Problem

8. Each atom in aluminum contributes about 3.5 conduction electrons. What is the drift speed in a 0.21-cm-diameter aluminum wire carrying 20 A?

Solution

As in Example 27-1, the drift speed of electrons in the wire is $v_d = I/(\frac{1}{4}\pi d^2)ne$, where n is the number density of conduction electrons. Using data from the text's inside back cover, we find $n = (3.5\,\text{electrons/ion})(2702\,\text{kg/m}^3) \div (26.98\,\text{u/ion} \times 1.66 \times 10^{-27}\,\text{kg/u}) = 2.11 \times 10^{29}\,\text{electrons/m}^3$, for aluminum. Thus, $v_d = 20\,\text{A}/\frac{1}{4}\pi(0.21\,\text{cm})^2 \times (2.11 \times 10^{29}\,\text{m}^{-3})(1.6 \times 10^{-19}\,\text{C}) = 0.171\,\text{mm/s}$.

Problem

10. The filament of the light bulb in Example 27-6 has a diameter of 0.050 mm. What is the current density in the filament? Compare with the current density in a 12-gauge wire (diameter 0.21 cm) supplying current to the light bulb.

Solution

The current density in the filament (Equation 27-3a, with numbers from Example 27-6) is $J = (0.833\,\text{A})/\pi(0.025\,\text{mm})^2 = 4.24 \times 10^8\,\text{A/m}^2$, in the direction of the current. A 12-gauge wire, carrying the same current, would have a current density smaller by a factor of the square of the ratio of the diameters, $(0.05\,\text{mm}/2.1\,\text{mm})^2 = 5.67 \times 10^{-4}$, or $J = 2.41 \times 10^5\,\text{A/m}^2$.

Problem

12. A piece of copper wire joins a piece of aluminum wire whose diameter is twice that of the copper. The same current flows in both wires. The density of conduction electrons in copper is $1.1 \times 10^{29}\,\text{m}^{-3}$; in aluminum it is $2.1 \times 10^{29}\,\text{m}^{-3}$. Compare (a) the drift speeds and (b) the current densities in each.

Solution

(a) Since the current is the same in both wires, but the cross-sectional area of the aluminum wire is four times that of the copper wire, Equation 27-3a gives $I_{Cu}/I_{Al} = 1 = n_{Cu}ev_{d,Cu}/4n_{Al}ev_{d,Al}$, or $v_{d,Cu}/v_{d,Al} = 4(2.1 \times 10^{29})/(1.1 \times 10^{29}) = 7.64$.
(b) The same equation also gives $J_{Cu}A_{Cu}/J_{Al}A_{Al} = 1$, or $J_{Cu}/J_{Al} = A_{Al}/A_{Cu} = 4$.
(see Table 27-1).

Problem

17. What electric field is necessary to drive a 7.5-A current through a silver wire 0.95 mm in diameter?

Solution

From Ohm's law (which applies to silver) and the definition of current density (which we assume is uniform in the wire) one finds $E = \rho J = \rho I/\frac{1}{4}\pi d^2 = (1.59 \times 10^{-8}\,\Omega \cdot \text{m})(7.5\,\text{A})/\frac{1}{4}\pi(0.95\,\text{mm})^2 = 0.168\,\text{V/m}$.

Problem

19. A 1.0-cm-diameter rod carries a 50-A current when the electric field in the rod is 1.4 V/m. What is the resistivity of the rod material?

Solution

If the rod has a uniform current density and obey's Ohm's law (Equations 27-3a and 4b), then its resistivity is $\rho = E/J = E/(I/\frac{1}{4}\pi d^2) = \frac{1}{4}\pi(10^{-2}\,\text{m})^2(1.4\,\text{V/m})/(50\,\text{A}) = 2.20 \times 10^{-6}\,\Omega \cdot \text{m}$.

Problem

23. The free-electron density in aluminum is 2.1×10^{29} m^{-3}. What is the collision time in aluminum?

Solution

We can estimate τ from Equation 27-5 and Table 27-1: $\tau = m_e/\rho n e^2 = (9.11 \times 10^{-31}$ kg$)(2.65 \times 10^{-8}$ $\Omega \cdot$ m$)^{-1} \times$

$(2.1 \times 10^{29}$ m$^{-3})^{-1}(1.6 \times 10^{-19}$ C$)^{-2} = 6.39 \times 10^{-15}$ s. (An explicit confirmation of the units is: $\dfrac{\text{kg}}{(\Omega \cdot \text{m})(\text{m}^{-3})(\text{C}^2)} =$

$\dfrac{\text{kg} \cdot \text{m}^2}{(\text{V/A})\text{C}^2} = \dfrac{\text{kg} \cdot \text{m}^2(\text{C/s})}{(\text{J/C})\text{C}^2} = \dfrac{\text{kg} \cdot \text{m}^2/\text{s}}{\text{kg} \cdot \text{m}^2/\text{s}^2} = \text{s}.\Big)$

Problem

34. A uniform wire of resistance R is stretched until its length doubles. Assuming its density and resistivity remain constant, what is its new resistance?

Solution

If the density (and mass) of the wire is constant, then so is its volume, $\ell A = $ constant. Thus, $R = \rho \ell/A = \rho \ell^2/\ell A = $ constant $\times \ell^2$, and doubling ℓ increases the resistance four times.

Problem

35. A cylindrical iron rod measures 88 cm long and 0.25 cm in diameter. (a) Find its resistance. If a 1.5-V potential difference is applied between the ends of the rod, find (b) the current, (c) the current density, and (d) the electric field in the rod.

Solution

(a) Equation 27-7 gives the resistance of a uniform object of Ohmic material, $R = \rho \ell/A = (9.71 \times 10^{-8}$ $\Omega \cdot$ m$)(88$ cm$) \div \frac{1}{4}\pi(0.25$ cm$)^2 = 17.4$ mΩ (see Table 27-1 for the resistivity of iron). (b) Equation 27-6 (Ohm's law) gives $I = V/R = 1.5$ V$/17.4$ m$\Omega = 86.2$ A. (c) Equation 27-3a gives $J = I/\frac{1}{4}\pi d^2 = 86.2$ A$/\frac{1}{4}\pi(0.25$ cm$)^2 = 17.6$ MA/m^2. (d) Equation 27-4b gives $E = \rho J = (9.71 \times 10^{-8}$ $\Omega \cdot$ m$)(17.6$ MA/m$^2) = 1.70$ V/m. (The quantities were calculated in the order queried; alternatively, in reverse order, $E = V/\ell$, $J = E/\rho$, $I = JA$, and $R = V/I$.)

Problem

44. A 4.5-W flashlight bulb draws 750 mA. (a) At what voltage does it operate? (b) What is its resistance?

Solution

(a) From Equation 27-8, $V = P/I = 4.5$ W$/750$ mA $= 6.0$ V. (b) From Equation 27-6, $R = V/I = 6.0$ V$/0.75$ A $= 8.0$ Ω.

Problem

47. What is the resistance of a standard 120-V, 60-W light bulb?

Solution

The bulb's resistance, from Equation 27-9b, is $R = V^2/P = (120$ V$)^2/60$ W $= 240$ Ω, at its operating temperature. (This equation, with average values of voltage and power, can be used for an ac-resistor.)

Problem

53. Two cylindrical resistors are made from the same material and have the same length. When connected across the same battery, one dissipates twice as much power as the other. How do their diameters compare?

Solution

At the same voltage, the ratio of the power dissipated is the inverse of the ratio of the resistances, which in turn, goes as the inverse of the square of the ratio of the diameters: $P_1/P_2 = (V^2/R_1)/(V^2/R_2) = R_2/R_1 = (\rho\ell\frac{1}{4}\pi d_2^2)/(\rho\ell\frac{1}{4}\pi d_1^2) = (d_1/d_2)^2$. We used Equation 27-9b for the power, and Equation 27-7 for the resistance. Thus, if $P_1 = 2P_2$, then $d_1 = \sqrt{2}\, d_2$.