PHYSICS 4C | QUIZ 1 | FALL QUARTER 2007
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Formulas:

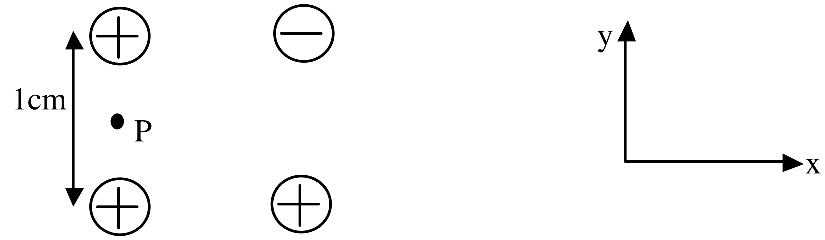
$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^{\ 2}} \hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E} \cdot d\vec{a} = 4\pi q_{enc} = 4\pi \int \rho dv$ Gauss' law   1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

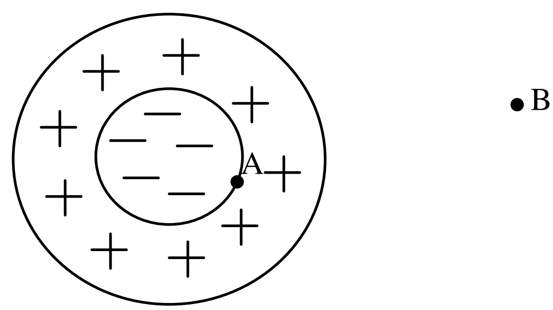
Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Problem 1 (10 pts)



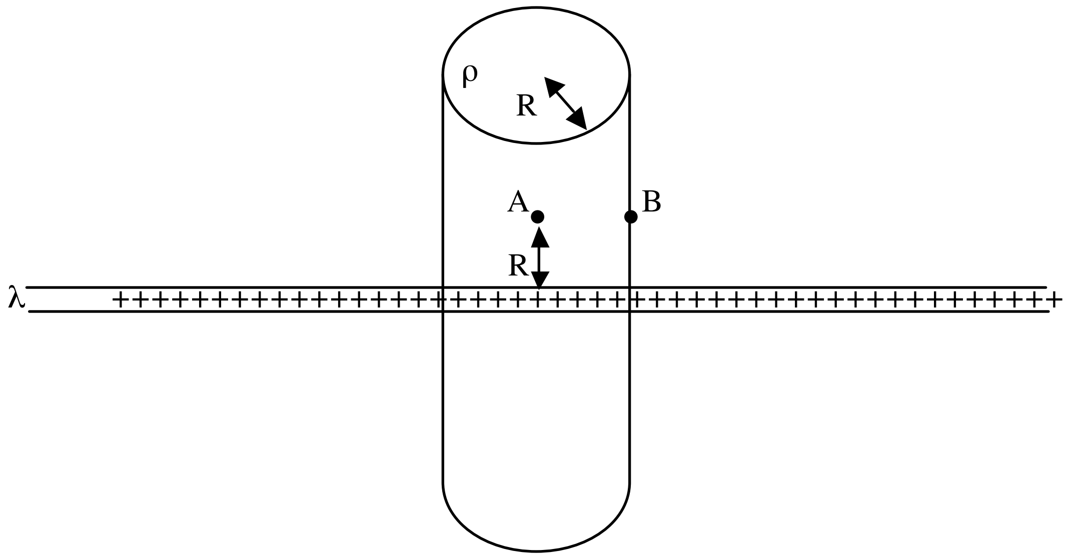
The four charges in the figure are arranged in a square of side length 1cm. Their magnitude is -1esu for the upper right charge and 1esu for the others.
(a) Find the magnitude of the electric field at the center of the square, in dynes/esu.
(b) Find the magnitude of the force acting on the negative charge, in dynes.
(c) Find the magnitude and direction of the electric field at the point in the figure midway between the two charges on the left side (indicated by the black dot).

Problem 2 (10 pts)



In the figure, the negatively charged sphere of radius R has uniform negative charge density -ρ. The positively charged shell has inner radius R and outer radius 2R and uniform positive charge density +ρ.
(a) What is the magnitude of the electric field at point A, on the surface of the sphere of radius R, in terms of ρ and R? Does it point in or out?
(b) At point B, the <u>magnitude</u> of the electric field is the same as at point A. Does it point in or out? How far is B from the center of the sphere? Give your answer in terms of R.
(c) Find points (other than at infinity) where the electric field resulting from this charge distribution is zero.

Problem 3 (10 pts)



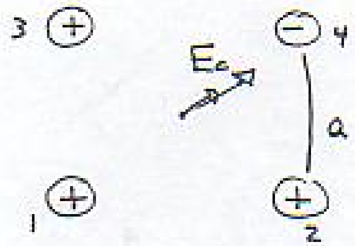
The infinitely long vertical cylinder in the figure has radius R and uniform volume charge density ρ. The horizontal infinite line of charge goes through the center of the cylinder and has uniform linear charge density λ.

(a) Find the magnitude of the electric field at point A, on the axis of the cylinder at distance R from the line of charge, in terms of ρ, λ and R.

(b) The electric field at point B, at the surface of the cylinder at distance R from the line of charge, is found to point in direction at a 45^0 angle from the horizontal direction. From this deduce the value of ρ in terms of λ and R.

Physics 4C — Quiz 1 Solutions — Fall 2007

Problem 1



distance from corner to center:

$$r = \frac{\sqrt{2}\,a}{2} = \frac{1}{\sqrt{2}}\,a$$

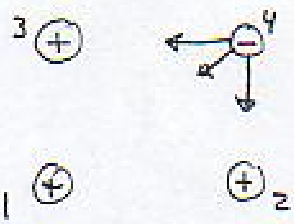
At the center, the fields of 2 and 3 cancel out.

$\vec{E}_c = \vec{E}_{c1} + \vec{E}_{c4}$, both point in the same direction.

$$E_{c1} = \frac{q_1}{r^2} = \frac{2q}{a^2} = E_{c4} \Rightarrow E_c = \frac{4q}{a^2}$$

For $q = 1\,esu$, $a = 1\,cm$, $\boxed{E_c = 4\,esu/cm^2 = 4\,\frac{dynes}{esu}}$ (a)

(b)



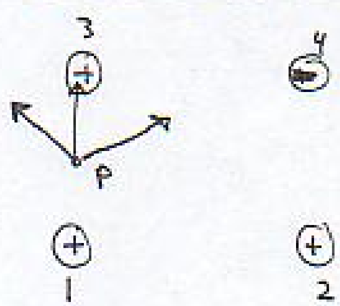
Clearly, the net force points in the direction of 1.

$$F_{43} = F_{42} = \frac{q^2}{a^2} \quad . \quad F_{41} = \frac{q^2}{2a^2}$$

total force is: $F_4 = (F_{43} + F_{42})\cos 45° + F_{41} = \sqrt{2}\,\frac{q^2}{a^2} + \frac{q^2}{2a^2} \Rightarrow$

$$\boxed{F_4 = \left(\sqrt{2} + \tfrac{1}{2}\right)\frac{q^2}{a^2} = 1.914\ \text{dynes}}$$

(c)



fields from 3 and 1 cancel out

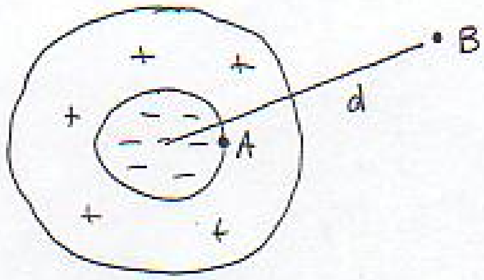
$$E_p = (E_{p2} + E_{p4})\cos 45° = 2\,E_{p2}\cos 45°$$

distance is $r = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{5}}{2}\,a$

$$\Rightarrow E_p = \sqrt{2}\,\frac{q^2 \cdot 2}{5a^2} = \frac{2\sqrt{2}}{5}\,\frac{q^2}{a^2} = \boxed{0.566\ \frac{dynes}{esu}}$$

$\boxed{\text{points in the +y direction}}$

Problem 2



Charge of inner sphere

$$\boxed{q_i = -\frac{4}{3}\pi\rho R^3}$$

At A, only charge inside matters.

$$E_A = \frac{q_{enc}}{R^2} = -\frac{4}{3}\pi\rho\frac{R^3}{R^2} \Rightarrow \boxed{E_A = -\frac{4}{3}\pi\rho R} \text{ (a) } \boxed{\text{points in}}$$

or $\boxed{E_A = \frac{q_i}{R^2}}$

(b) Charge of outer shell is:

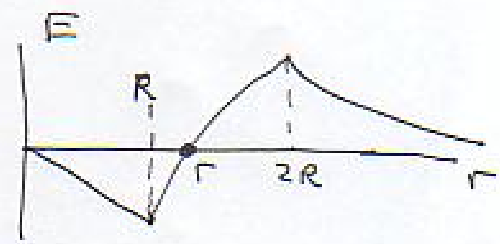
$$q_o = \frac{4}{3}\pi\rho\left[(2R)^3 - R^3\right] = -7q_i$$

So the total field at B, at distance d from center, is

$$E_B = \frac{q_i}{d^2} + \frac{q_o}{d^2}$$ sign is opposite to E_A

$$\Rightarrow -\frac{q_i}{R^2} = \frac{q_i - 7q_i}{d^2} \Rightarrow d^2 = 6R^2 \Rightarrow \boxed{d = \sqrt{6}\,R}$$

(c) The field as function of r looks like

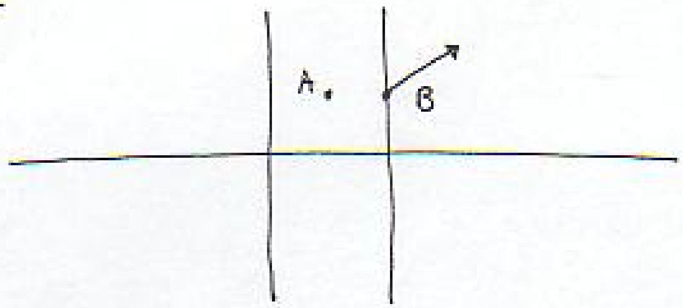


It is 0 when the net enclosed charge vanishes.

$$\frac{4}{3}\pi\rho R^3 = \frac{4}{3}\pi\rho(r^3 - R^3) \Rightarrow r^3 = 2R^3 \Rightarrow$$

$\Rightarrow \boxed{r = 2^{1/3}R}$ on the sphere of that radius, $E = 0$.

Problem 3



field at point A is only due to line of charge:

$$\boxed{E_A = \frac{2\lambda}{R}} \quad (a)$$

(b) it means the field from the cylinder has same magnitude as that of the charge.

For cylinder:

$$\int \vec{E}_c \cdot d\vec{a} = 4\pi q_{enc}$$

$$E_c \cancel{2}\cancel{\pi} R \cdot \cancel{l} = \cancel{4}^{2}\pi \cdot \cancel{\pi} R^2 \cdot \cancel{l} \cdot \rho \Rightarrow E_c = 2\pi R \rho$$

Hence $2\pi R\rho = \frac{2\lambda}{R} \Rightarrow \boxed{\rho = \frac{\lambda}{\pi R^2}}$