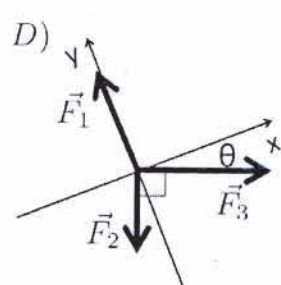# SOLUTIONS

Name: _____

Worksheet 3: Forces

1 Free Body Diagrams

1.1 Newton's Second Law

A)   B)   C)   D)
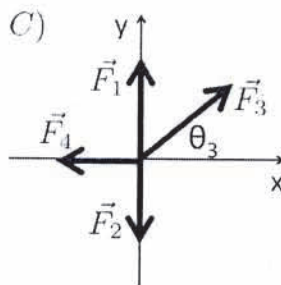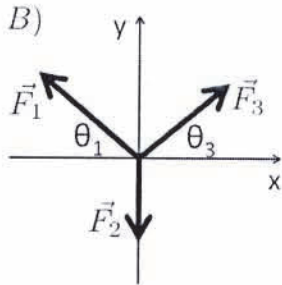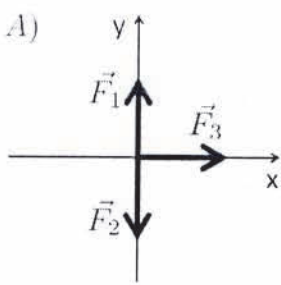
Sum up the forces in the x and y directions:

A)

$$ma_x = F_3$$

$$ma_y = F_1 - F_2$$

B)

$$ma_x = F_3\cos\theta_3 - F_1\cos\theta_1$$

$$ma_y = F_3\sin\theta_3 + F_1\sin\theta_1 - F_2$$

C)

$$ma_x = F_3\cos\theta_3 - F_4$$

$$ma_y = F_1 + F_3\sin\theta_3 - F_2$$

D)

$$ma_x = F_3\cos\theta - F_2\sin\theta$$

$$ma_y = F_1 - F_3\sin\theta - F_2\cos\theta$$

E)   F)   G)
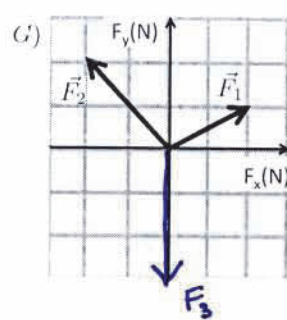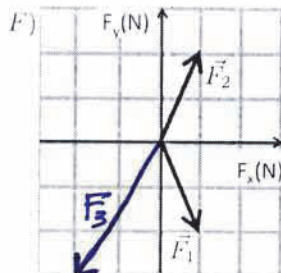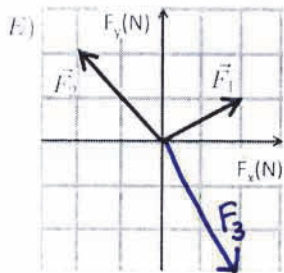
Three forces act on a 1 kg object to give it the following acceleration. Note that the grid has units of 1 N. Draw in the missing vector ($\vec{F}_3$).

E)

$a_x = 2 \text{ m/s}^2$ $F_{3x} = 2N$

$a_y = 0 \text{ m/s}^2$ $F_{3y} = -3N$

F)

$a_x = 0 \text{ m/s}^2$ $F_{3x} = -2N$

$a_y = -3 \text{ m/s}^2$ $F_{3y} = -3N$

G)

The object moves with constant velocity.

$F_{3x} = 0$

$F_{3y} = -3N$

1.2 Components of forces on an inclined plane.

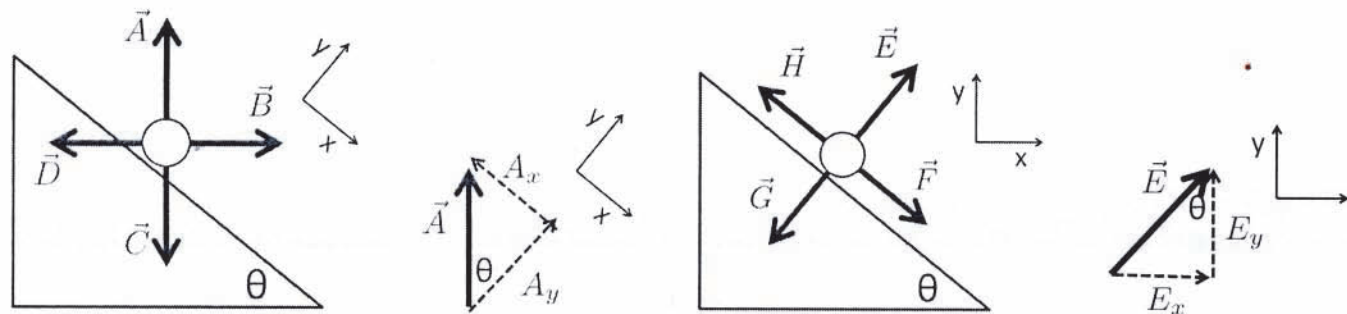For each vector, sketch the direction of its components in the given coordinate system and write out their magnitudes. $\vec{A}$ and $\vec{E}$ are worked out as examples for you. The hardest part is identifying where θ fits in the right triangle where you break up the vector into components.

A) $A_x = -|A| \sin \theta, \qquad A_y = |A| \cos \theta$

E) $E_x = |E| \sin \theta, \qquad E_y = |E| \cos \theta$

B)
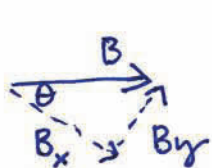
$B_x = |B| \cos \theta$

$B_y = |B| \sin \theta$

F)

$F_x = |F| \cos \theta$

$F_y = -|F| \sin \theta$

C)

$C_x = |C| \sin \theta$

$C_y = -|C| \cos \theta$

G)

$G_x = -|G| \sin \theta$

$G_y = -|G| \cos \theta$

D)

$D_x = -|D| \cos \theta$

$D_y = -|D| \sin \theta$

H)

$H_x = -|H| \cos \theta$

$H_y = |H| \sin \theta$

1.3 Identifying forces on an object
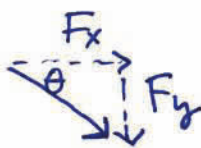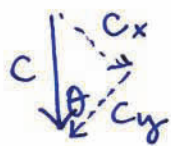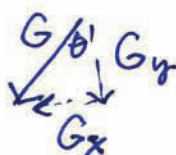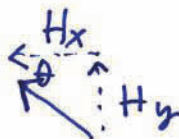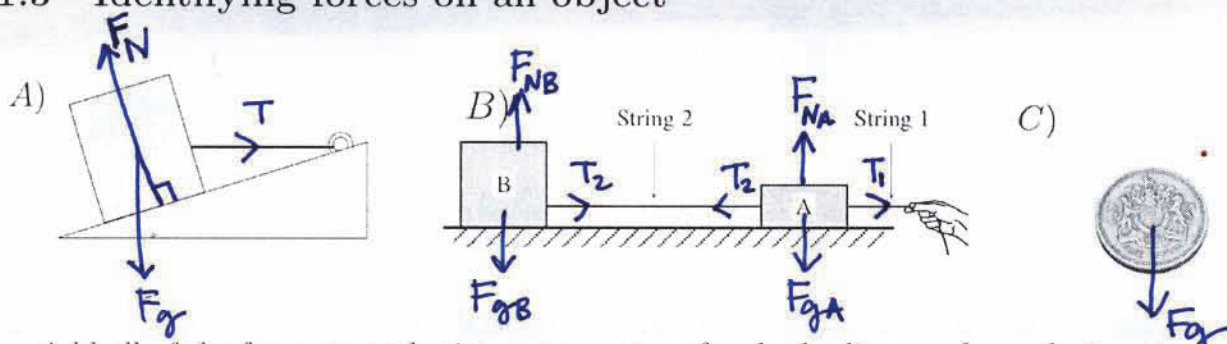
A)

B) String 2   String 1   C)

Add all of the forces to each picture to create a free body diagram for each situation.

A) A block is held in place on a frictionless incline by a massless string, as shown. The string is horizontal.

B) Two blocks are connected by a string. Block A is pulled by another string so that it moves across a table with increasing speed. There is no friction

C) A coin is flicked up in the air and is on its way up. Air resistance is negligible.

2 Elevator Problems
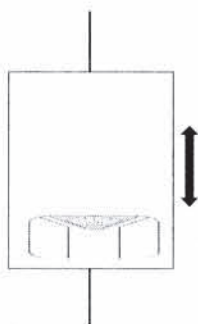
— In each case

$N - mg = ma$

— Direction of $\vec{a}$ tells you whether $N > mg$, $N < mg$, or $N = mg$

You are in an elevator with a scale in it which is on the surface of the planet Earth. For each situation, decide whether or not the scale reads your weight, something greater than your weight, or something less than your weight. [Hint: The scale will read the same as the normal force you feel from the scale.]

A) You are ascending and speeding up.

$a \uparrow$ greater than

B) You are ascending and slowing down.

$a \downarrow$ less than

C) You are descending and speeding up.

$a \downarrow$ less than

D) You are descending and slowing down.

$a \uparrow$ greater than

E) You are ascending at a constant speed.

$a = 0$ Same weight

F) You are descending with an acceleration equal to 9.8 m/s².

less than (actually the scale will read 0)

3 Friction

3.1 Races

Top view

$\vec{F}_0$ → A

Frictionless table

$\vec{F}_0$ → B

First mark
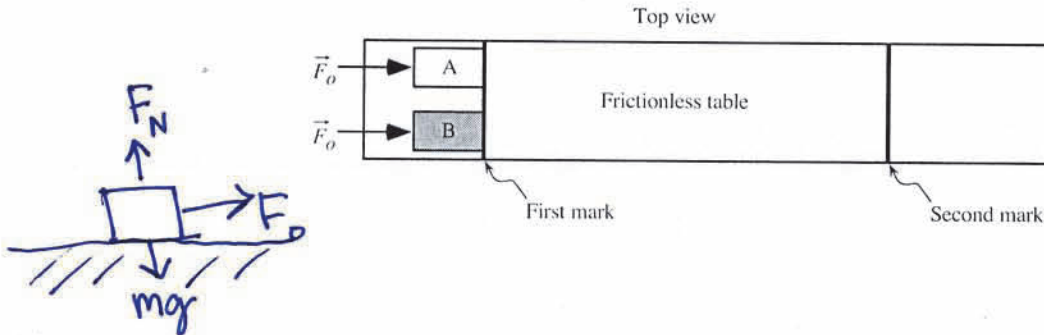
Second mark

A) Masses A and and B are on a frictionless track and given equal forces $\vec{F}_0$ that are constantly applied for the duration of the motion. Mass A is half the mass of mass B such that $m_A = 2 \cdot m_B$. The distance between the first and second mark is D. Which mass will reach the second mark first? [*Challenge:* By how much time will it beat the other mass?]

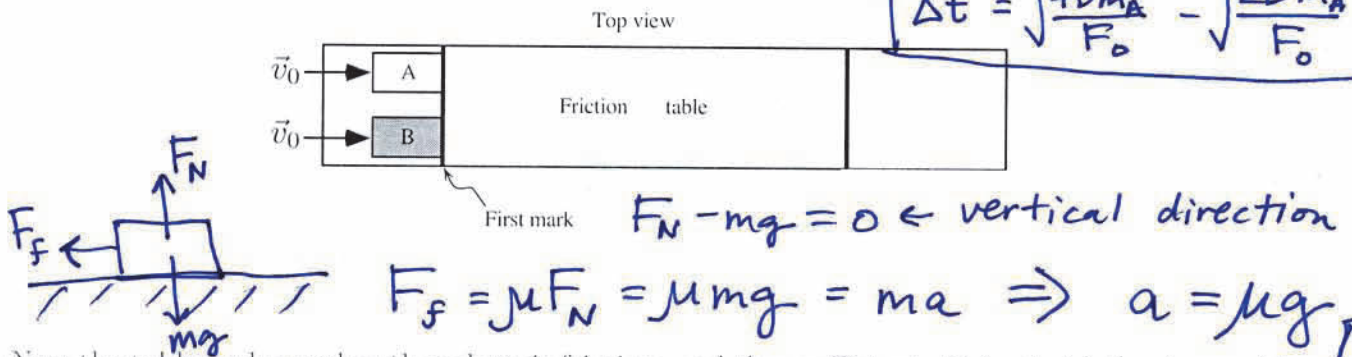mass A : $\quad F_0 = m_A a_A \quad$ if $\quad m_A = m_B/2$ $\qquad \boxed{\text{mass A will win}}$

mass B : $\quad F_0 = m_B a_B$

$$a_A = \frac{F_0}{m_A} > \frac{F_0}{m_B} = a_B$$

Challenge : $D = \frac{1}{2}at^2$, $\quad t = \sqrt{\frac{2D}{a}}$, $\quad t_A = \sqrt{\frac{2Dm_A}{F_0}}$, $\quad t_B = \sqrt{\frac{4Dm_A}{F_0}}$

$$\boxed{\Delta t = \sqrt{\frac{4Dm_A}{F_0}} - \sqrt{\frac{2Dm_A}{F_0}}}$$

Top view

$\vec{v}_0$ → A

Friction   table

$\vec{v}_0$ → B

First mark

$F_N - mg = 0 \leftarrow$ vertical direction

$$F_f = \mu F_N = \mu mg = ma \implies a = \mu g$$

B) Now the table is changed so that there is friction, and the coefficient of kinetic friction is μ_k. Also this time, a force is only applied before the blocks cross the first mark, so that the initial velocity that is given the blocks is the same and equal to v_0. Which block will stop first? [*Challenge:* How far apart will the blocks be when they stop moving?]

~~The force of friction is larger on mass B so it will stop first.~~

~~...~~

The acceleration due to kinetic friction is independent of mass → they stop at the same place.

3.2 Free body diagrams

A)

F_S F_P

F_N ← m

F_g

B)

F_P

F_N ← m

F_S

F_g

C)

F_N

F_F

mg

Force of friction in the direction of motion for rolling without slipping
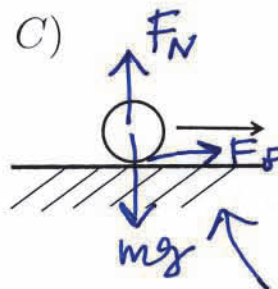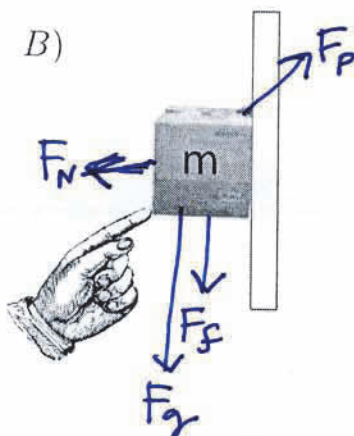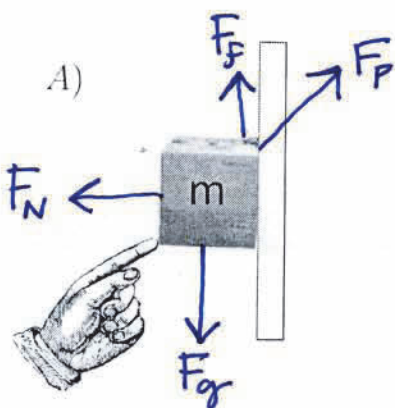
Add all of the forces to each picture to create a free body diagram for each situation.

A) A box is held in place on a vertical wall by the force of a hand. There is friction between the box and the wall. The hand exerts just enough force so that the box does not slide *down* the wall

B) Here the hand exerts just enough force so that if the push was greater the box would slide *up* the wall.

C) A tire rolls to the right on a horizontal surface

The force of friction points opposite the direction of the motion that would happen if there was no friction

4 Newton's Third Law

4.1 Scenarios

Draw a free body diagram for each situation and answer the question posed.

A) A heavier and a lighter ice skater, Bob and Alice, stand facing each other. Putting his hands on Alices shoulders, Bob pushes off so that the two ice sckaters end up moving in opposite directions. Which force, if either, do you think is greater during the push-off: The force exerted by the Bob on Alice, or the force exerted by Alice on Bob? Who will move away faster?

F_{AB} ← B   A → F_{BA}

The forces are the same but Alice will move faster because $a = \frac{F}{m}$

B) A box of mass m is placed on the back of a truck that is moving in the positive x direction with an acceleration $\vec{a}$. The box does not move relative to the truck. If the coefficient of static friction is μ_s, how much can the truck accelerate before the box starts to move?
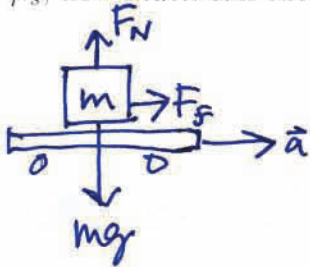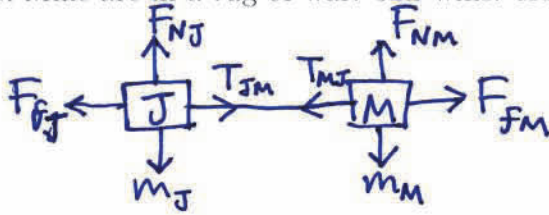
$$F_N - mg = 0$$

$$F_{f_s} = \mu_s F_N = \mu_s mg \geq ma \quad \leftarrow \text{condition for not moving}$$

$$\boxed{a > \mu_s g \quad \text{for movement}}$$

C) Jeff and Mike are in a tug-of-war. Jeff wins. How do the forces on Jeff compare to the forces on Mike?

$$T_{JM} = T_{MJ}$$

the tension in the rope is the same

$$F_{f_J} > F_{f_M}$$

the force of friction on Jeff must be bigger than the force of friction on Mike

4.2 Free body diagrams
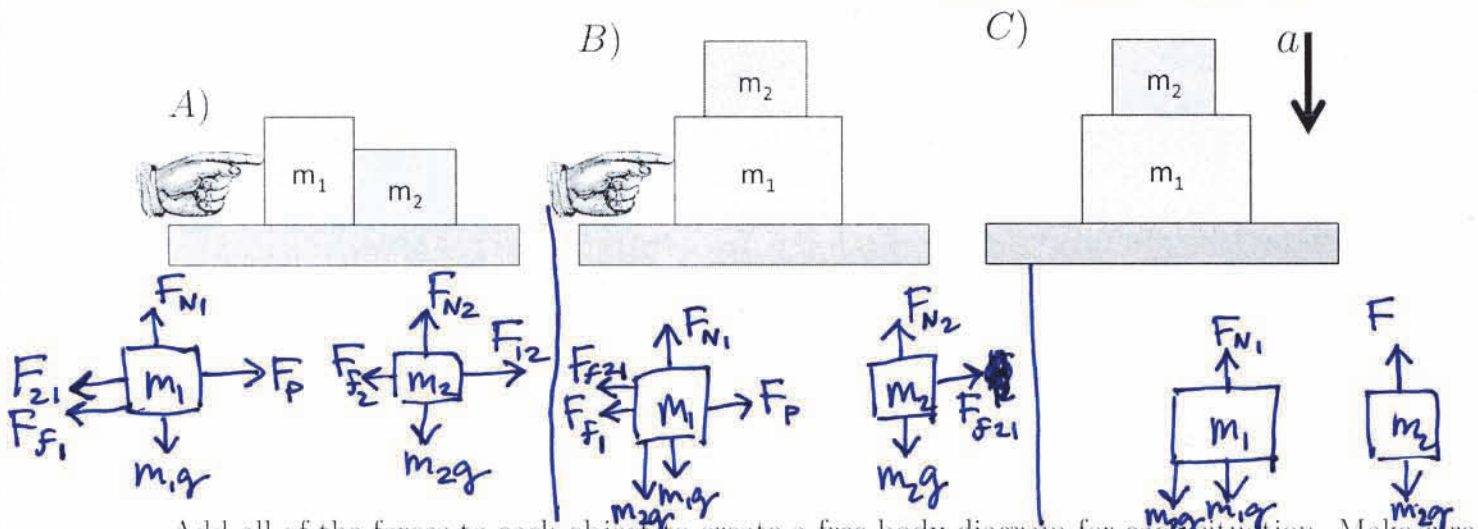
Add all of the forces to each object to create a free body diagram for each situation. Make sure you specify for each force who is acting on whom. You should redraw the objects separately.

A) A hand pushes a mass m_1 that pushes a mass m_2 over surface with friction.

B) A hand pushes a mass m_1 and there is friction between the two masses and between the mass and the surface.

C) Mass m_1 and m_2 are in an elevator that is accelerating downwards

5 Ropes and Pulleys

A)

$T = 3 \cdot 2.45$
$+ 3 \cdot 9.8$
$T = 37N$

$m_1 g - T = m_1 a$

$m_1 g$ $m_2 g$ $T - m_2 g = m_2 a$

$m_1 g + m_2 g = m_1 a + m_2 a$

$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$

$a = \frac{9.8 \; m/s^2}{4} = 2.45 \; m/s^2$

B)

$a = \frac{[m_1 g \sin\theta - m_2] g}{[m_1 g \sin\theta + m_2]}$

$T = m_2 (g + a)$
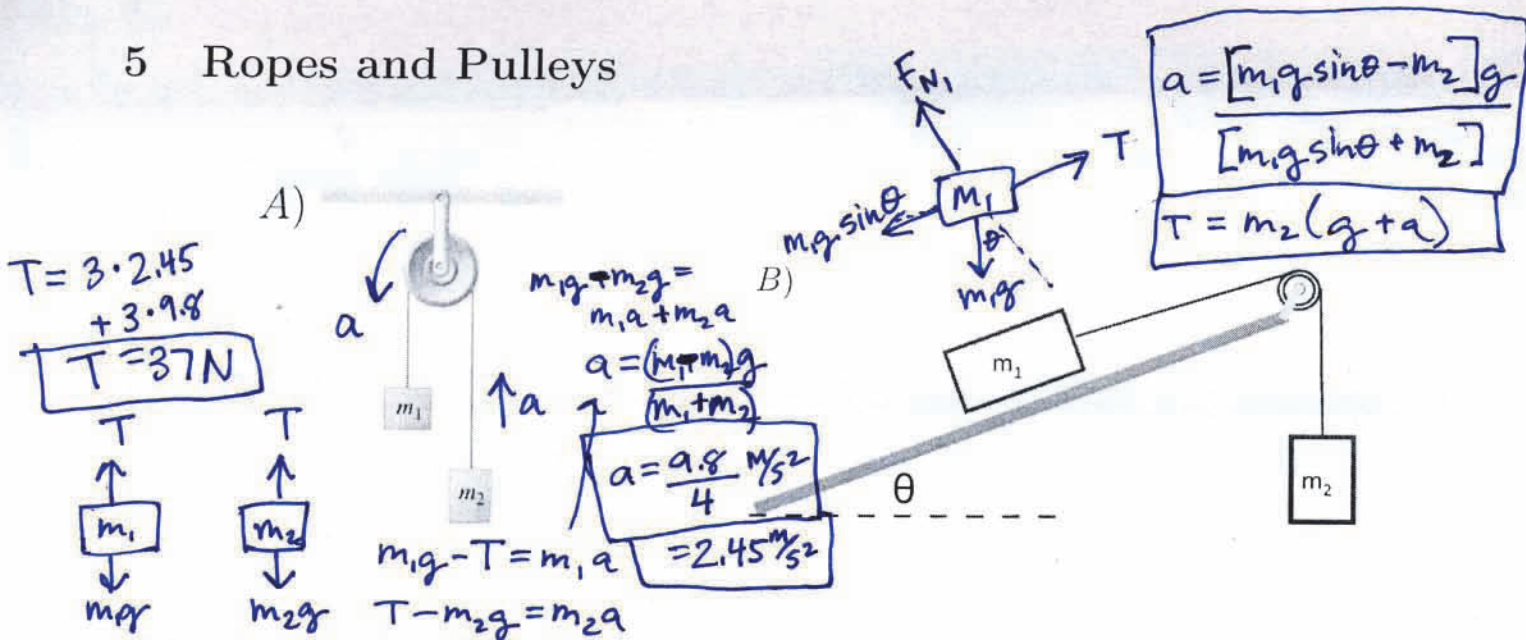
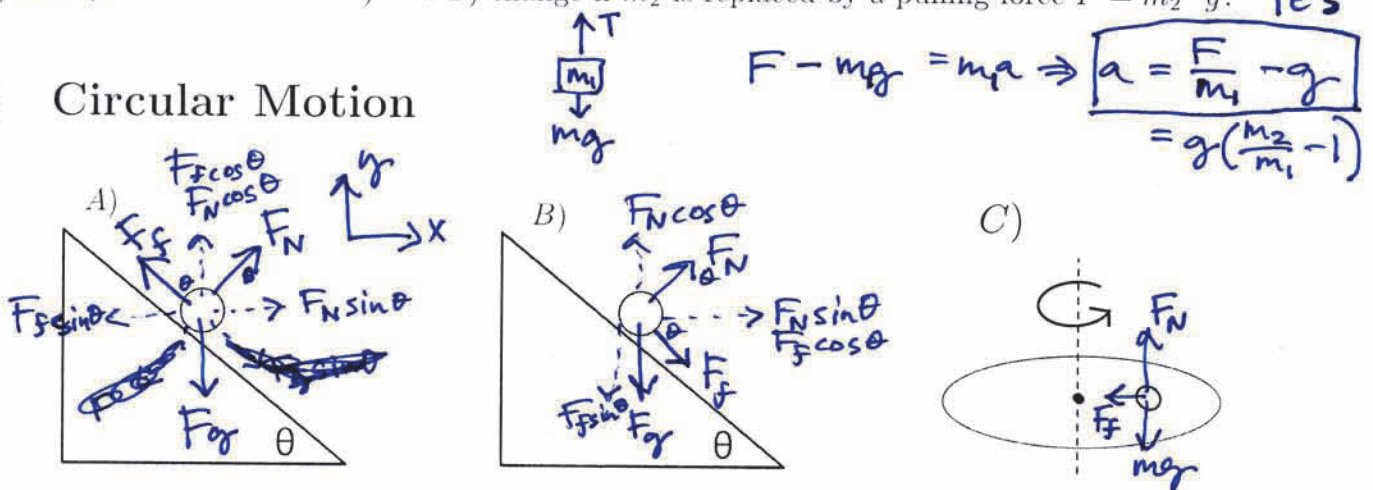A) For the Atwood machine shown, $m_1 = 5$ kg and $m_2 = 3$ kg. Will the boxes accelerate? If so in what directions? Draw a separate free-body diagram for each box. Be sure the vector lengths indicate the relative sizes of the forces. How fast do the boxes accelerate? What is the tension in the rope?

B) Repeat the same questions for the Atwood machine on an incline. Ignore friction.

C) Do the answers in A) and B) change if m_2 is replaced by a pulling force $F = m_2 \cdot g$? **Yes**

$F - mg = m_1 a \Rightarrow a = \frac{F}{m_1} - g$

$= g\left(\frac{m_2}{m_1} - 1\right)$

6 Circular Motion

A)

$F_F \cos\theta$
$F_N \cos\theta$
$F_f \sin\theta$
$F_N \sin\theta$
F_g
θ

B)

$F_N \cos\theta$
$F_N \sin\theta$
$F_f \cos\theta$
$F_f \sin\theta$ F_g
θ

C)

$r = 10 cm$

F_N
F_F
mg

Draw a free body diagram and write out Newton's second law.

A) A car drives around a banked curve of radius R with friction coefficient μ_s at a speed v that is just slow enough that it doesn't slip *downwards*.

B) A car drives around a banked curve of radius R with friction coefficient μ_s at a speed v that is just fast enough that it doesn't slip *upwards*.

C) An ant is standing 10 cm from the center of a record player spinning with angular frequency ω that has friction coefficient μ_s.

$x: \; F_N \sin\theta - F_f \sin\theta = \frac{mv^2}{R}$

$y: \; F_N \cos\theta + F_f \cos\theta - F_g = 0$

$F_f = \mu_s F_N$

$x: \; F_N \sin\theta + F_f \cos\theta = \frac{mv^2}{R}$

$y: \; F_N \cos\theta - F_f \sin\theta - F_g = 0$

$y: \; F_N - mg = 0$

$x: \; F_f = m\omega^2 r$