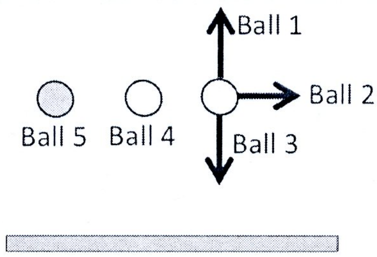
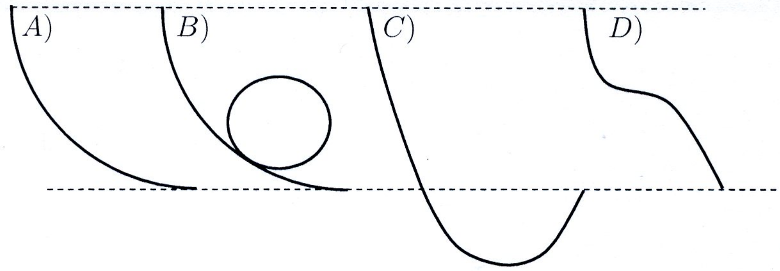
SOLUTIONS

Name: ______________________ 3 Digit Code: __________

Worksheet 4: Energy

1 Mechanical Energy





(*left*) Three balls are fired simultaneously with *equal* speeds from the same height above the ground. Ball 1 is fired straight up, Ball 2 is fired horizontally, and Ball 3 is fired straight down. Ball 4 and Ball 5 are released from same height as the others but are dropped from rest. All the balls have the same mass except Ball 5 which is much heavier than the others. Rank in order, from largest to smallest, their speeds v_1, v_2, v_3, v_4, and v_5 as they hit the ground. Next rank the time that they are in the air, T_1, T_2, T_3, T_4, and T_5. Use Kinematics and Energy conservation to make your argument.

$\boxed{v_1 = v_2 = v_3 > v_4 = v_5}$

$\boxed{T_1 > T_2 = T_4 = T_5 > T_3}$

- Balls 1, 2, 3 have initial $KE = \frac{1}{2}mv^2$ that is the same. Direction of v_0 does not change this
- Only v_{0y} determines time in the air, horizontal velocity and mass do not matter
- Balls 4 and 5 will land with the same velocity, because mass cancels $\boxed{\cancel{m}gh = \frac{1}{2}\cancel{m}v^2}$

(*right*) Four balls of equal mass are released from rest at the top of four frictionless ramps, each with different curvature but the same starting and ending heights. Ramp A is the most direct, Ramp B is a loop-the-loop, Ramp C goes down and then comes up, and Ramp D levels off a bit before coming down. Rank in order, from largest to smallest, their speeds v_A, v_B, v_C and v_D as they come out of each track. Next rank the time that they are on the track, T_A, T_C, and T_D (don't worry about T_B). Use Kinematics and Energy conservation to make your argument.

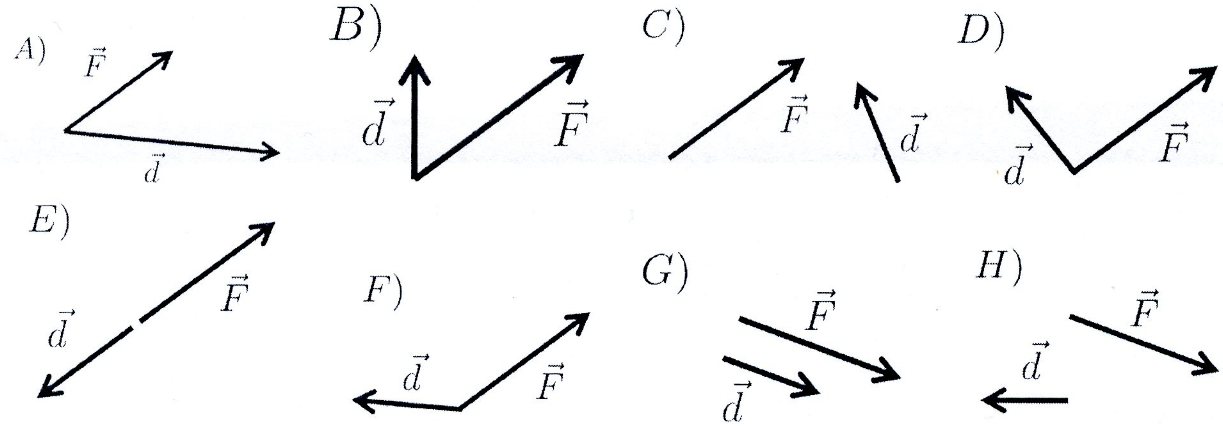
$\boxed{v_A = v_B = v_C = v_D}$

$\boxed{T_D > T_A > T_C}$

- Path does not matter because there is no friction, only Δh determines v_f because they are all dropped from rest
- Path C gains additional KE by going lower, thus gaining a speed advantage in the middle. For the same reason, Path D should be slowest.

2 Work

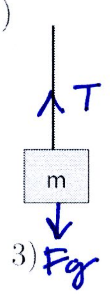
An object experiences a force while undergoing the displacement shown. Is the work done positive (+), negative (-), or zero (0)?



A) + B) + C) + D) 0 E) − F) − G) + H) −

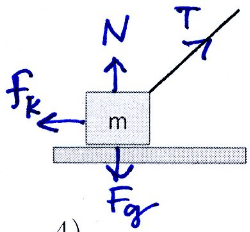
Draw a free body diagram along with a vector for the displacement. List all the forces in the table provided along with the sign of the work done by that force on the object.

1)



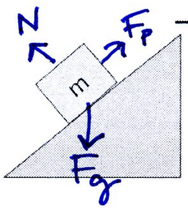
Force	Sign of work
T	+
Fg	−

2)



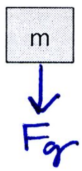
Force	Sign of work
T	+
N	0
fk	−
Fg	0

3)



Force	Sign of work
Fp	+
N	0
Fg	−

4)

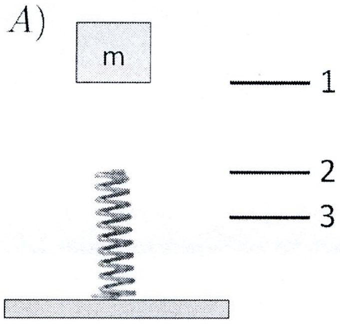


Force	Sign of work
Fg	+

1) A rope pulls a box up.
2) You pull a box with a rope at a 45° angle with the horizontal across a floor with friction.
3) You push a box up a frictionless ramp.
4) You throw a ball in the air and it's on its way up.

3 Energy Account Book

A)



B)

A) A heavy object is released from rest at position 1 above a spring. It falls and contacts the spring at position 2. The spring achieves maximum compression at position 3. Fill in the table below to indicate whether each of the quantities are +, −, or 0. Pay careful attention to signs.

	$1 \to 2$	$2 \to 3$	$1 \to 3$
change in kinetic energy (of mass + spring), ΔKE	+	−	0
change in gravitational potential energy (of mass + spring), ΔPE_g	−	−	−
change in spring potential energy (of mass + spring), ΔPE_s	0	+	+
work done by gravity on mass, W_g	+	+	+
work done by spring on mass, W_s	0	−	−

B) A weightlifter lifts a 100 kg dumbbell to a height of 2 m in 3 seconds. Then he holds it in place for 10 seconds. This time plug in actual values in the units desired.

	from ground to 2 m	holding in place
change in dumbbell kinetic energy ΔKE [J]	0	0
change in dumbbell potential energy, ΔPE_g[J]	1960 J	0
work done by gravity on dumbbell, W_g [J]	−1960 J	0
work done by man on dumbbell, W_s [J]	+1960 J	0
Power exerted by man, P [W]	653 W	0

$$mgh = (100\,kg)(9.8\,m/s^2)(2\,m) = 1960\,J$$

$$P = \frac{\Delta E}{\Delta t} = \frac{1960\,J}{3\,s} = 653\,W$$

4 Manipulating equations

For each of these situations you are either comparing between two scenarios of the same measurements (i.e. m_A vs. m_B) or looking at changes over time for the same scenario (i.e. $\Delta x = x_f - x_i$). Solve for the algebraic expression that is asked, but also describe the physical situation that is being considered.

Example) $\boxed{F_s = -k \cdot \Delta x}$ → If $k_A = k_B$ and $\Delta x_A = 2 \cdot \Delta x_B$, what is F_{sA}/F_{sB}?

When the same spring is extended twice as much, the force required is doubled.
→ $F_{sA}/F_{sB} = 2$

A) $\boxed{KE = \frac{1}{2}mv^2}$ → If $m_A = m_B/2$ and $KE_A = 8 \cdot KE_B$, what is v_A/v_B? $v = \sqrt{\frac{2\,KE}{m}}$

$\boxed{\frac{v_A}{v_B} = 4} \leftarrow \frac{v_A}{v_B} = \sqrt{\frac{2(8KE_B)}{m_B/2}} \cdot \sqrt{\frac{m_B}{2\,KE_B}}$

If mass A is half of mass B but it has 8 times the Kinetic Energy, the velocity of A is 4 times that of B

B) $\boxed{PE_s = \frac{1}{2}kx^2}$ → If $k_A = 2 \cdot k_B$ and $\Delta x_A = \Delta x_B/2$, what is $\Delta PE_{sA}/\Delta PE_{sB}$?

$\frac{\Delta PE_{sA}}{\Delta PE_{sB}} = \frac{1}{2}(2k_B)\left(\frac{\Delta x_B}{2}\right)^2 \Big/ \frac{1}{2}k_B(\Delta x_B)^2 = \boxed{\frac{1}{2}}$

If spring constant A is twice that of spring constant B but it is extended half as much, it will have half the stored potential energy

C) $\boxed{PE_g = mgh}$ → If $\Delta h_A = \Delta h_B$ and $\Delta PE_{gA} = 10 \cdot \Delta PE_{gB}$, what is m_A/m_B?

$m = \frac{PE}{gh}$; $\frac{m_A}{} = \frac{\Delta PE_A}{g\Delta h_A} = \frac{10\Delta PE_B}{g\Delta h_B} = \boxed{10}\, m_B$

If two masses are the same height but one has 10 times the gravitational PE, the mass must be 10 times larger

D) $\boxed{E_{tot} = KE + PE_g}$ → If $\Delta E_{tot} = 0$, $m_i = m_f$, and $\Delta h = h_f - h_i$, What is ΔKE?

$\Delta E_{tot} = \Delta KE + \Delta PE = 0$

$\boxed{\Delta KE = -\Delta PE = -mg\Delta h}$

E) $\boxed{W = F\Delta x}$ → If $\Delta x_A = 5 \cdot \Delta x_B$, and $W_A = W_B$, What is F$_A$/F$_B$?

$F = \frac{W}{\Delta x}$, $\frac{F_A}{F_B} = \frac{W_A}{\Delta x_A} \cdot \frac{\Delta x_B}{W_B} = \frac{\Delta x_B}{5\Delta x_B} = \boxed{\frac{1}{5}}$

$E_{fA} = 2E_{iA}$ and $E_{fB} = 4E_{iB}$

F) $\boxed{P = \frac{\Delta E}{\Delta t}}$ → If $E_{iA} = E_{iB}$, $\Delta t_A = \Delta t_B$ and ~~$E_{fA} = E_{fB}/4$~~, what is P_A/P_B?

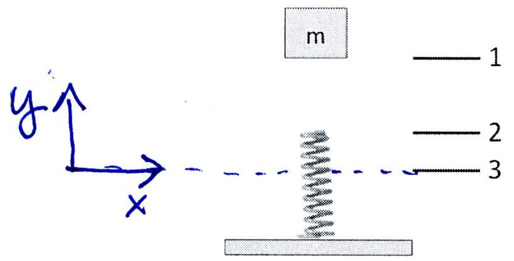
$\Delta E_A = E_{fA} - E_{iA} = E_{iA}$

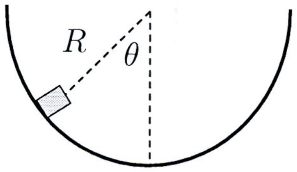
$\Delta E_B = E_{fB} - E_{iB} = 3E_{iB}$ $\frac{P_A}{P_B} = \frac{E_{iA}/\Delta t_A}{3E_{iB}/\Delta t_B} = \boxed{\frac{1}{3}}$

5 Solving Problems with Conservation of Energy

For each problem follow the following steps:

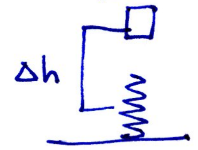
1. Pick a coordinate system with an origin (this is up to you to decide).
2. Draw separate pictures for the initial and final condition.
3. List the Knowns and Unknowns. Work out any important geometry in the problem
4. Write out the total energy at the initial and final position.
5. Solve algebra.





(*left*) A) You drop a mass of 2 kg from rest that is 5 m above a spring (position 1 → 2). The spring compresses 2 m (position 2 → 3). What is the spring constant of the spring?

initial



known: $\Delta h = 7m$

final

known: $\Delta y = 2m$

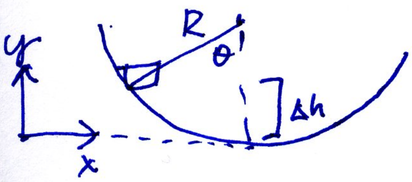
$E_i = mgh = (2\,kg)(9.8\,m/s^2)(7m) = 137\,J$

$E_f = \frac{1}{2}k(\Delta y)^2 = k\cdot(2m)^2\frac{1}{2} = 2m^2 k$

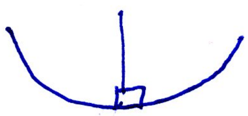
$E_i = E_f \Rightarrow k = \frac{137\,J}{2\,m^2} = \boxed{69\,\frac{kg}{s^2}}$

(*right*) B) A box of mass m slides around on a frictionless, hemispherical bowl with radius R. Suppose the cube is release at an angle θ. What is the box's speed at the bottom of the bowl?

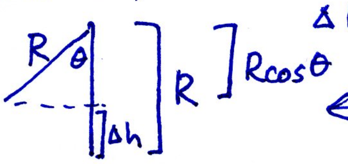
initial



final



known: R, θ ⇒ must find Δh:



$\leftarrow \Delta h = R - R\cos\theta$

$E_i = mgh = mgR(1-\cos\theta)$

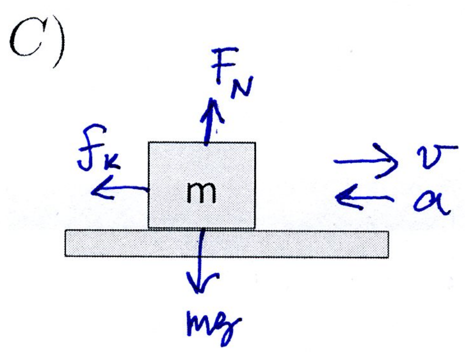
$E_f = \frac{1}{2}mv^2$

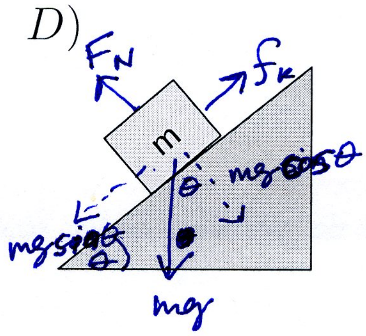
$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gR(1-\cos\theta)$

$v^2 = 2gR(1-\cos\theta)$

$\boxed{v = \sqrt{2gR(1-\cos\theta)}}$

[*Try solving C) and D) by using the Work/Energy theorem. You might want to check that you can also solve them using Newton's second Law and Kinematics equations.*]





C) A 10 kg box with an initial velocity of 5 m/s slides across a table with coefficient of kinetic friction 0.3. How much distance will it travel before it comes to a rest?

Work/Energy

$E_i = \frac{1}{2}mv^2 \qquad E_f = 0$

$W = f_k \Delta x$

$f_k = \mu_k F_N = \mu_k mg = (0.3)(10\text{kg})(9.8\,\text{m/s}^2)$

$= 29.4\text{ N}$

$E_i = W$

$\Delta x = \dfrac{\frac{1}{2}mv^2}{f_k} = \dfrac{\frac{1}{2}(10\text{kg})(5\,\text{m/s})^2}{29.4\text{N}}$

$\boxed{\Delta x = 4.3\text{ m}}$

Newton 2^{nd}/Kinematics

$f_k = 29.4\text{N}$

$a = \dfrac{29.4\text{N}}{10\text{kg}} = 2.94\,\text{m/s}^2$

$v_f^2 = v_i^2 + 2a\Delta x$ ($v_f = 0$)

$\Delta x = \dfrac{(5\,\text{m/s})^2}{2(2.94\,\text{m/s}^2)}$

$= 4.3\text{m}$ ✓

D) A 100 kg box is at a height of 5 m. It slides down a ramp that is at an incline of 50°. The coefficient of kinetic friction is 0.50. What is the final velocity of the box when it reaches the bottom of the ramp?

(see solutions to last week's quiz for N 2^{nd}/Kinematics solution)

Work/Energy

$KE_i = 0 \qquad PE_i = mgh$

$KE_f = \frac{1}{2}mv_f^2 \qquad PE_f = 0$

$W = f_k \Delta x$

$f_k = \mu_k F_N = \mu_k mg\cos\theta$

$\Delta x = \dfrac{h}{\sin\theta}$

$W = (\mu_k mg\cos\theta)\left(\dfrac{h}{\sin\theta}\right)$

$E_i = E_f + W$

$mgh = \frac{1}{2}mv_f^2 + (\mu_k mg\cos\theta)\left(\dfrac{h}{\sin\theta}\right)$

$\Rightarrow v_f^2 = 2gh\left(1 - \mu\dfrac{\cos\theta}{\sin\theta}\right) \Rightarrow v_f = \sqrt{2(9.8)(5)\left(1 - 0.5\left(\dfrac{0.64}{0.77}\right)\right)}$

$= \boxed{7.56\,\text{m/s}}$