Physics 1C

Lecture 28C

"*In quantum mechanics we have found a region of the universe where the human brain is simply unable to be comfortable.*"

--James Trefil

Outline

CAPE and extra credit

Wave-particle duality

Electron diffraction

The Uncertainty Principle

Quiz 4 Info

It will be a Scantron test that covers Chapters 27 and 28.

There will be 10 problems on this quiz.

A list of equations, constants, and conversions will be provided on the last page.

You are to write the version of your test on the Scantron form.

You will use the Quiz Code Number that you were assigned at the first Quiz.

You are expected to abide by UC Policy on Integrity of Scholarship.

Wave Particle Duality

Experiments such as the photoelectric effect, the Compton effect, and double-slit interference helped to describe light’s true nature: a wave and a particle.

Light has a dual nature, exhibiting both wave and particle characteristics.

This means a photon can have energy given by:

$$E = hf = h\frac{c}{\lambda}$$

And that photons can have a momentum given by:

$$p = \frac{E}{c} = \frac{1}{c}\left(h\frac{c}{\lambda}\right) = \frac{h}{\lambda}$$

Wave Particle Duality

Then in 1923, Louis de Broglie (Sorbonne, Paris) proposed a very interesting hypothesis:

Because photons have wave and particle characteristics, perhaps all forms of matter have both properties.

de Broglie proposed that subatomic particles, like protons and electrons, will have wavelengths just like photons.

de Broglie suggested that a particle of mass *m* and velocity *v* would have a wavelength of:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

← This is known as the **de Broglie wavelength** of a particle.

Wave Particle Duality

de Broglie also postulated that **matter waves** have **frequencies** that can be found as:

$$f = \frac{E}{h}$$

But de Broglie <u>only hypothesized</u> about matter waves, what kind of experiment could we design to demonstrate the wave nature of particles (like electrons)?

We could shoot electrons through a double slit apparatus and observe if there is a resulting interference pattern (similar to the pattern we observed for light).

Wave Particle Duality

Davisson and Germer (US) performed this experiment in 1927.

They scattered low energy electrons from a nickel target in a vacuum.

From this experiment they found (by accident!) interference patterns and calculated a wavelength for the electron.

This wavelength agreed with the theoretical de Broglie wavelength.

This confirmed the wave nature of electrons.

This was the first experimental confirmation of the de Broglie hypothesis.

Clicker Question 28C-1

We have discussed two wavelengths associated with the electron, the Compton wavelength and the de Broglie wavelength. Which is an actual physical wavelength associated with the electron?

A) the de Broglie wavelength

B) the Compton wavelength

C) both wavelengths

D) neither wavelength

Examples

A. Calculate the de Broglie wavelength of an electron (m_e = 9.11x10^{-31}kg) moving with a speed of 1.00x10^7 m/s.

Answer: $\lambda = h/(m_e v)$ = 0.0727 nm

This λ is close to the characteristic wavelength of x-rays.

It is also on the order of the spacing of atoms in the sodium chloride lattice.

B. Calculate the de Broglie wavelength for a rock of mass 50.0 g thrown with a speed of 40 m/s.

Answer: 3.31x10^{-34} m

This wavelength is much smaller than the rock. Thus, the wave properties (e.g. diffraction effects) of large-scale objects cannot be observed.

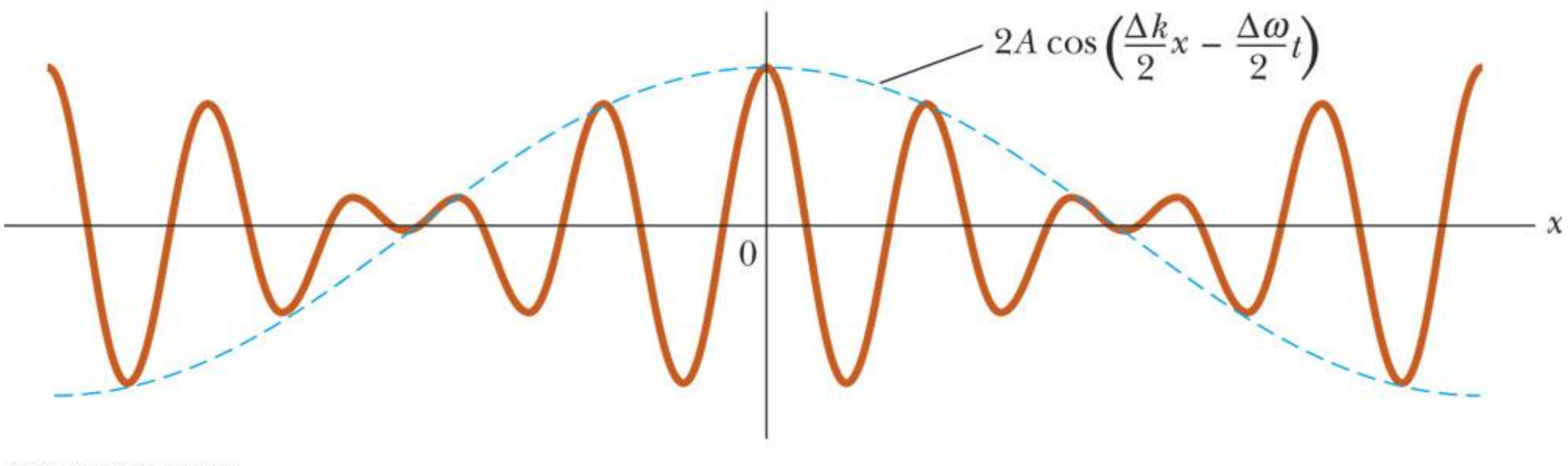
Wave Particle Duality

Can consider a particle as a wave packet

composed by many individual waves that add together (Action Figure).

The blue line represents the envelope function

This envelope can travel through space



Wave Particle Duality

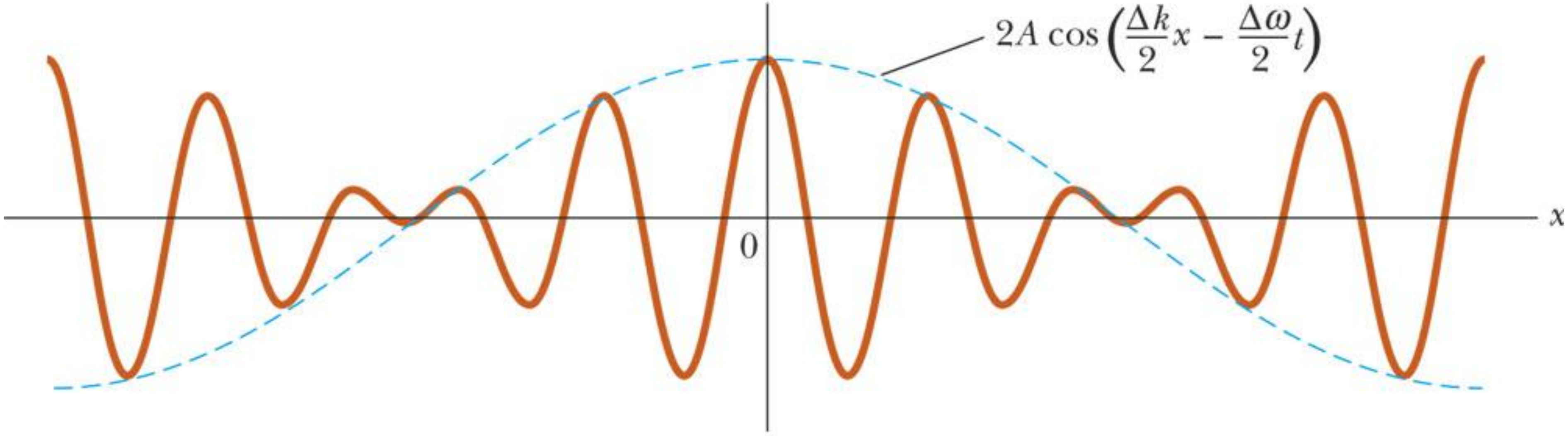
These waves can also travel much as a particle can travel with a velocity v_g (Action figure)

Phase speed - rate of advance of a crest on

$$v_{phase} = \omega / k$$

Group speed - speed of the wave packet itself :

$$v_g = d\omega / dk$$



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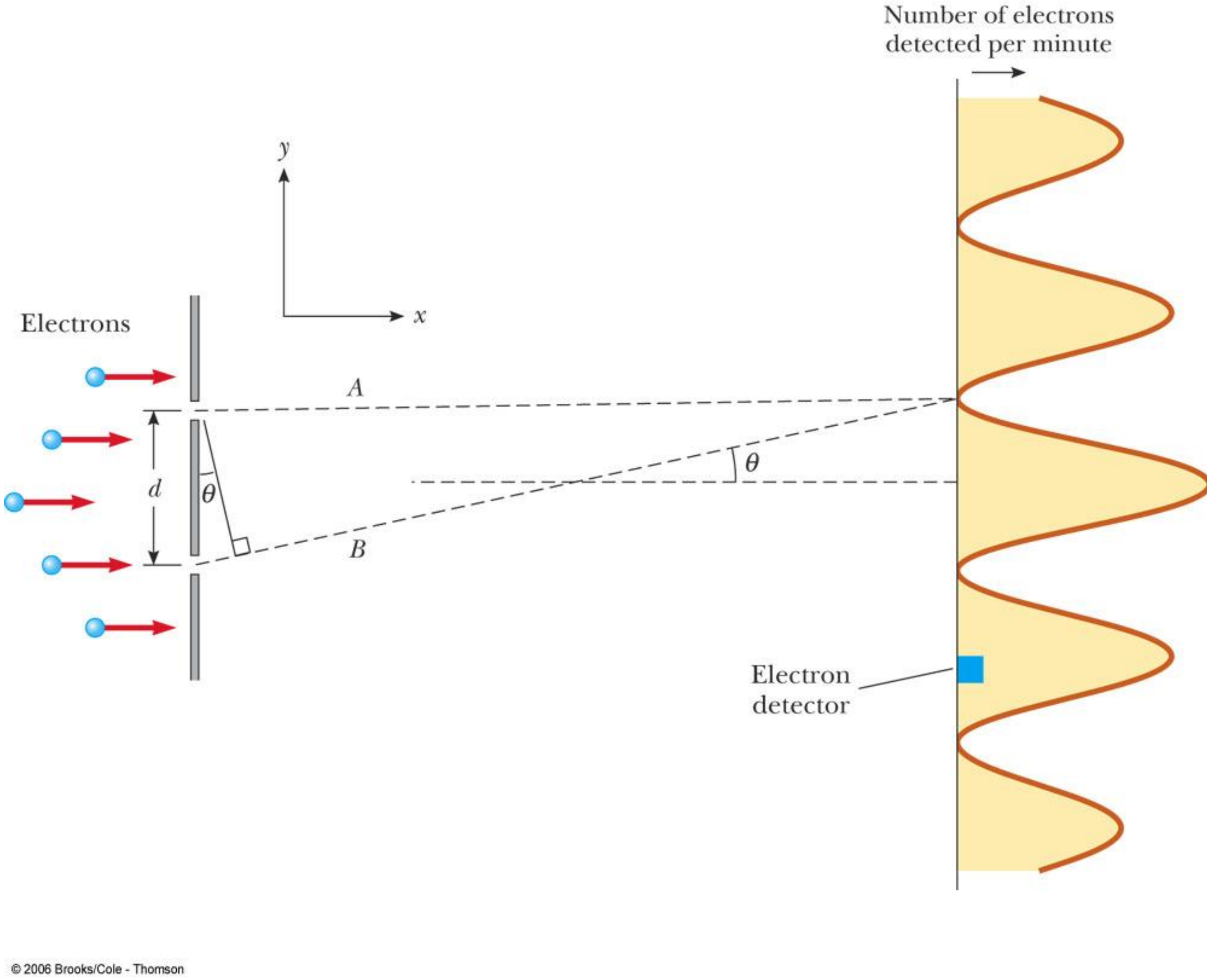
Electron Diffraction

Consider a parallel beam of monoenergetic electrons that is incident on a double slit.

Assume that the slit width are small compared to the electron wavelength.

An electron detector is positioned at a distance *L*>>*d*.



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A typical wave interference pattern for the electron counts per minute appears.

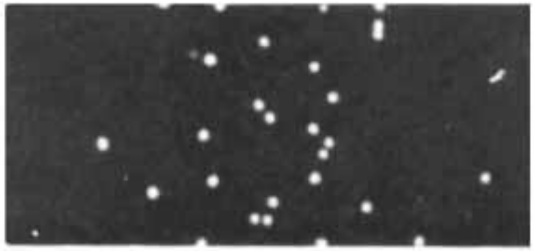
Electron Diffraction

It is clear that electrons are interfering, which is a distinct wave-like behavior.

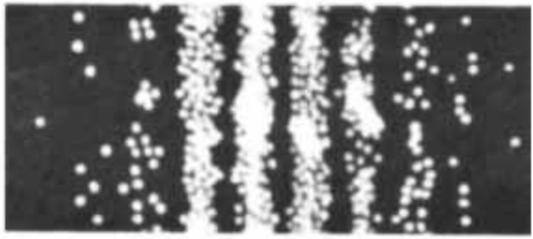
If the experiment is carried out at lower electron beam intensities, the interference pattern is still observed if the exposure is sufficiently long.

As in Chapter 27, we can use the waves in interference model to find the angular separation θ between the central maximum and the first minimum: $d \sin \theta = \lambda/2$.

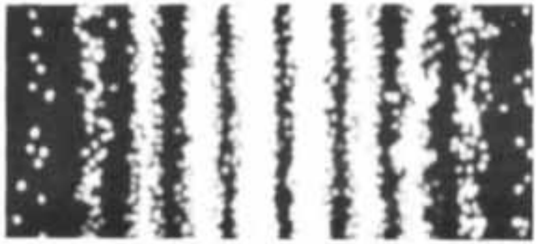
Because for electrons $\lambda = h/p_x$ we get for small θ: $\sin \theta \approx \theta = h/(2p_x d)$



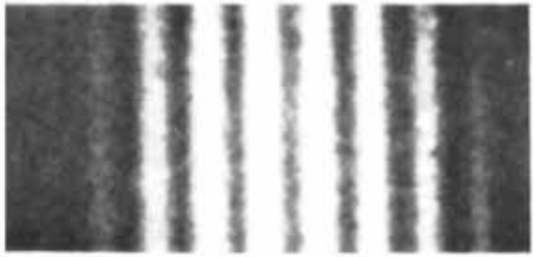
(a) After 28 electrons



(b) After 1000 electrons



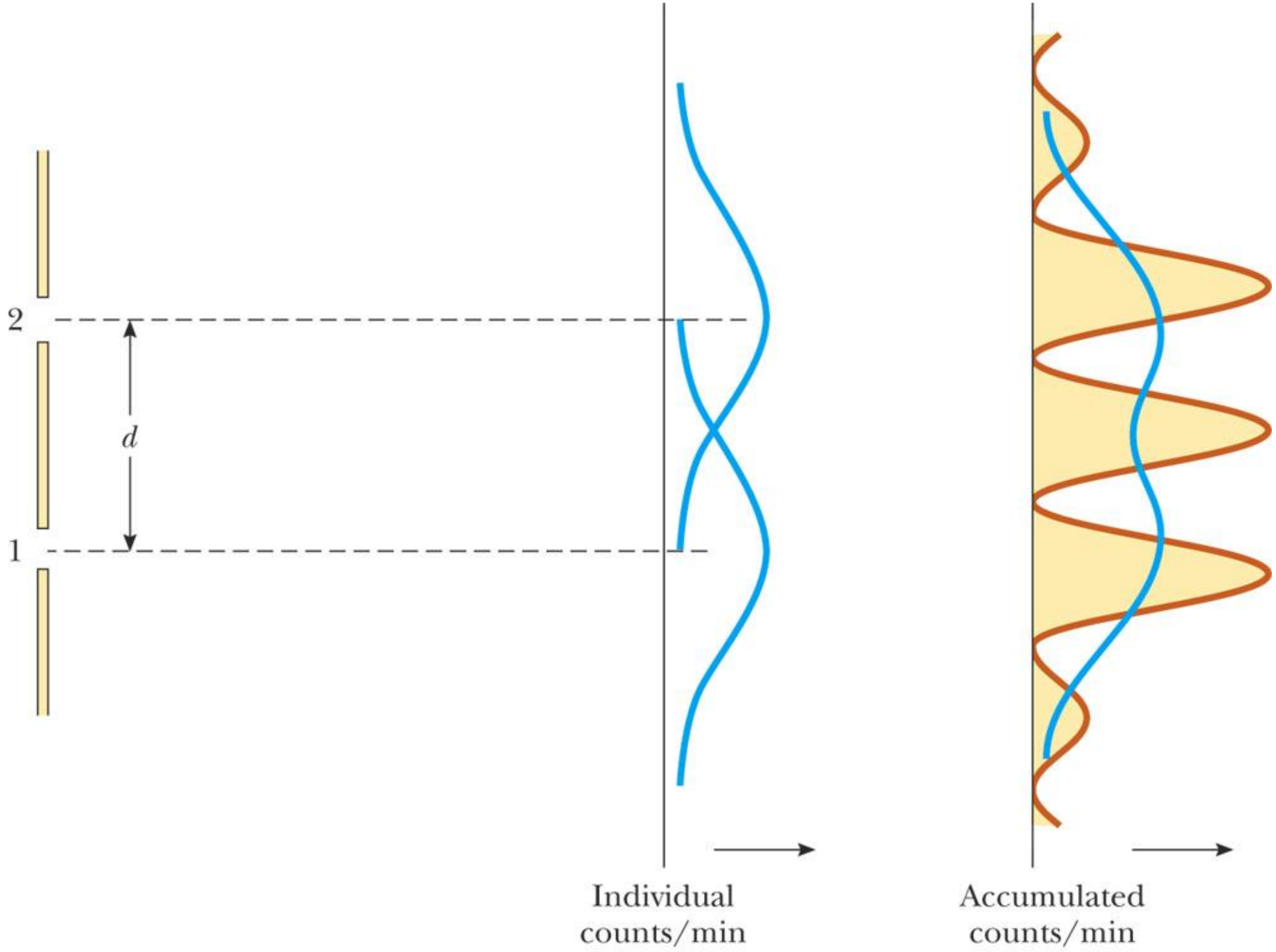
(c) After 10000 electrons



(d) Two-slit electron pattern

Electron Diffraction

The resultant pattern was not the sum of two single slit interference patterns but showed interference between the slit.



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Electron Diffraction

The electrons are detected as particles at a localized spot at some instance in time, but the probability of arrival at that spot is determined by finding the intensity of two interfering waves.

We conclude that an electron interacts with both slits simultaneously.

It is impossible to determine which slit the electron goes through.

We can only say that the electron passes through both slits!

If we attempt to experimentally determine which slit the electron goes through, the interference pattern is destroyed.

The same argument applies to photons.

The Uncertainty Principle

When measurements are made, the experimenter is always faced with experimental uncertainties in the measurements.

In classical mechanics, there can be measurements with arbitrarily small uncertainties (no limit).

Yet quantum mechanics predicts that a barrier to measurements with ultimately small uncertainties does exist.

When taking the measurement of position, x, the uncertainty of the measurement is given by Δx.

When taking the measurement of momentum, p_x, the uncertainty of the measurement is given by Δp_x.

The Uncertainty Principle

In 1927, Werner Heisenberg unveiled his uncertainty principle:

The product of the uncertainty of the position of a particle (Δx) and the uncertainty of the linear momentum of the particle (Δp_x) can never be smaller than $h/4\pi$.

Mathematically, this becomes:

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

Basically, the uncertainty principle states that **it is physically impossible to simultaneously measure the exact position and the exact linear momentum of a particle**.

The Uncertainty Principle

The uncertainty principle can be extended to deal with energy uncertainties as well.

In this case, position and linear momentum are replaced with energy and time such that:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

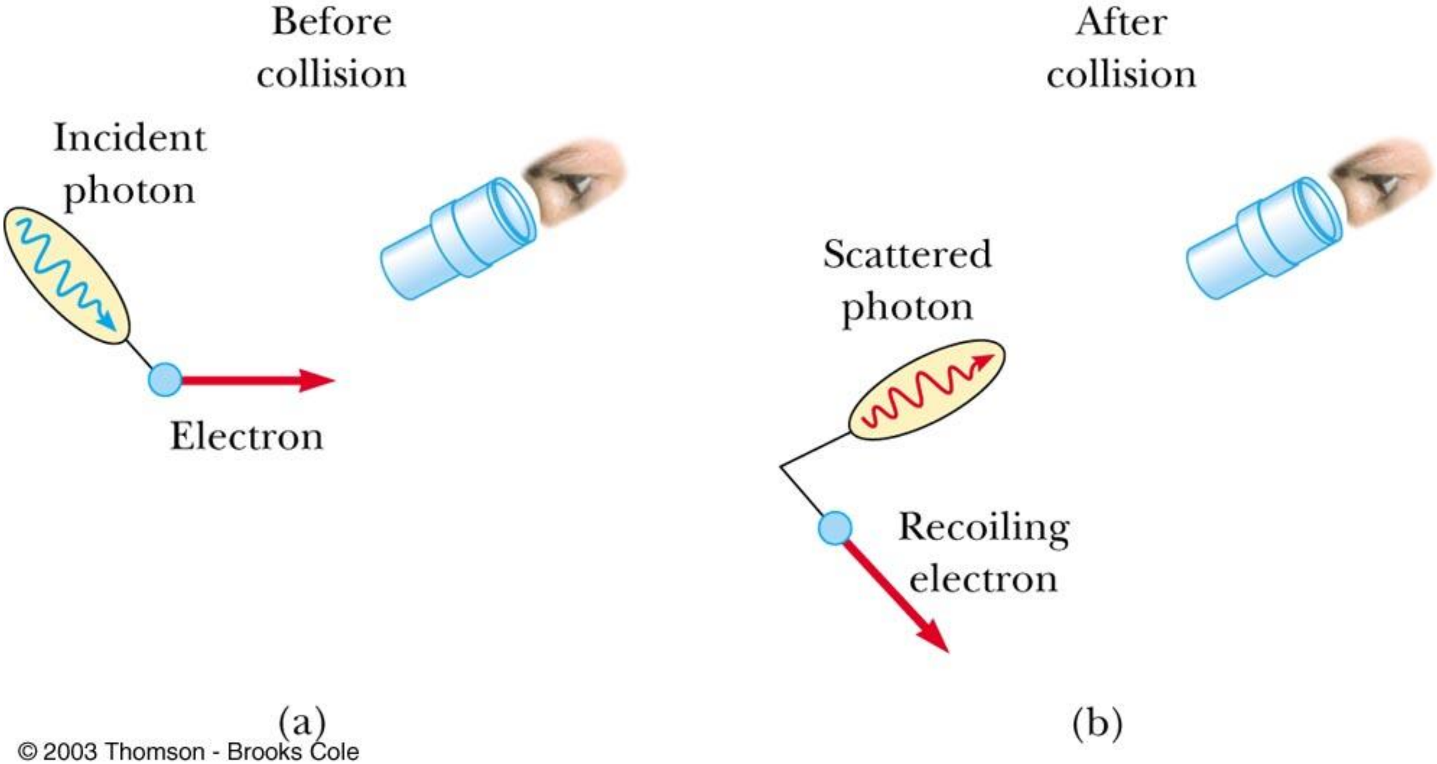
In this form, the uncertainty principle states that it is physically impossible to measure the exact energy of a particle over an accurate period of time.

The Uncertainty Principle

One way to consider the implications of the uncertainty principle is to think about how to measure the position and linear momentum of an electron with a very powerful microscope.

In order to locate the electron accurately, at least one photon must bounce off of it.

During this interaction, momentum is transferred from the photon to the electron.



The Uncertainty Principle

Therefore, the light that allows you to accurately locate the electron changes the momentum of the electron (the maximum change is $\Delta p_x = h/\lambda$).

To minimize the momentum change, we could use longer wavelength photons.

But because the photon also has wave properties, we can determine the electron position only within one wavelength of the photon, $\Delta x = \lambda$.

Consequently, the position and the momentum of the electron cannot both be known precisely at the same time: $\Delta p_x \Delta x \geq h$.

Apart from the numerical factor $1/4\pi$, this formula is the same as Heisenberg’s more precise result.

Clicker Question 28C-2

According to Heisenberg’s uncertainty principle, the more accurately we know about a subatomic particle’s momentum, the less we know about its precise:

A)

B)

C) speed

D)

$\Delta p_x \Delta x \geq h$

E) energy

For Next Time (FNT)

Prepare for Quiz # 4

Quiz will cover Chapter 27 – diffraction and thin films – and chapter 28 – through uncertainty principle

Finish reading Chapter 28

Homework for Chapter 28