Normal Distributions
Rejection of Data
+
RLC Circuits

Lecture 4

Physics 2CL

Summer 2010

Outline

- Reminder of simple uncertainty propagation formulae
- Hidden useful formula for estimating uncertainties
- More on distributions
  - They tell you more than you may think
- How to tell if data is bad
- What to do about it
- Physics for experiment # 2
  - RLC circuits
  - Resonance
  - damping

How to estimate uncertainty in calculated parameter

- Method 1: Propagation of uncertainty
  - Estimate uncertainty in reading instrument
    - For example, finite scale on ruler or guide
    - Quoted uncertainty (tolerance) of circuit components
  - Only valid for statistical uncertainties
    - Make sure instrument is calibrated
    - Use a known (standard) to test reading

How to minimize uncertainty

- Can always estimate uncertainty, but equally important is to minimize it
- How can you reduce uncertainty in circuit measurements
  - Use better components
  - Maximize waveform on oscilloscope
  - Have both partners read value – demand consistency
  - Measure multiple times

Summary of propagation formulae

summation

$$q = x + y - z$$

$$\delta q = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

multiplication

$$q = x \times y \div z$$

$$\varepsilon q = \sqrt{(\varepsilon x)^2 + (\varepsilon y)^2 + (\varepsilon z)^2}$$

constants

$$q = Bx$$

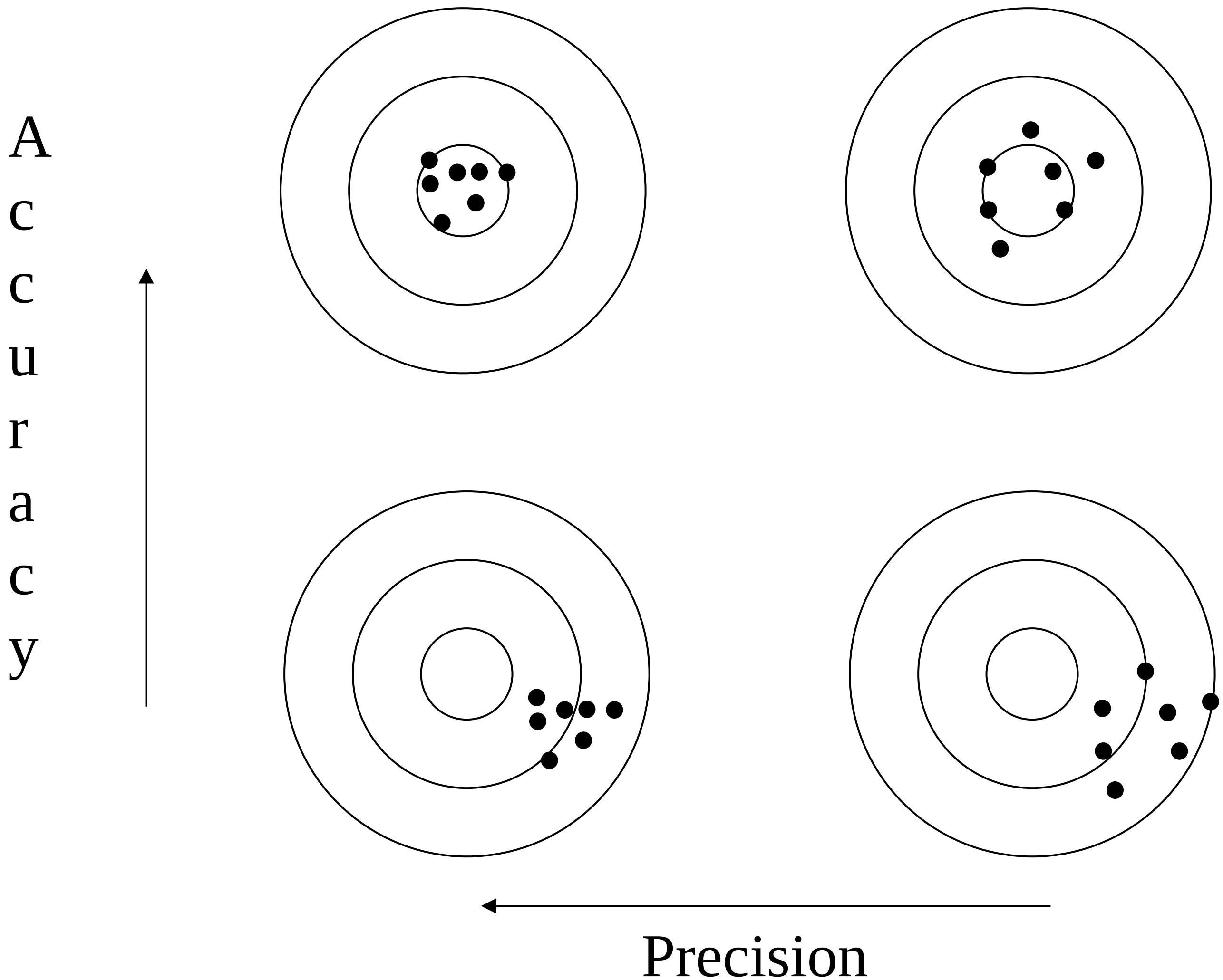
$$\delta q = |B| \delta x$$

$$\varepsilon q = \varepsilon x$$

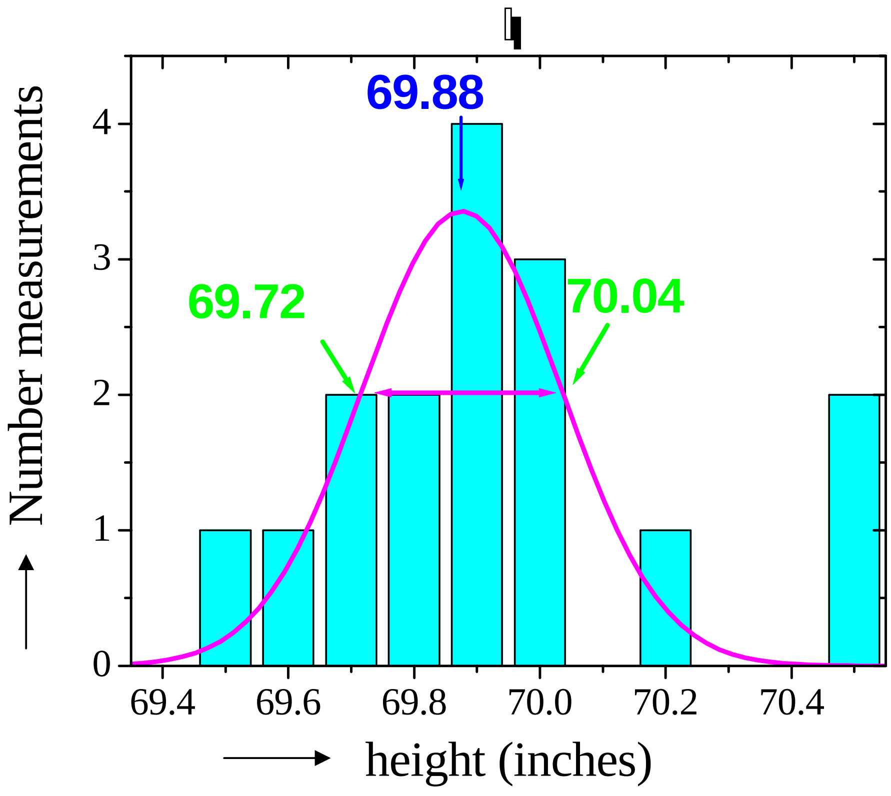
Useful concept for complicated formula

- Often the quickest method is to calculate with the extreme values
  - $q = q(x)$
  - $q_{max} = q(\overline{x} + \delta x)$
  - $q_{min} = q(\overline{x} - \delta x)$
  - □ $\delta q = (q_{max} - q_{min})/2$ (3.39)

Accuracy vs. Precision

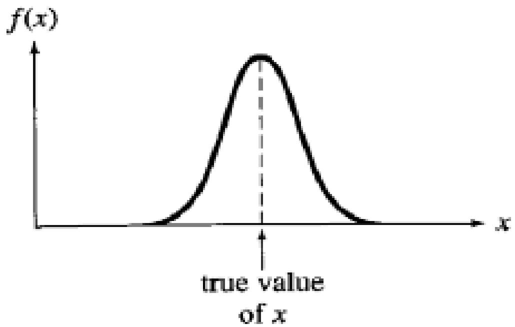


Gaussian Distribution (Chapter5 Taylor)



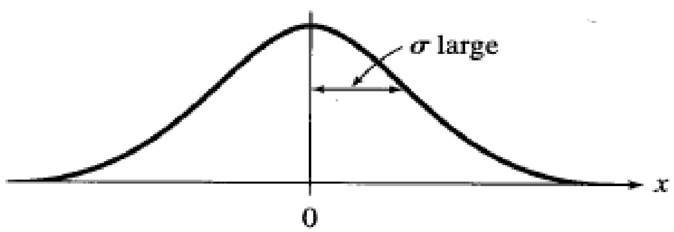
$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(x-\overline{x})^2 / 2\sigma^2)$$

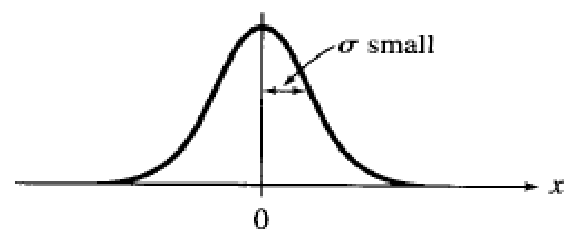
Gaussian Distribution



the limiting distribution for a measurement subject to many small random errors is bell shaped and centered on the true value of x

the mathematical function that describes the bell-shape curve is called the <u>normal distribution</u>, or <u>Gauss function</u>





prototype function

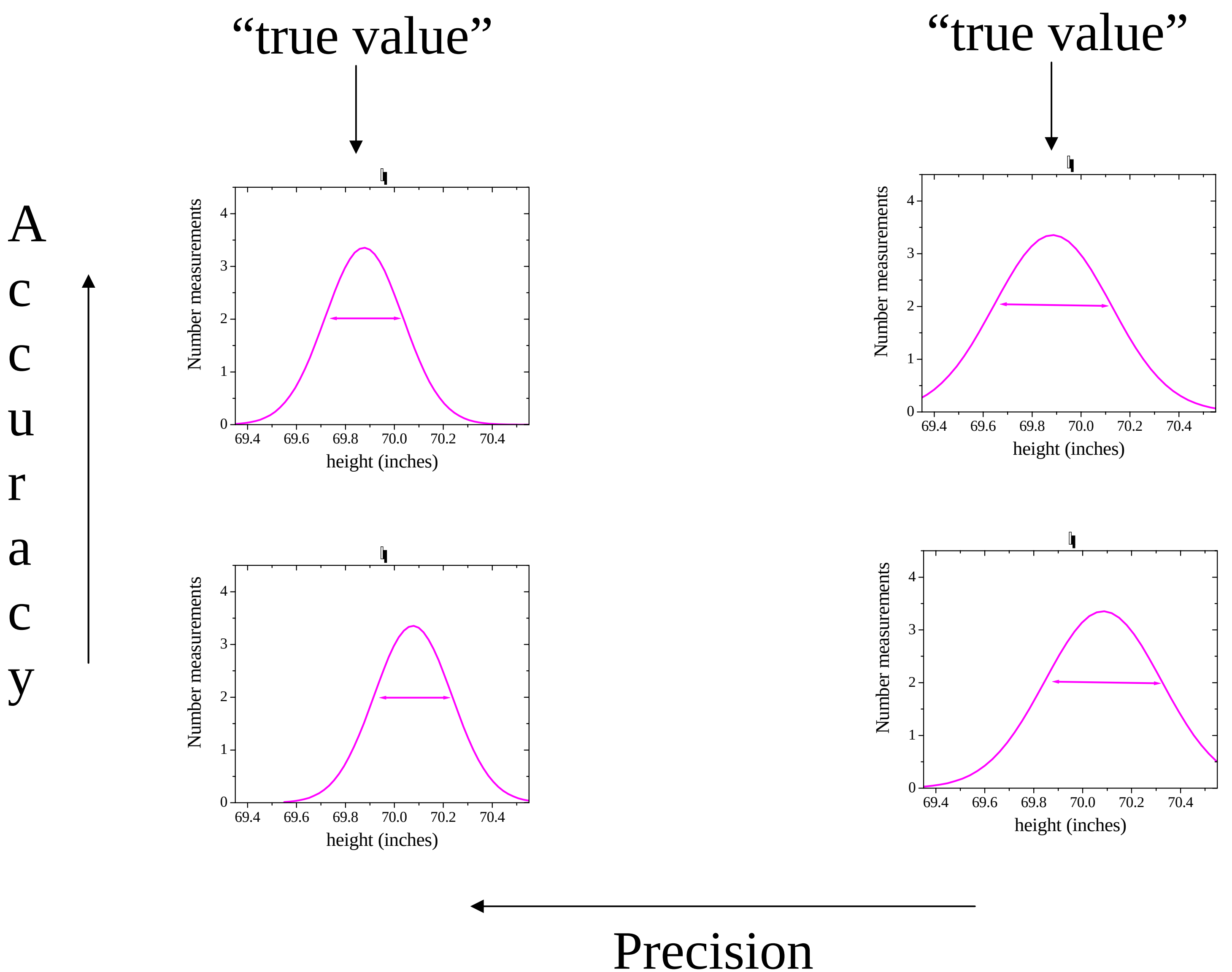
$$e^{-x^2/2\sigma^2}$$

$$e^{-(x-X)^2/2\sigma^2}$$

σ – width parameter

X – true value of x

Accuracy vs. Precision



Drawing a Histogram

1. Determine the range of your data by subtracting the smallest number from the largest one.

2. The number of bins should be approximately $\sqrt{N}$ and the width of a bin should be the range divided by $\sqrt{N}$.

3. Make a list of the boundaries of each bin and determine which bin each of your data points should fall into.

4. Draw the histogram. The y axis should be the number of values that fall into each bin.

5. Sometimes this procedure will not produce a good histogram. If you make too many bins the histogram will be flat and too few bins will not show the curve on either side of the maximum. You might need to play around with the number of bins to produce a better histogram.

The Gaussian Distribution

- A bell-shaped distribution curve that approximates many physical phenomena - even when the underlying physics is not known.
- Assumes that many small, independent effects are additively contributing to each observation.
- Defined by two parameters: Location and scale, i.e., mean and standard deviation (or variance,σ^2).
- Importance due (in part) to central-limit theorem:

  The sum of a large number of independent and identically-distributed random variables will be approximately normally distributed (i.e.,following a Gaussian distribution, or bell-shaped curve) if the random variables have a finite variance.

The Gauss, or Normal Distribution

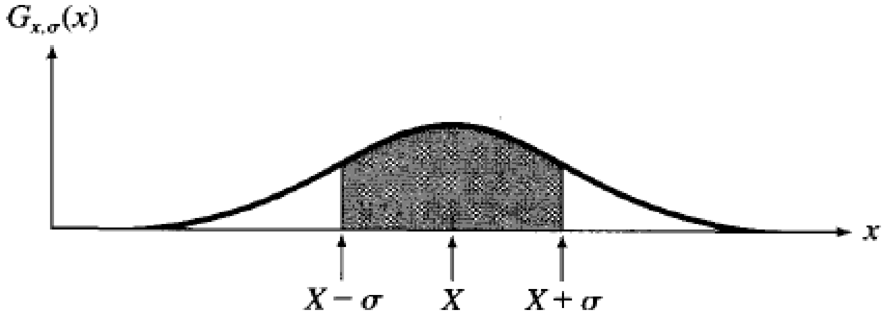
normalize $e^{-(x-X)^2/2\sigma^2}$ → $\int_{-\infty}^{+\infty} f(x)dx = 1$

↓

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$

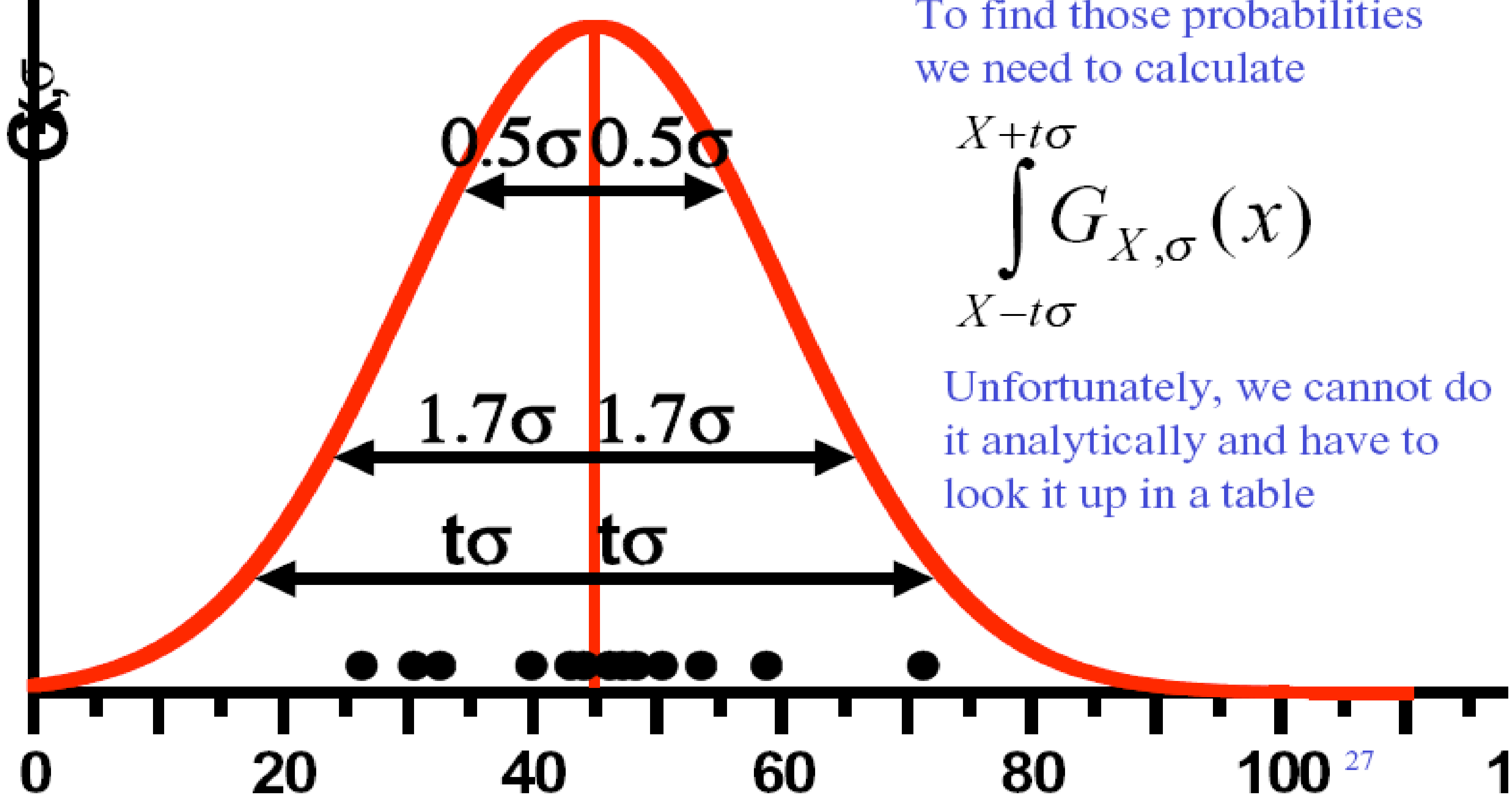
standard deviation σ_x = width parameter of the Gauss function σ

the mean value of x = true value X



What about the probabilities to find a point within 0.5σ from X, 1.7σ from X, or in general $t\sigma$ from X?

$$G_{X,\sigma} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2 / 2\sigma^2}$$

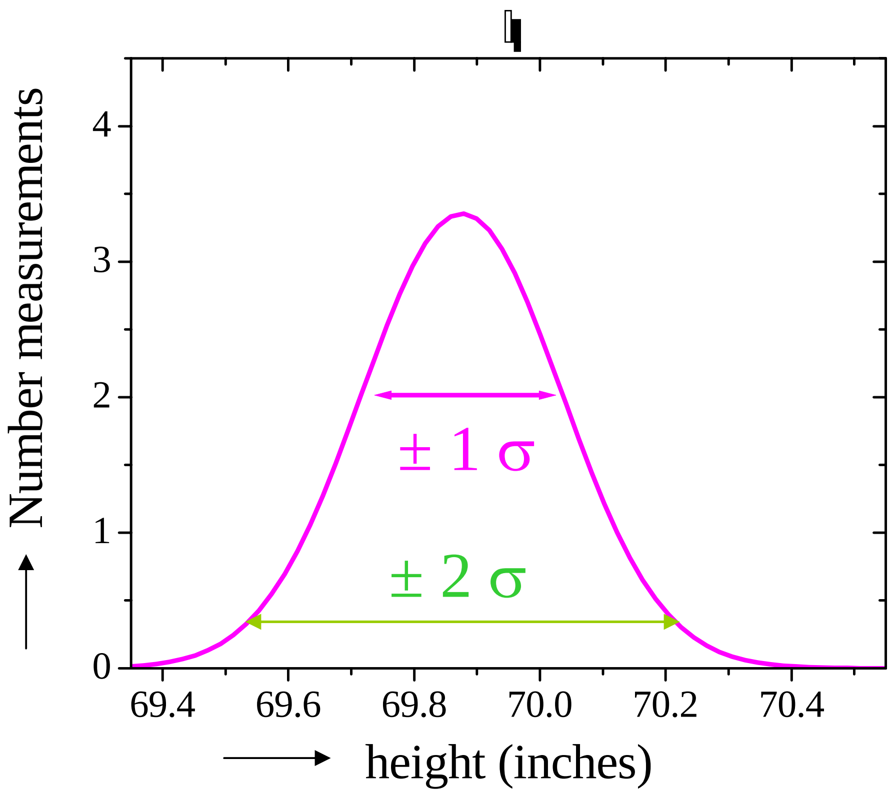


To find those probabilities we need to calculate

$$\int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)$$

Unfortunately, we cannot do it analytically and have to look it up in a table

Width of Distribution



68 %

95 %

Probability within 1.0 σ – App. A

t=1

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72

Probability within 1.47 σ – App. A

t=1.47

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	[illegible]	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									

Rejection of Data ?

- For series of measurements expect Gaussian limiting distribution
- Can we use this to identify suspect data point?
- Make use of the derived probabilities and ask how likely it would be to see a data point deviate by $t\ \sigma$

Chapter 6 - Rejection of Data ?

- Consider series – 3.8s, 3.5s, 3.9s, 3.9s, 3.4s, 1.8s
- Reject 1.8s ?
  - Bad measurement
  - New effect
    - Something new
- Make more measurements so that it does not matter

How different is the data point?

- From series obtain
  - $\langle x \rangle = 3.4\text{s}$
  - $\sigma = 0.8\text{s}$
- How does 1.8s data point apply?
- How far from average is it?
  - $x - \langle x \rangle = \Delta x = 1.6\text{ s} = 2\,\sigma$
- How probable is it?
  - $\text{Prob}\,(|\Delta x| > 2\,\sigma) = 1 - 0.95 = 0.05$

Chauvenet’s Criterion

- Given our series, what is prob of measuring a value 2 σ off ?
  - Multiply Prob by number of measurement
  - Total Prob = 6 x 0.05 = 0.3

- If chances < 50% discard

Strategy

- $t_{sus} = \Delta x$ (in σ)
- Prob of x outside Δx
- Total Prob = N x Prob
- If total Prob < 50% then reject

Refinement

- When is it useful
  - Best to identify suspect point
  - remeasure
- When not to reject data
  - When repeatable
  - May indicate insufficient model
  - Experiment may be sensitive to other effects
  - May lead to something new (an advance)

Rejection of other data points

- If more than one data point suspect, consider that model is incorrect
- Look at distribution
- Additional analysis
  - Such as χ^2 testing (chapter 12)
    - Makes use of expected bin distributions
  - Remeasure/ repeatable
  - Determine circumstances were effect is observed.

Experiment #2
Oscillations and Damping RLC Circuit

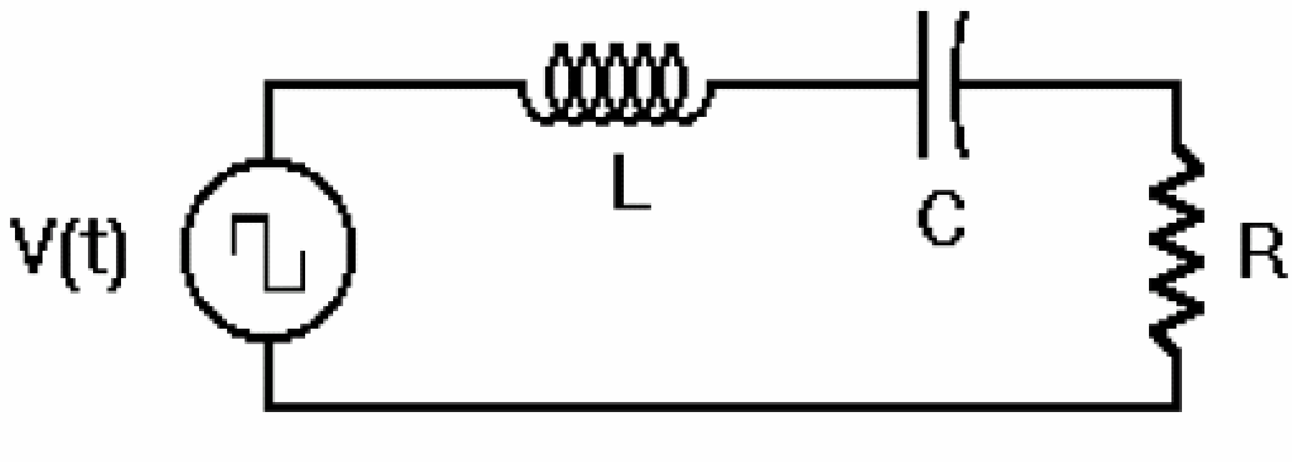


Figure 1 *LRC circuit for this experiment*

Analysizing RLC Circuit

Kirchoff's Law

$$V(t) = V_L + V_C + V_R$$

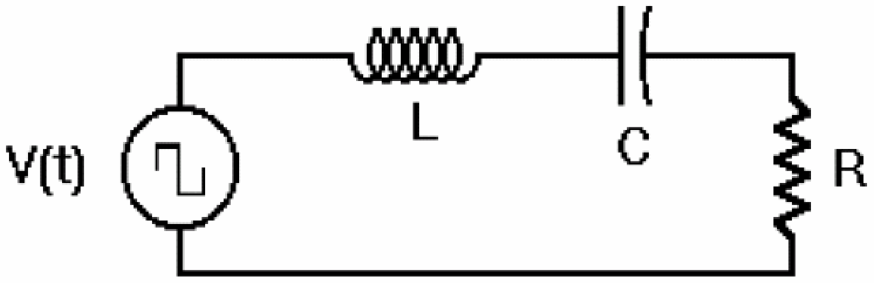


Figure 1 *LRC circuit for this experiment*

$$V_R = IR$$

$$V_C = \frac{1}{C}\int I dt$$

$$V_L = L\frac{dI}{dt}$$

$$V(t) = IR + \frac{1}{C}\int I dt + L\frac{dI}{dt}$$

RLC Circuit Response

Differentiate

$$\frac{dV}{dt} = \frac{I}{C} + \frac{RdI}{dt} + L\frac{d^2 I}{dt^2}$$

Square Wave input

$$I + RC\frac{dI}{dt} + LC\frac{d^2 I}{dt^2} = 0$$

RLC Circuit Response

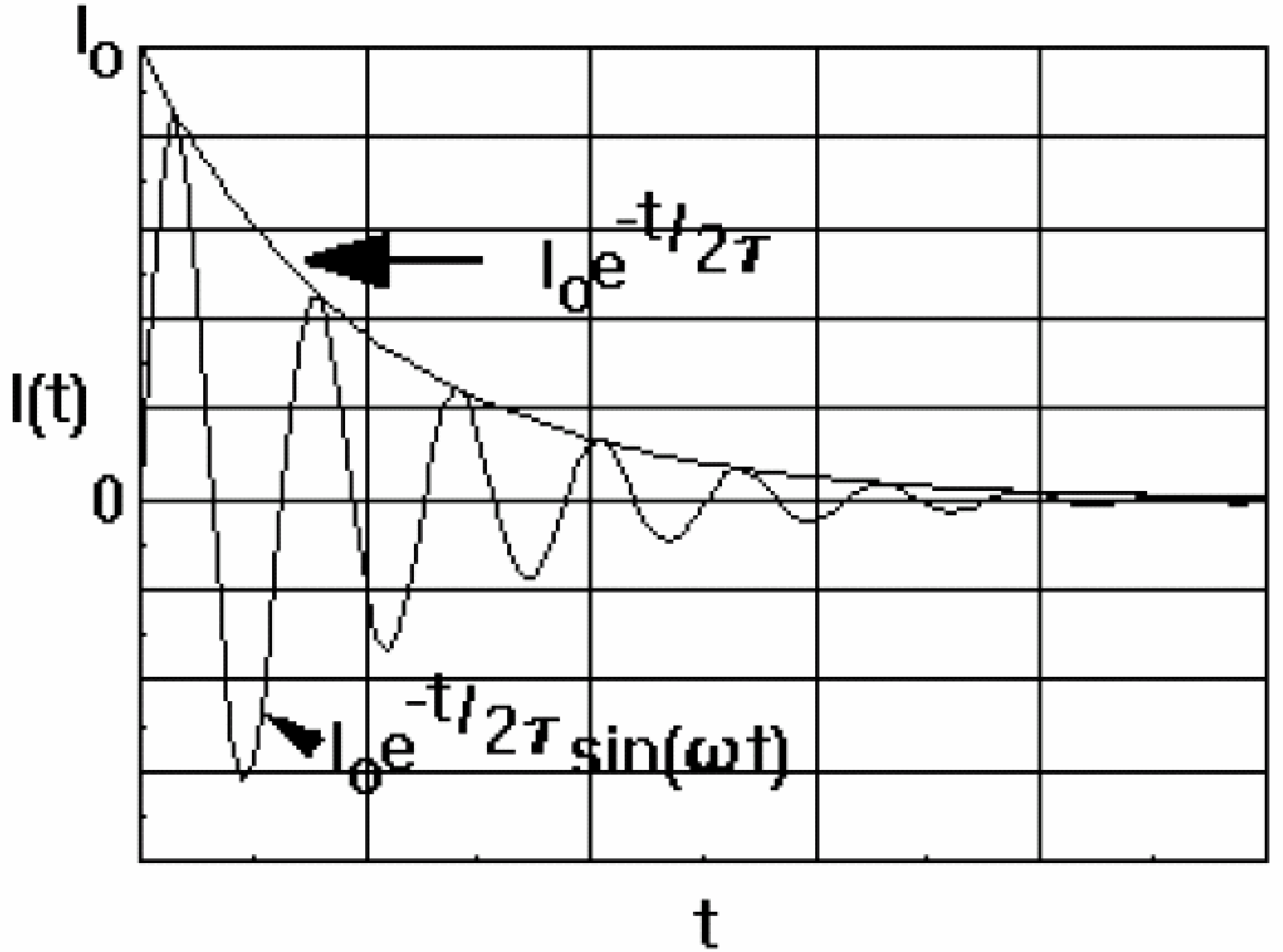
Solve

$$I(t) = I_0 e^{\frac{-R}{2L}t} \sin \omega t$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2} = \omega_0^2 - \frac{1}{4\tau^2}$$

$$\omega_0^2 = \frac{1}{LC} \qquad \tau = \frac{L}{R}$$

Graph of RLC Circuit Response



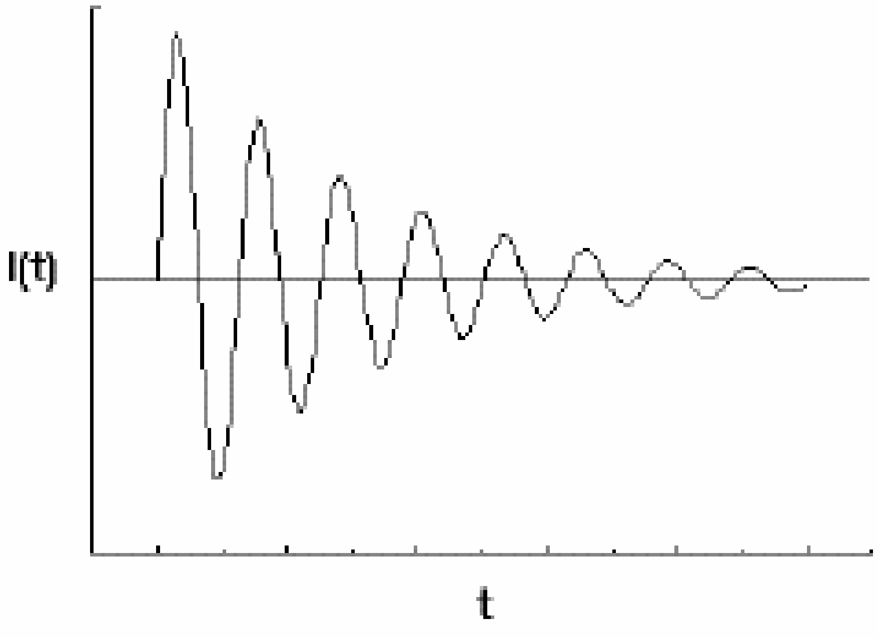
Critical Damping

Define critical damping time constant

$$\tau = \tau_C = \frac{1}{2\omega_0}$$

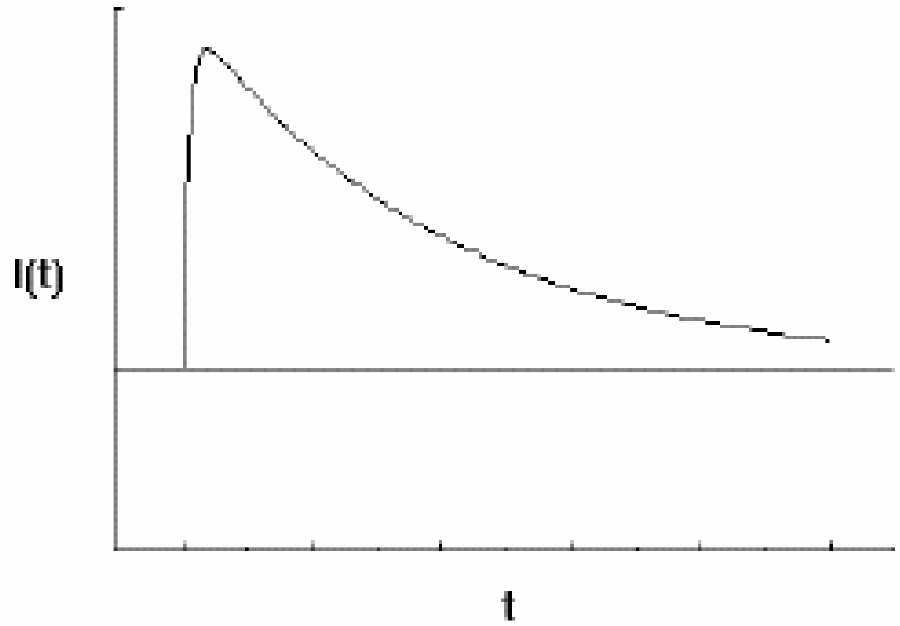
No oscillations observed

Three Regimes for Damping



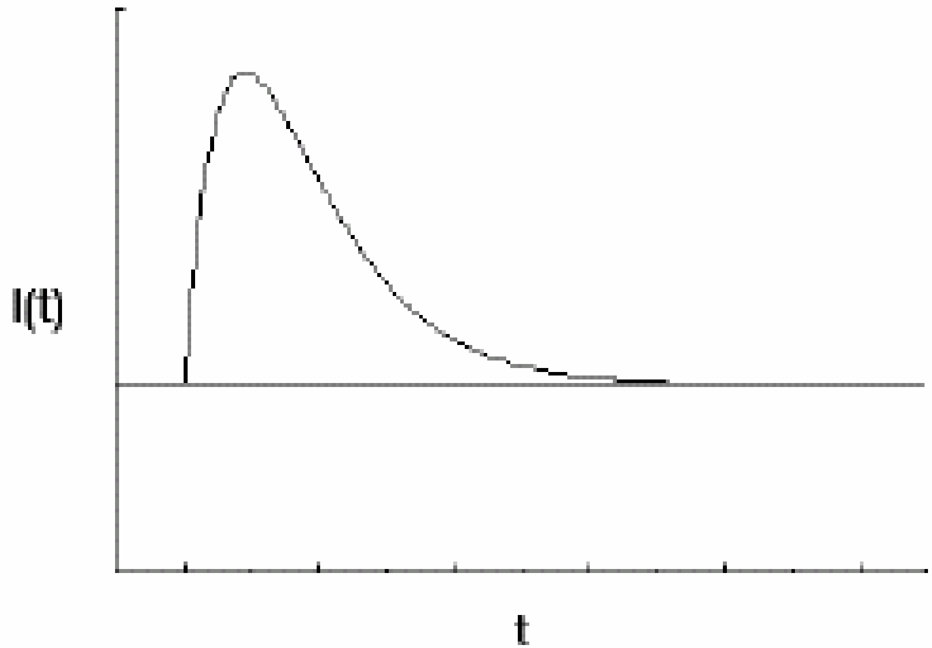
Underdamped($\tau > \tau_c$)

Three Regimes for Damping



Overdamped($\tau<\tau_c$)

Three Resimes for Damping



Critical ($\tau=\tau_c$)

Energy Storage and Dissipation

Quality Factor

$$Q = \omega_0 \tau = \omega_0 \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Lab Objectives

1) Determine ω and Q

2) Achieve Critical Damping

3) Determine unknown L

Summary

- Average

$$\bar{x} = \frac{\sum x_i}{N}$$

- Standard deviation

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

- Standard deviation of the mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

- Suspect data

$P_{TOT} < 50\%$

Remember

- Lab Writeup
- Read next week’s lab description, do prelab
- Homework 3 (Taylor 5.1, 5.20, 5.36)
- Read Taylor through chapter 5 and chapter 8