End of Chapter 6 and Chapter 7 Homework Solutions

## Chapter 6.

26.

**a.** If  $m_2$  is stationary, the tension force must balance the gravitational force. Therefore:

$$F_T = m_2 g \tag{1}$$

We used the notation  $F_T$  for the tension force because we will need the letter T in the next section for the period.

**b.** This tension force is providing the centripital force for the mass  $m_1$  moving in uniform circular motion. Therefore:

$$m_1 \frac{v^2}{R} = m_2 g \tag{2}$$

$$\Rightarrow v = \sqrt{\frac{m_2 g R}{m_1}} \tag{3}$$

The period is therefore:

$$T = \frac{2\pi R}{v} = 2\pi \sqrt{\frac{m_1 R}{m_2 g}} \tag{4}$$

36. For the minimum speed to keep the water from falling out, at the top of the circle, the centrifugal force must balance the gravitational force in the non-inertial frame of the bucket. Looking at it from an inertial frame of reference, the gravitational acceleration must provide exactly the centripetal acceleration for the water to move in a uniform circle (this is only valid for the point at the very top of the circle). Thus:

$$g = \frac{v^2}{r} \tag{5}$$

$$\Rightarrow v = \sqrt{gr} = \sqrt{9.8 \frac{m}{s^2} * .85m} = 2.9 \frac{m}{s}$$
 (6)

Speeds faster than this will also keep the water in the bucket, and in that case, the normal force between the bucket and water will provide the additional centripetal force necessary.

## Chapter 7.

**14.** We need to put **A** and **B** into component form (draw a picture to help you with the projections):

$$\mathbf{A} = 10\cos(30^\circ)\hat{\mathbf{i}} + 10\sin(30^\circ)\hat{\mathbf{j}} \tag{7}$$

$$\mathbf{B} = -4\sin(10^\circ)\hat{\mathbf{i}} + 4\cos(10^\circ)\hat{\mathbf{j}} \tag{8}$$

$$\mathbf{A} = 5\sqrt{3}\hat{\mathbf{i}} + 5\hat{\mathbf{j}} \tag{9}$$

$$\mathbf{B} = -.69\hat{\mathbf{i}} + 3.93\hat{\mathbf{j}} \tag{10}$$

a. We can do the rest of the problem simply:

$$\mathbf{A} \cdot \mathbf{B} = (5\sqrt{3} * -.69) + (5 * 3.93) = 13.7 \tag{11}$$

b.

$$\mathbf{C} \cdot \mathbf{D} = (5.6 * 1.9) + (-3.1 * 7.2) = -11.7 \tag{12}$$

C.

$$\mathbf{B} \cdot \mathbf{C} = (-.69 * 5.6) + (3.93 * -3.1) = -16.0 \tag{13}$$

**26.** We use the formula  $W = \int_{x_1}^{x_2} kx dx = \frac{1}{2} kx^2 \Big|_{x_1}^{x_2}$ .

a.

$$W = \frac{1}{2}kx^2 \Big|_{0}^{.1m} = \frac{1}{2}(200)(.1^2) = 1J$$
 (14)

b.

$$W = \frac{1}{2}kx^2 \Big|_{1m}^{.2m} = \frac{1}{2}(200)(.2^2 - .1^2) = 3J$$
 (15)

Note that it takes *more* work to stretch the spring from 10cm to 20cm than it does to stretch it to 10cm from equilibrium. This is because farther from the equilibrium position, the spring is applying a stronger force and therefore more work is required to stretch it an equal distance.

**36.** At every point, the force of gravity acts on the ant with force  $-mg\hat{\mathbf{j}}$ . The ant moves at constant radius R, and moves through a quarter circle from  $\theta=0$  to  $\theta=\frac{\pi}{2}$  ( $\theta$  here is defined as the angle above the horizontal with my origin at the center of the log). This means that force of gravity projected onto the direction of motion of the ant (draw a picture) is:

$$F_q = -mg\cos(\theta) \tag{16}$$

Therefore, the work done by gravity is:

$$W_g = \int_0^{\frac{\pi}{2}} (-mg\cos(\theta))Rd\theta \tag{17}$$

We must not forget to include the factor of R in the integral  $(dW = \mathbf{F} \cdot d\mathbf{r})$ , and  $d\mathbf{r} = Rd\theta\hat{\theta}$ . Therefore:

$$W_g = -mgR\sin(\theta)|_0^{\frac{\pi}{2}} = -mgR \tag{18}$$