{ball}}{(m_{ball} + m_{wood})} v_{ball} = \frac{2(0.012\text{ kg})}{(0.012\text{kg} + 0.1\text{kg})}(20\text{ m/s})$$

$$= (0.214)(20\text{ m/s})$$

$$= \boxed{4.28\text{ m/s}}$$

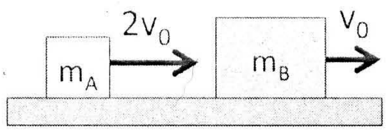
Mass A has has velocity $2\,v_0$ moving to the right and mass B, which is three times as mas- e, has velocity v_0 also to the right. Mass A starts out to the left of mass B, but catches up and llides with mass B elastically. What are the final velocities of mass A and mass B?

$m_A = m_A \qquad m_B = 3m_A$

$v_{0A} = 2v_0 \qquad v_{B_0} = v_0$

$$v_{Af} = \left(\frac{-2m_A}{4m_A}\right) v_{0A} + \left(\frac{6\, m_A}{4\, m_A}\right) v_{0B}$$

$$= -\frac{1}{2}(2v_0) + \frac{3}{2}v_0 = \boxed{\frac{1}{2}v_0}$$



$$v_{Bf} = \left(\frac{2m_A}{4m_A}\right) v_{0A} + \left(\frac{2\, m_A}{4\, m_A}\right) v_{0B}$$

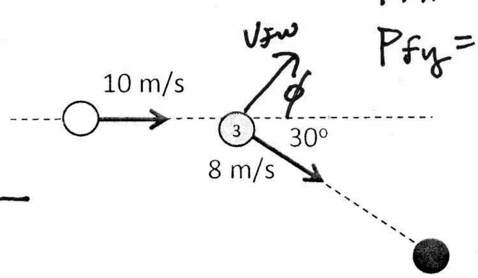
$$= \frac{1}{2}(2v_0) + \frac{1}{2}(v_0) = \boxed{\frac{3}{2}v_0}$$

Billiard balls are all 160 g. If you shoot the white ball at the number 3 ball (which is initially at st) with an initial velocity of 10 m/s so that the number 3 ball has a velocity of 8 m/s and goes o the corner pocket, what is the final speed and direction of the white ball? Assume the collision perfectly elastic.

$P_{ix} = m(10\text{ m/s}) \qquad P_{iy} = 0$

$P_{fx} = m(8\text{ m/s})\cos(30°) + P_{Wfx}$

$P_{fy} = m(8\text{ m/s})\sin(30°) + P_{Wfy}$



$P_{ix} = P_{fx}$

$M_W V_{Wx} = m(10\text{ m/s}) - m(6.93\text{ m/s})$

$W_{Wx} = 10 - 6.93 = \boxed{3.07\text{ m/s}}$

$P_{iy} = P_{fy}$

$M_W V_{Wy} = 0 - m(-4\text{ m/s})$

$V_{Wy} = \boxed{4\text{ m/s}}$

$$V_{Wf} = \sqrt{(3.07)^2 + (4)^2}$$

$$= \boxed{5.04\text{ m/s}}$$

$$\phi = \tan^{-1}\left(\frac{v_{Wy}}{v_{Wx}}\right) = \tan^{-1}\left(\frac{4}{3.07}\right) = \boxed{52.5°}$$

4 Inelastic collisions

A) A 12.0 g ball of clay is used to knock over a 100 g wood post in a carnival game. The ball hits the post with 20 m/s and the collision is perfectly inelastic. What is the final velocity of the wood post and clay?

$$m_{ball} v_{ball} = (m_{ball} + m_{wood}) v_f$$

$$v_f = \frac{0.012}{(0.012 + 0.1)} 20 \text{ m/s}$$

$$= \boxed{2.14 \text{ m/s}}$$

B) Mass A has has velocity 2 v_0 moving to the right and mass B, which is three times as massive, has velocity v_0 also to the right. Mass A starts out to the left of mass B, but catches up and collides with mass B inelastically. What is the final velocity of mass A and mass B?

$$m_A v_{A0} + m_B v_{B0} = (m_A + m_B) v_f$$

$$v_f = \frac{m_A (2v_0) + 3m_A (v_0)}{4 m_A} = \boxed{\frac{7}{4} v_0}$$

C) A 1500 kg car is rolling at 2.0 m/s. You would like to stop the car by firing a 10 kg blob of sticky clay at it. How fast should you fire the clay?

$$m_c v_c + m_{clay} v_{clay} = 0$$

$$v_{clay} = \frac{-m_c v_c}{m_{clay}} = \frac{-(1500 \text{ kg})(2.0 \text{ m/s})}{10 \text{ kg}} = \boxed{300 \text{ m/s}}$$

D) A 50 kg archer, standing on frictionless ice, shoots a 100 g arrow at a speed of 100 m/s. What is the recoil speed of the archer?

$$0 = m_A v_R + m_{Arrow} v_{Arrow}$$

$$v_R = -\frac{m_A}{m_{Arrow}} v_{Arrow} = -\frac{0.1 \text{ kg}}{50.0 \text{ kg}} 100 \text{ m/s} = \boxed{-0.2 \text{ m/s}}$$

E) Dan is gliding on his skateboard at 4 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5 kg.

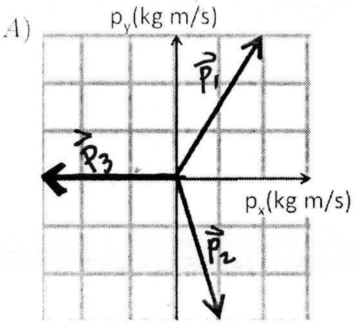
$$(m_D + m_S) 4 \text{ m/s} = m_D v_{Df} + m_S 8 \text{ m/s}$$

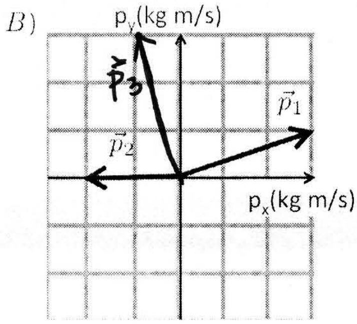
$$v_{Df} = \left(\frac{m_D + m_S}{m_D}\right) 4 \text{ m/s} - \left(\frac{m_S}{m_D}\right) 8 \text{ m/s}$$

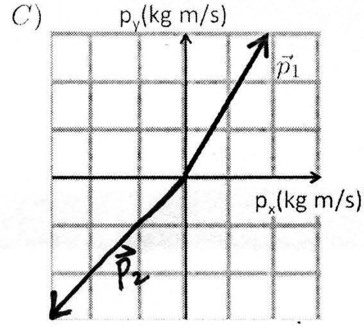
$$= \left(\frac{50 \text{ kg} + 5 \text{ kg}}{50 \text{ kg}}\right) 4 \text{ m/s} - \left(\frac{5 \text{ kg}}{50 \text{ kg}}\right) 8 \text{ m/s}$$

$$= 4.4 \text{ m/s} - 0.8 \text{ m/s} = \boxed{3.6 \text{ m/s}}$$

Draw the missing momentum vector for the description of the collision.







A) An object initially at rest explodes into three fragments. Draw $\vec{p}_3$.

An 2 kg object moving in the positive direction y direction with a velocity ~~x~~ 2 m/s explodes into 3 fragments. Draw $\vec{p}_3$.

C) The initial momentum of object 1 is shown. Draw the initial momentum of object 2 if the two collide inelastically end up with a final x momentum -1 kg m/s.

5 Rotational Kinematics

Determine the signs (+ or −) for ω and α.

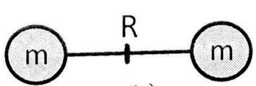
A) Counterclockwise, speeding up.
ω +
α +

B) Clockwise, speeding up.
ω −
α −

C) Counterclockwise, slowing down.
ω +
α −

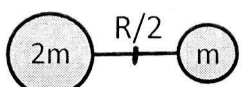
D) Clockwise, slowing down.
ω −
α +

6 Moment of Inertia



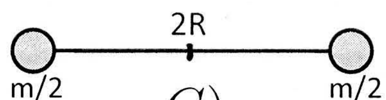
A)

$I_A = 2m(R/2)^2 = \frac{mR}{2}$



B)

$I_B = 2m(\frac{R}{\cancel{2}4})^2 + m(\frac{R}{4})^2$
$= mR^2/8 + mR^2/16 = \frac{3}{16}mR^2$



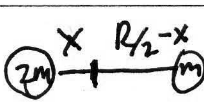
C)

$2(m/2)R^2 = mR^2$

Rank the moments of inertia I_A, I_B, I_C about the midpoint of each connecting rod: $\boxed{I_C > I_A > I_B}$

What is the moment of inertia I_B about its center of mass?

Find CoM :



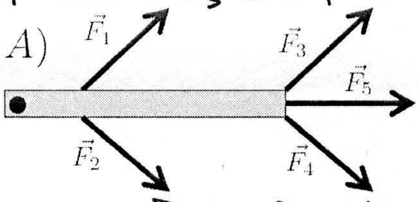
$2mx = m(\frac{R}{2} - x)$
$3mx = \frac{mR}{2}$
$x = R/6$

$I_{B,com} = 2m(\frac{R}{6})^2 + m(\frac{R}{3})^2 = \boxed{\frac{mR}{3}}$

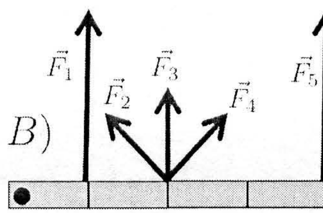
7 Torque

For each case, rank the torques from most negative to most positive.

τ_4 and τ_2 are negative, $\tau_5 = 0$
τ_1 and τ_3 are positive

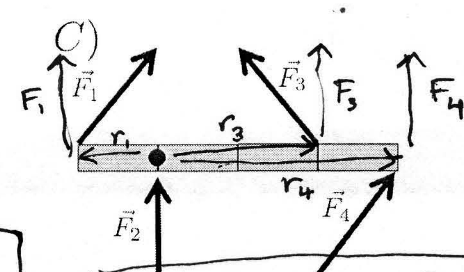


$\boxed{\vec{\tau}_4, \vec{\tau}_2, \vec{\tau}_5, \vec{\tau}_1, \vec{\tau}_3}$



$\boxed{\vec{\tau}_2 = \vec{\tau}_4, \ \vec{\tau}_1 = \vec{\tau}_3, \ \vec{\tau}_5}$

all are positive



$\boxed{\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3, \vec{\tau}_4}$

τ_1 is ~~positive~~ negative
τ_2 is zero, τ_3 and τ_4 are positive

A) Five forces with equal magnitude are applied to a door and we are looking at it with a birds-eye-view.

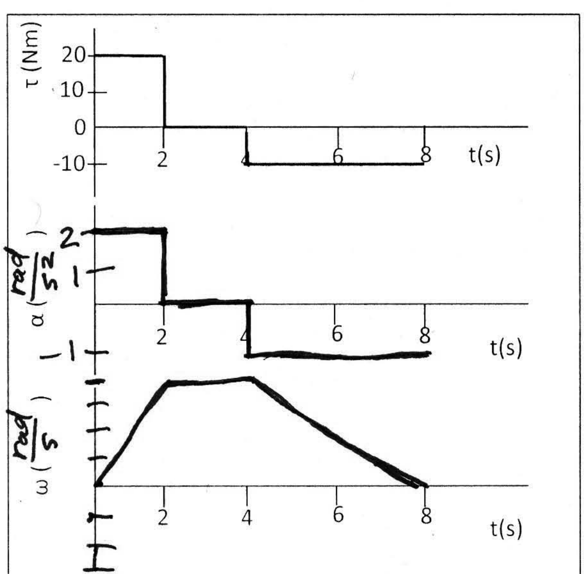
B) The door has been divided into four equal segments. $\vec{F}_1$ and $\vec{F}_2$ are twice as strong as the other three.

C) Here the pivot point has moved. The forces are all equal in magnitude.

The top graph shows the torque on a rotating wheel as a function of time. The wheel's moment of inertia is 10 kg m^2. Draw graphs of α vs t and ω vs t assuming $\omega_0 = 0$.

$\vec{\tau} = I\vec{\alpha}$

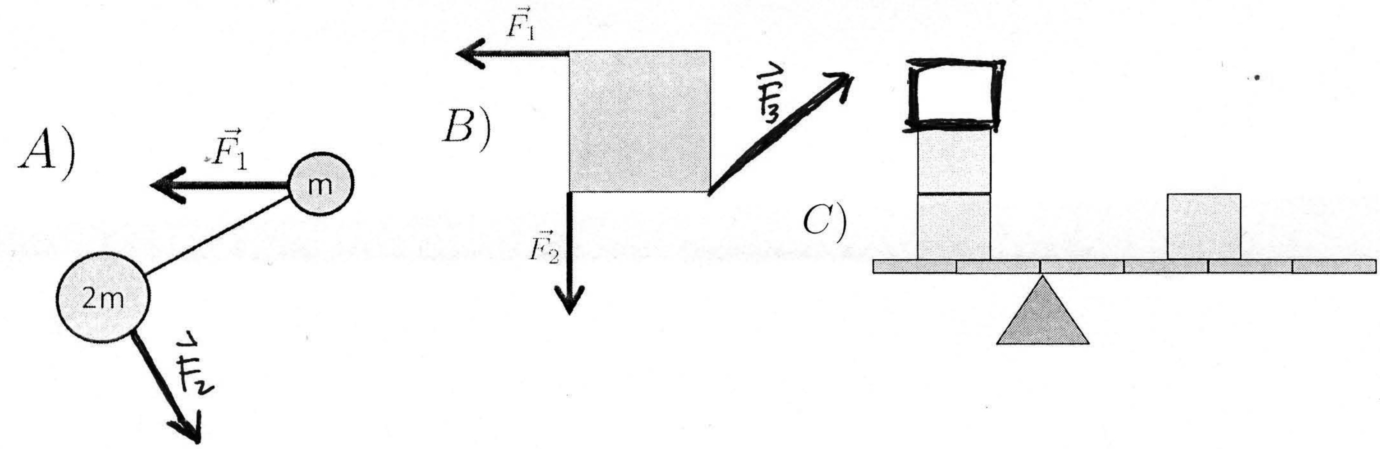
$\alpha = \frac{20}{10} = 2 \frac{rad}{s^2}$



$\Delta\omega = \alpha \Delta t$
$= (2 \frac{rad}{s^2})(2s)$
$= 4 \frac{rad}{s}$

$\Delta\omega = (-1 \frac{rad}{s^2})(4s)$
$= -4 \frac{rad}{s}$

8 Rotational equilibrium



A) Draw a force on the heavier mass such that the dumbbell will have translational motion but not rotational motion.

Forces $\vec{F}_1$ and $\vec{F}_2$ have the same magnitude. They are applied to the corners of the square plate shown. Draw and label a single force vector F_3 to create total static equilibrium.

C) The see-saw shown has the same mass as the boxes put on top of it. Add a single box to the see-saw so that it will be in equilibrium.

9 Angular momentum

A) A hoop of mass M and radius R is rotating with angular speed 60 rpm about its axis. What would be its angular speed if its mass suddenly doubled? What if its radius doubled without changing its mass?

$L = I\omega \qquad I = MR^2$

$$\left.\begin{array}{l} L_i = MR^2\omega_i \\ L_f = 2MR^2\omega_f \end{array}\right\} \boxed{\omega_f = \frac{\omega_i}{2}} \qquad \left.\begin{array}{l} L_i = MR^2\omega_i \\ L_f = M(2R)^2\omega_f \end{array}\right\} \boxed{\omega_f = \frac{\omega_i}{4}}$$

B) A disk of mass M and radius R is rotating with angular speed 60 rpm about its axis. A wad of clay of mass m is dropped on dropped on the outer radius. What is the new angular speed? How much energy has been lost?

$$L_i = \tfrac{1}{2}MR^2\omega_i \qquad L_f = \left(\tfrac{1}{2}MR^2 + mR^2\right)\omega_f \qquad \omega_f = \frac{\frac{1}{2}MR^2\omega_i}{\frac{1}{2}MR^2 + mR^2} = \boxed{\frac{M\omega_i}{M+2m}}$$

10 Conservation of Energy with Rotation

A solid cylinder and a solid sphere of equal mass and equal radius roll without slipping down a ramp. ~~Which will~~ have a larger final velocity at the bottom of the ramp?

Equations

$I_c = \frac{1}{2}MR^2$

$I_s = \frac{2}{5}MR^2$

$\omega = \frac{V}{r}$

$$E_{ic} = E_{is}$$

$$E_{fc} = \tfrac{1}{2}I_c\omega_c^2 + \tfrac{1}{2}Mv_{cf}^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\tfrac{V_{cf}}{R}\right)^2 + \tfrac{1}{2}Mv_{cf}^2 = \tfrac{3}{4}MV_{cf}^2$$

$$E_{fs} = \tfrac{1}{2}I_s\omega_s^2 + \tfrac{1}{2}Mv_{sf}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\left(\tfrac{V_{sf}}{R}\right)^2 + \tfrac{1}{2}MV_{sf}^2 = \tfrac{7}{10}MV_{sf}^2$$

$$V_{cf}^2 = \frac{4E_{ci}}{3M}, \quad V_{sf}^2 = \frac{10E_{si}}{7M} \Rightarrow$$ Sphere has higher final velocity

11 Fill out your course evaluation online!