The Nature of Scientific Progress

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Error Analysis

Lecture 3

Physics 2CL

Summer 2010

Outline

- How scientific knowledge progresses:
  - Replacing models
  - Restricting models
- What you should know about error analysis (so far) and more
- Limiting Gaussian distribution
- Back to series and parallel circuits
- Preview of Exp. 1
- Reminder

Lab Objectives, This Week

- Models
  - Extend knowledge of resistors, capacitors and inductors to a circuit
  - Testing/limiting
    - Models of circuit analysis
    - Models for circuit responses
- Physics
  - Capacitor charging/discharging
  - Oscillation and Damping
  - Resonance
- More complicated analysis

Models

- Invented
- Properties correspond closely to real world
- Must be testable

How Models Fit Into Process of Doing Science

- Science is a process that studies the world by:
  - Limiting the focus to a specific topic (***making a choice***)
  - Observing (***making a measurement***)
  - Refining Intuitions (***making sense***) Creating
  - Extending (***seeking implications***) Predicting
  - Demanding consistency (***making it fit***) Refining or Replacing
  - Community evaluation and critique

How Models Change

- If models disagree with observation, we change the model
  - Refine - add to existing structure
  - Restrict - limit scope of utility
  - Replace - start over

Refining

- Original model consistent w/ new observations, but not complete
- Extend model to account for new observations
- May include new concepts

  e.g. Model of interaction between charged objects; to include interactions between charged & uncharged add concept of induced charge

Restriction

- New model correct in situations where old isn't
- New model agrees w/ old over some range

⇨ Old still useful in limited range

e.g. General relativity vs. classical gravitational theory

Replacement

- Old model can’t be extended consistently
- Replace entire model

⇨ Earlier observations provide limits for new model

e.g. Geocentric vs heliocentric models for solar system

Random and independent?

Yes

- Estimating between marks on ruler or meter
- Releasing object from ‘rest’
- Mechanical vibration
- o **Judgment**
- o **Problems of definition**

No

- End of ruler screwy
- Reading meter from the side (speedometer effect)
- Scale not zeroed Reaction time delay
- o **Calibration**
- o **Zero**

Random & independent errors:

$$q = x + y - z$$
$$\delta q = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$
$$\delta q \le \delta x + \delta y + \delta z$$

$$q = Bx$$
$$\delta q = |B|\delta x$$
$$\frac{\delta q}{|q|} = \frac{\delta x}{|x|}$$

$$q = x \times y \div z$$
$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2}$$
$$\frac{\delta q}{|q|} \le \frac{\delta x}{|x|} + \frac{\delta y}{|y|} + \frac{\delta z}{|z|}$$

$$q = q(x, y, z)$$
$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$
$$\frac{\delta q}{|q|} \le \left|\frac{\partial q}{\partial x}\right|\delta x + \left|\frac{\partial q}{\partial y}\right|\delta y + \left|\frac{\partial q}{\partial z}\right|\delta z$$

Propagation in formulas

Independent

Propagate error in steps

For example:

$$q = \frac{x}{y - z}$$

- First find

$$p = y - z$$

$$\delta p = \sqrt{(\delta y)^2 + (\delta z)^2}$$

- Then

$$q = \frac{x}{p}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta p}{p}\right)^2}$$

Propagation in formulas

Dependent

Variable(s) appear more than once

⇨ Errors compensate

For example:

$$\boxed{q = \frac{x}{x - z}}$$

Use general propagation formula:

$$q = q(x, y, z)$$

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

An Important Simplifying Point

$$h = \frac{1}{2} g t^2$$

$$g = 2h/t^2, \delta h/h = 5\%, \delta t/t = 0.1\%$$

Requires random & ind. errors!

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(2\frac{\delta t}{t}\right)^2}$$

$$\delta g/g = \sqrt{5\%^2 + (2 \times 0.1\%)^2}$$

$$\delta g/g = 0.050039984 = 5\%$$

- Often the error is dominated by error in least accurate measurement

⇨Simplifies calc.

⇨Suggests improvements in experiment

More Complicated Example

What is error in

$$q = Bx - y^n$$

Start with

$$\delta q^2 = (\delta(Bx))^2 + (\delta(y^n))^2$$

$$\delta(Bx) =$$

$$B\,\delta(x)$$

Example continued

$\delta(y^n) =$

$= \varepsilon(y^n) * y^n$

$= n\ \varepsilon(y)\ y^n = n\ y^{n-1}\ \delta y$

Then,

$\delta q^2 = (B\ \delta x)^2 - (n\ y^{n-1}\ \delta y)^2$

Analyzing Multiple Measurements

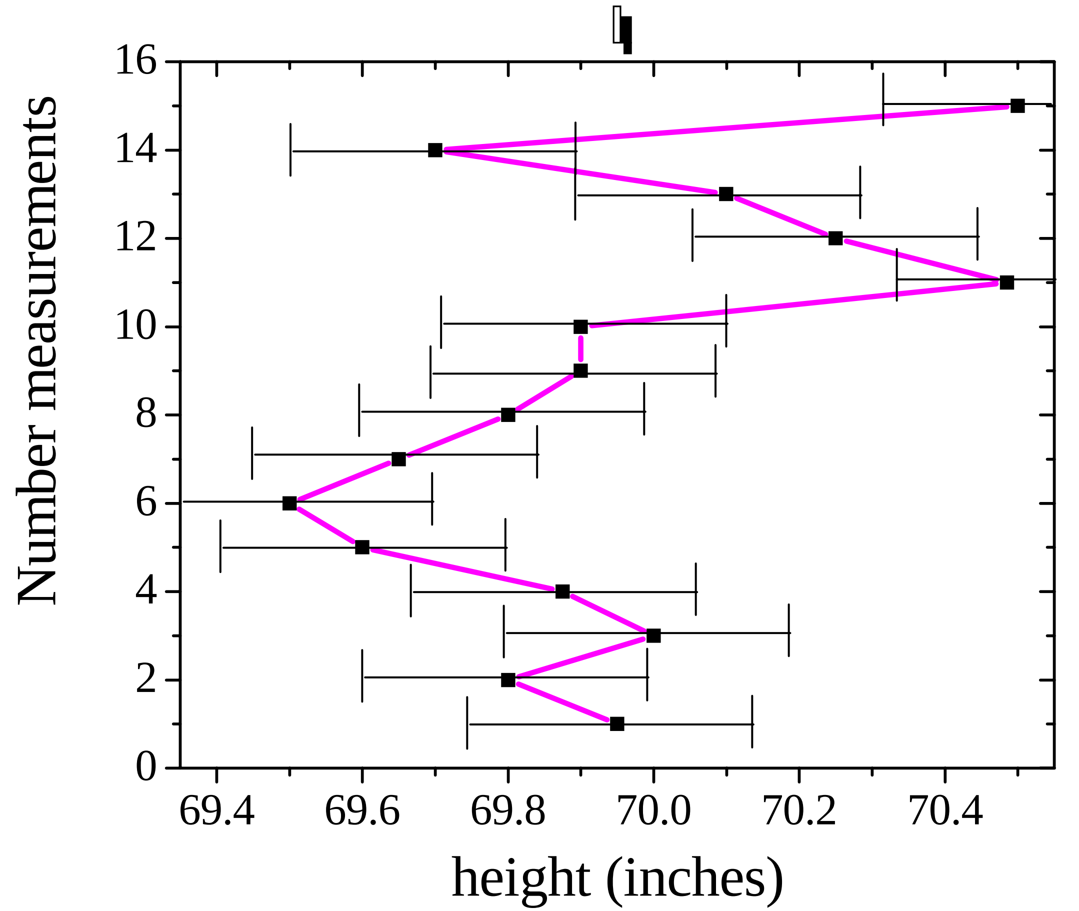
- Repeat measurement of *x* many times
- Best estimate of *x* is average (mean)

$$x_1, x_2, \cdots, x_N$$

$$x_{best} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

$$\bar{x} = \frac{\sum x_i}{N}$$

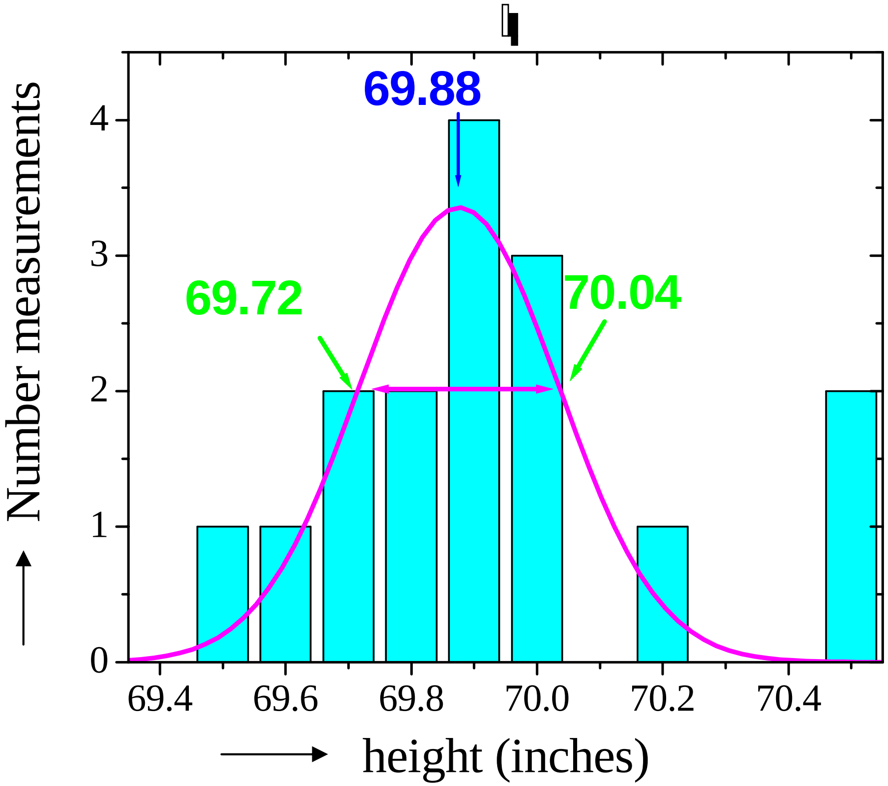
Repeated Measurements



How are Measured Values Distributed?

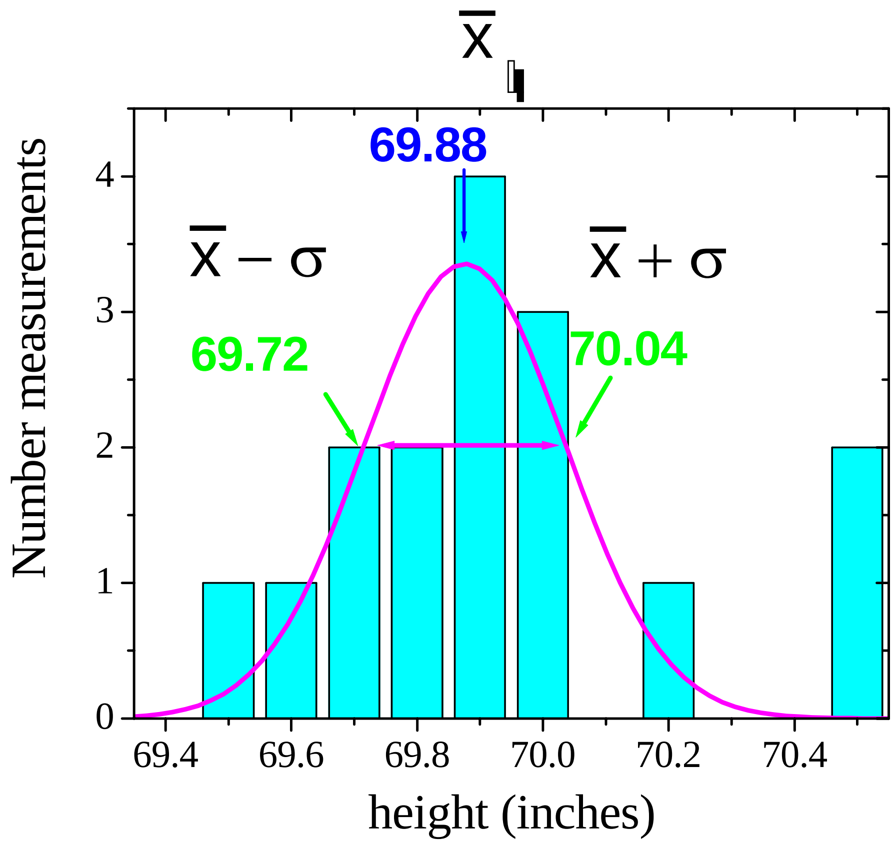
- If errors are random and independent:
  - Expect most values near true value
  - Expect few values far from true value

⇨ Assume values are distributed *normally*



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\overline{x})^2 / 2\sigma^2\right)$$

Normal Distribution



$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-(x-\bar{x})^2 / 2\sigma^2\right)$$

Error of an Individual Measurement

- How precise are measurements of *x*?
- Start with each value's deviations from mean
- Deviations average to zero, so square, then average, then take square root
- ~68% of time, x_i will be w/in σ_x of true value

$$d_i \equiv x_i - \bar{x}$$

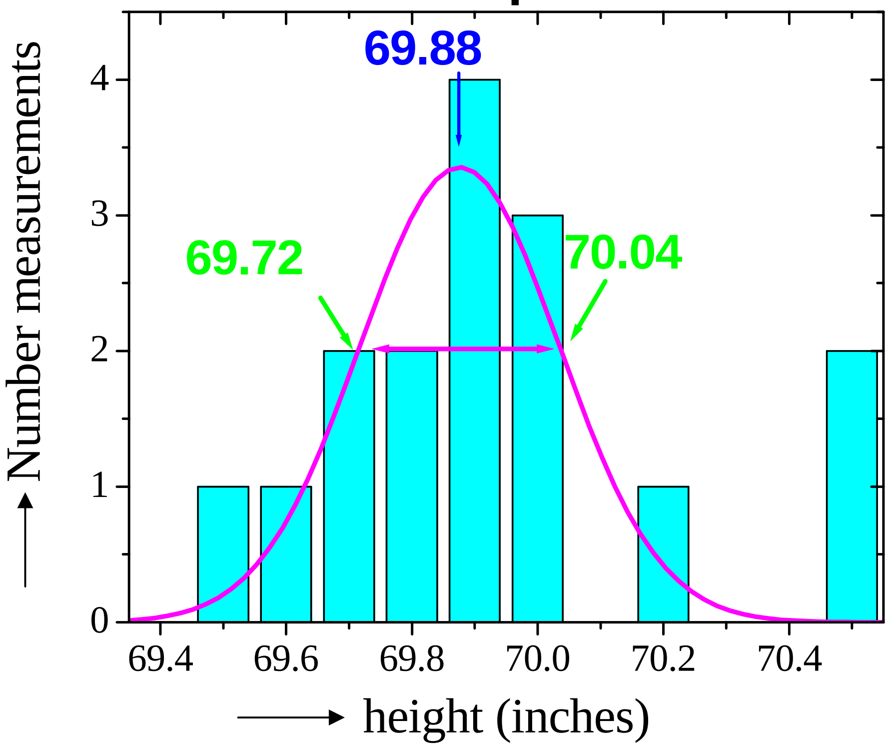
$$\bar{d} = 0$$

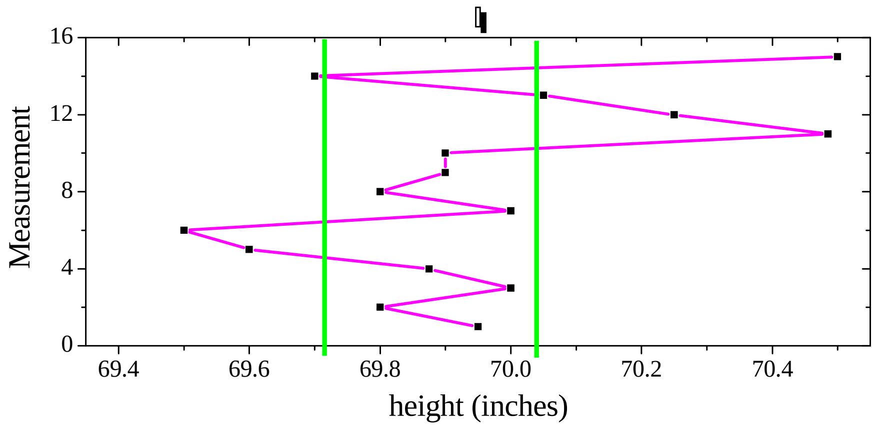
$$\sigma_x \equiv \sqrt{\overline{(d_i)^2}}$$

$$= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Take σ_x as error in individual measurement - called standard deviation

Standard Deviation





Error of the Mean

- Expect error of mean to be lower than error of the measurements it's calculated from
- Divide SD by square root of number of measurements
- Decreases slowly with more measurements

Standard Deviation of the Mean (SDOM)

or

Standard Error

or

Standard Error of the Mean

$$\boxed{\sigma_{\bar{x}} = \sigma_x \big/ \sqrt{N}}$$

Summary

- Average

$$\bar{x} = \frac{\sum x_i}{N}$$

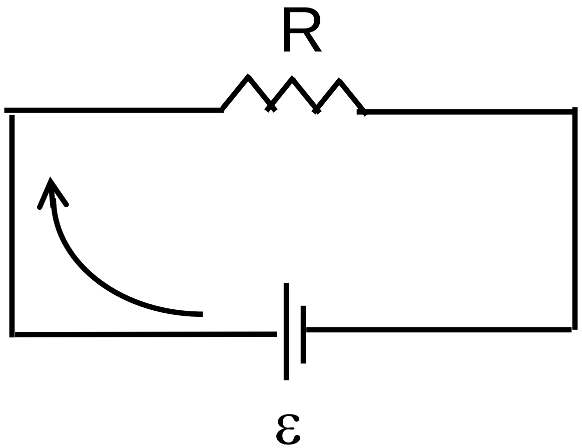
- Standard deviation

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

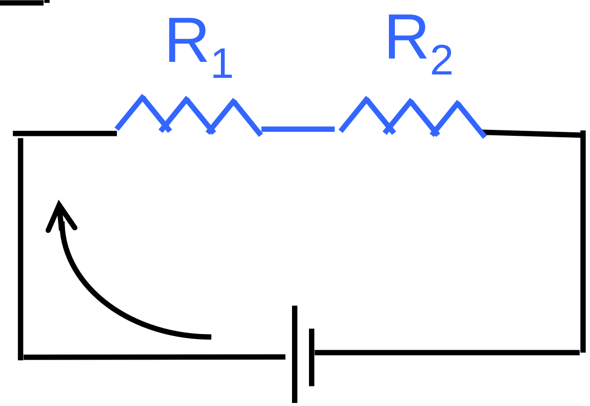
- Standard deviation of the mean

$$\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$$

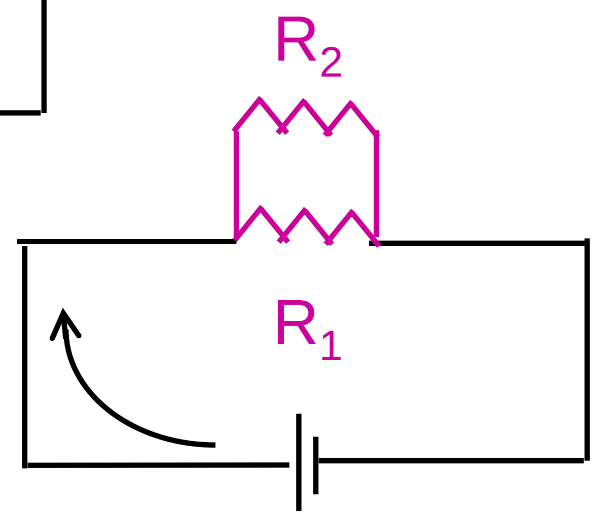
Series and Parallel



Simple circuit



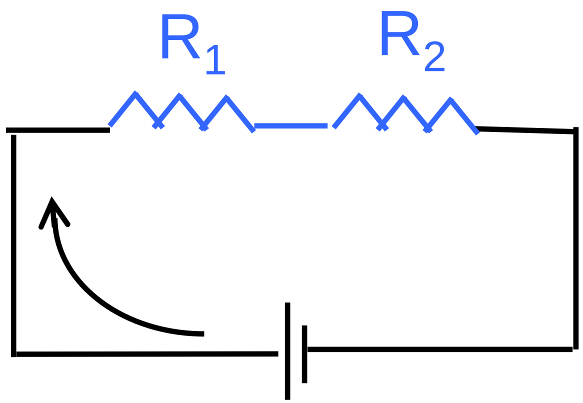
series



parallel

Uncertainties - Series and Parallel

series

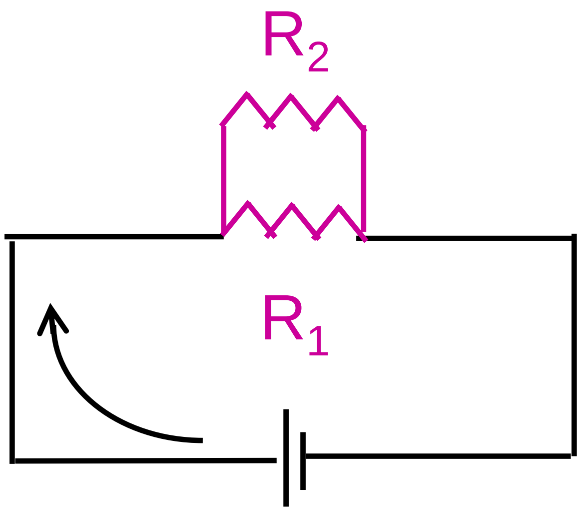


$$R_{TOTAL} = R_1 + R_2$$

$$\delta R_{TOTAL} = ??$$

$$\delta R_{TOTAL} = \sqrt{\delta R_1^2 + \delta R_2^2}$$

parallel



$$1/R_{TOTAL} = 1/R_1 + 1/R_2$$

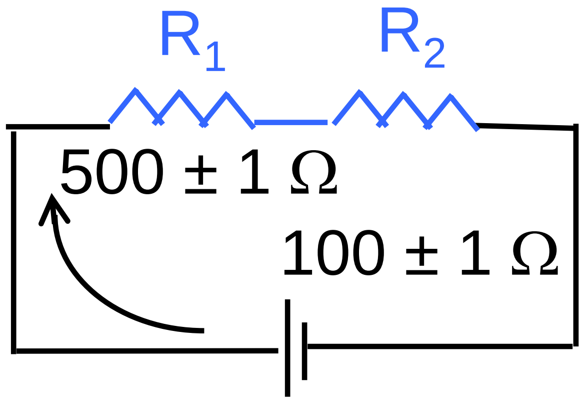
$$R_{TOTAL} = R_2 * R_1/(R_1 + R_2)$$

$$\varepsilon R_{TOTAL} = ??$$

$$\varepsilon R_{TOTAL} = \sqrt{(\delta R_T/\delta R_1) * \delta R_1)^2 + (\delta R_T/\delta R2) * \delta R_2)^2}$$

Quoting Uncertainties

series

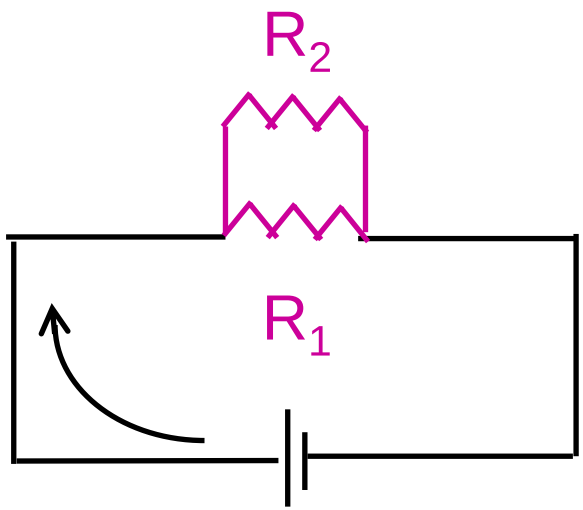


$$R_{TOTAL} = R_1 + R_2$$

$$\delta R_{TOTAL} = \sqrt{\delta R_1^{\,2} + \delta R_2^{\,2}}$$

$$R_{TOTAL} = (600 \pm 1)\ \Omega$$

parallel



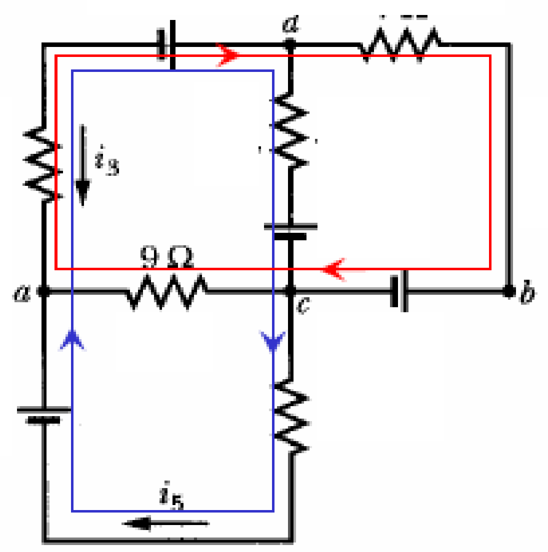
$$R_{TOTAL} = R_2 * R_1/(R_1 + R_2)$$

$$R_{TOTAL} = 83\ \Omega$$

$$\varepsilon R_{TOTAL} = \sqrt{(\delta R_T/\delta R_1)*\delta R_1)^2 + (\delta R_T/\delta R2)*\delta R_2)^2}$$

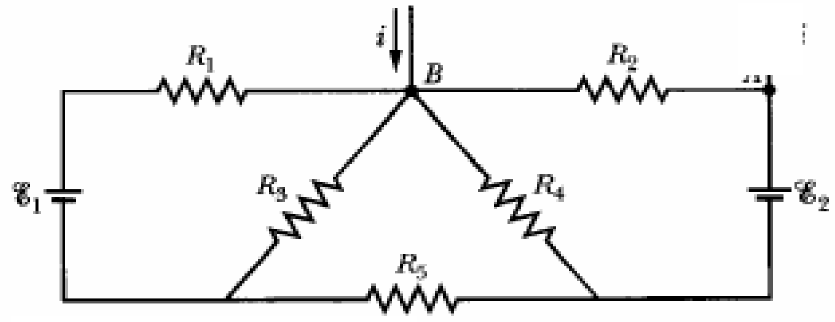
Circuit Analysis

Loop rule



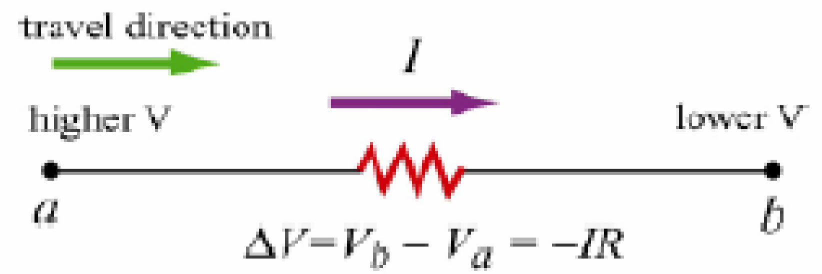
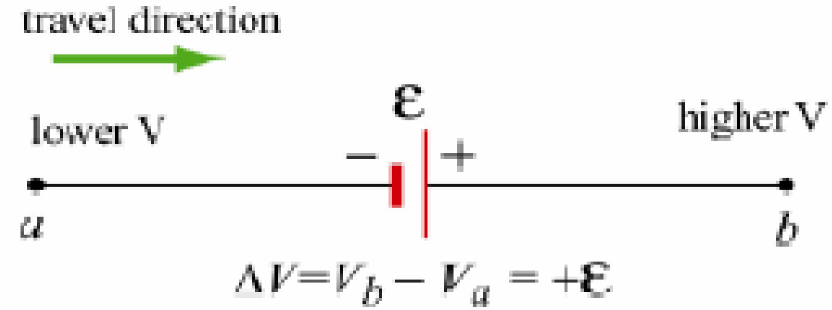
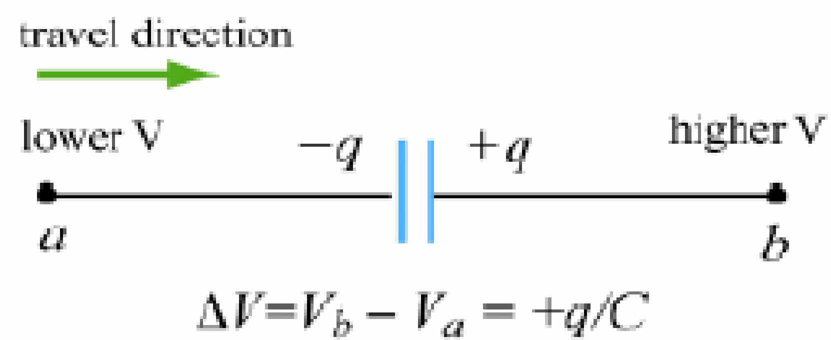
$$\sum_k \Delta V_k = \sum_j E_j$$

Junction rule

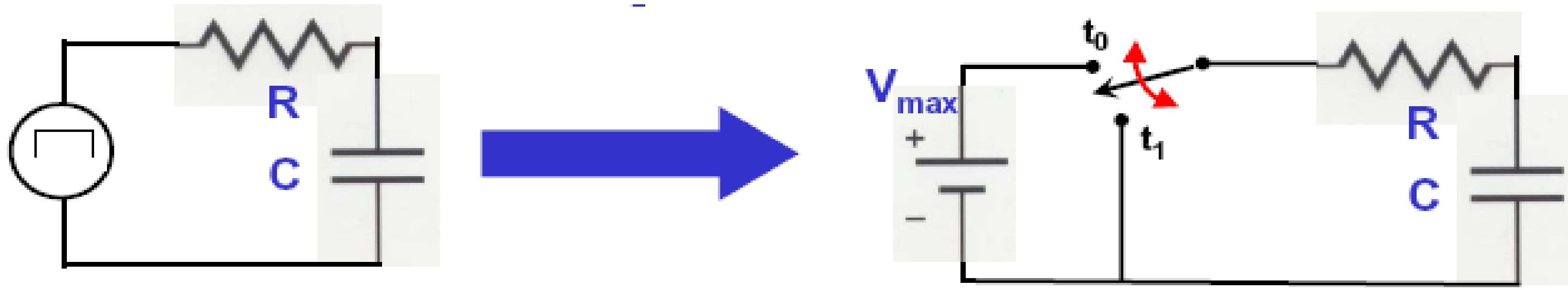


$$\sum_k i_k = 0$$

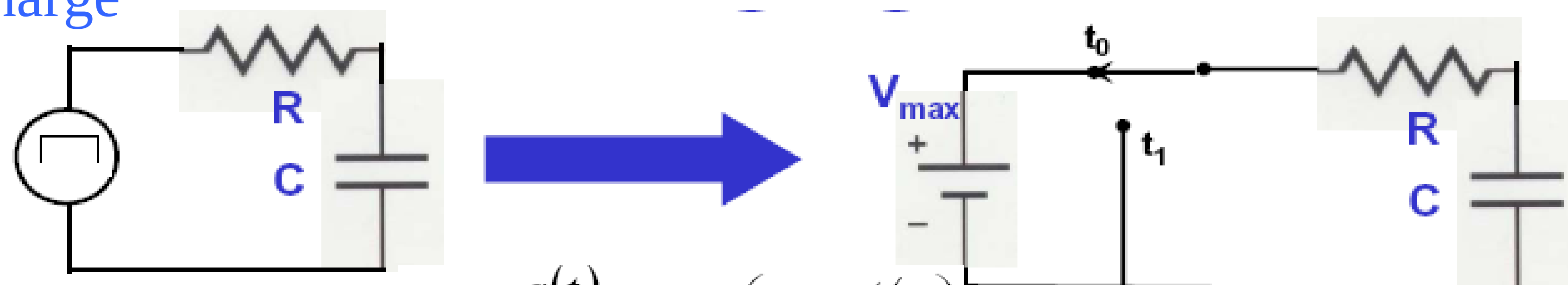
Kirchoff’s Rules

resistor	
emf source	
capacitor	

Circuit Exp. 1

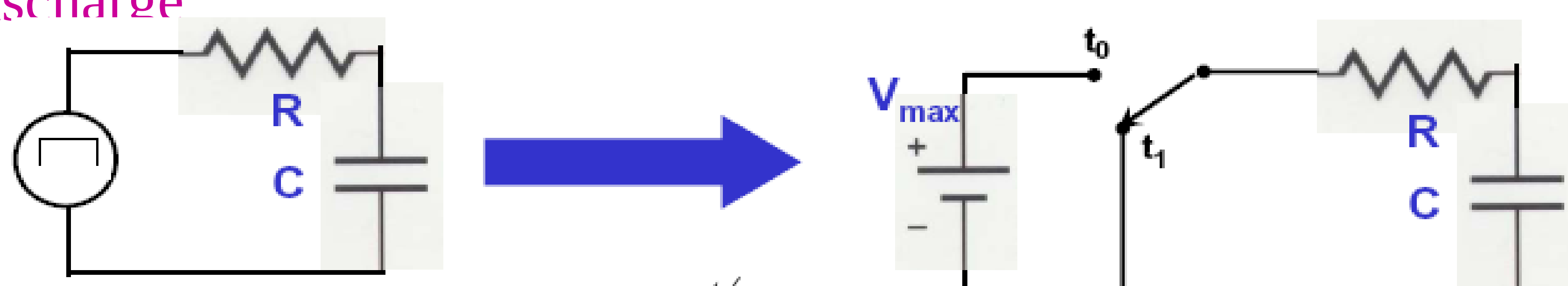


Charge



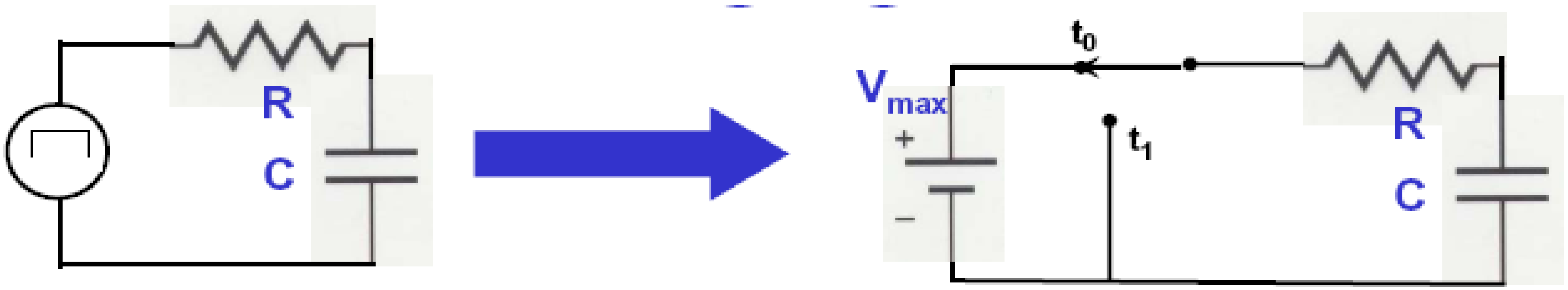
$$V_c = \frac{q(t)}{C} = V_{\max}\left(1 - e^{-t/RC}\right)$$

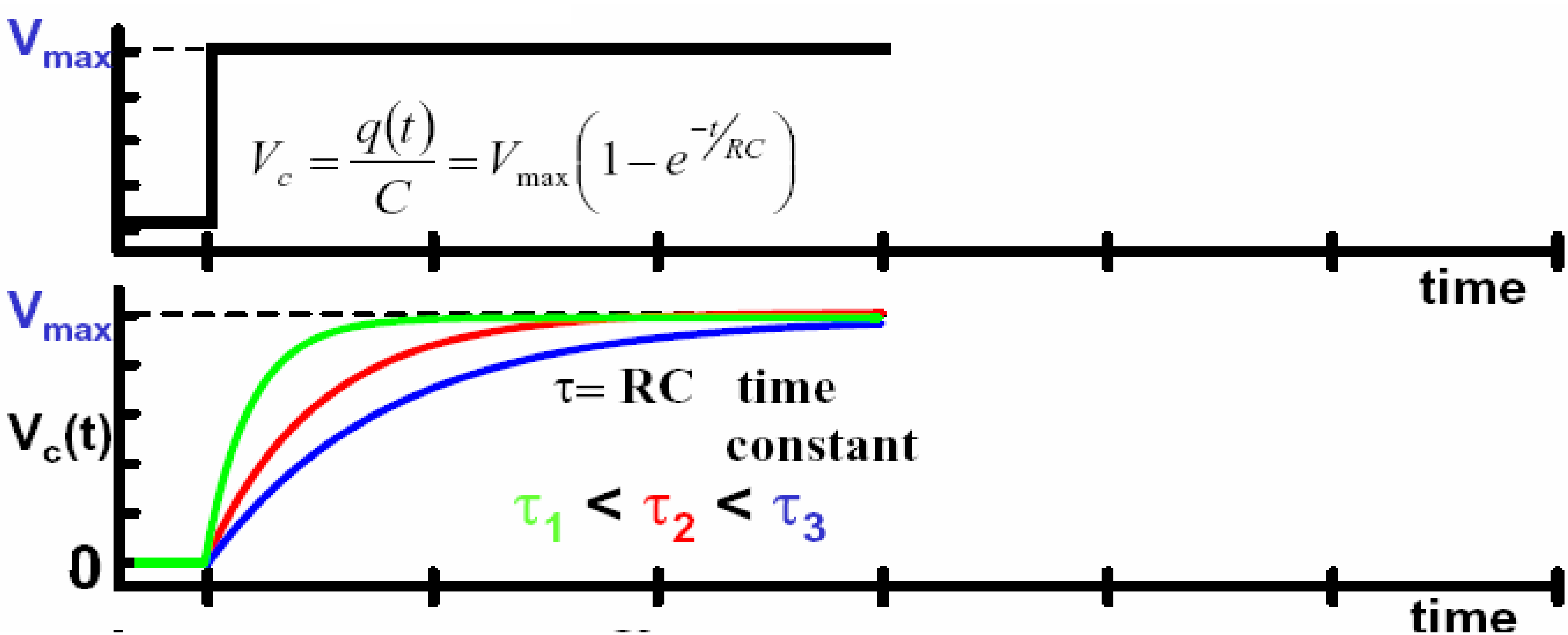
Discharge



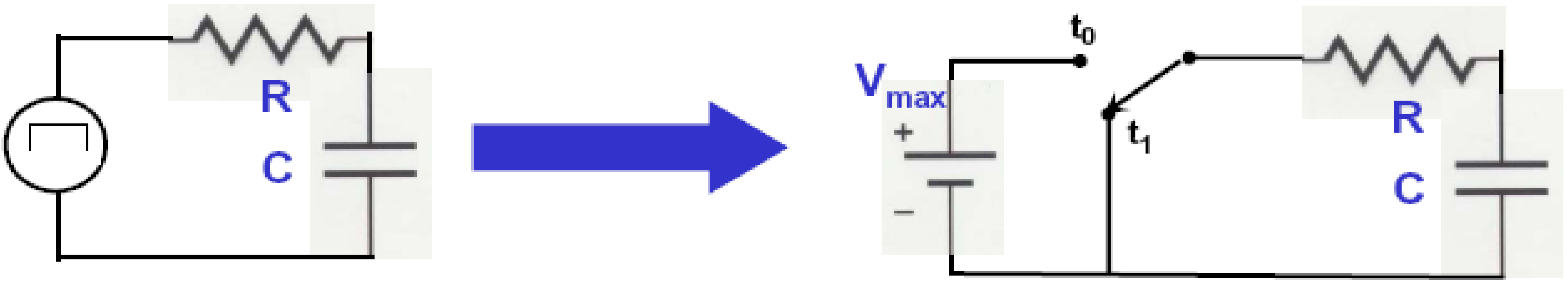
$$V_C(t) = V_{\max} e^{-t/RC}$$

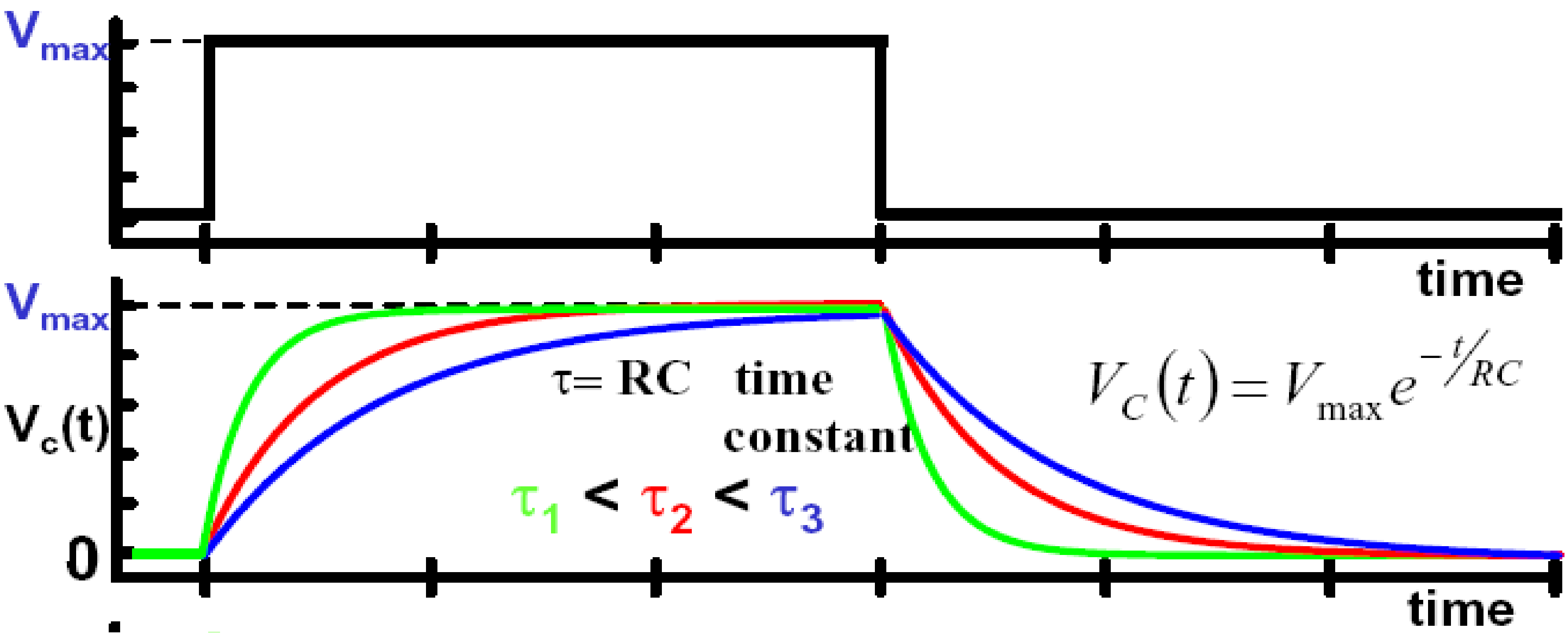
Charge Capacitor



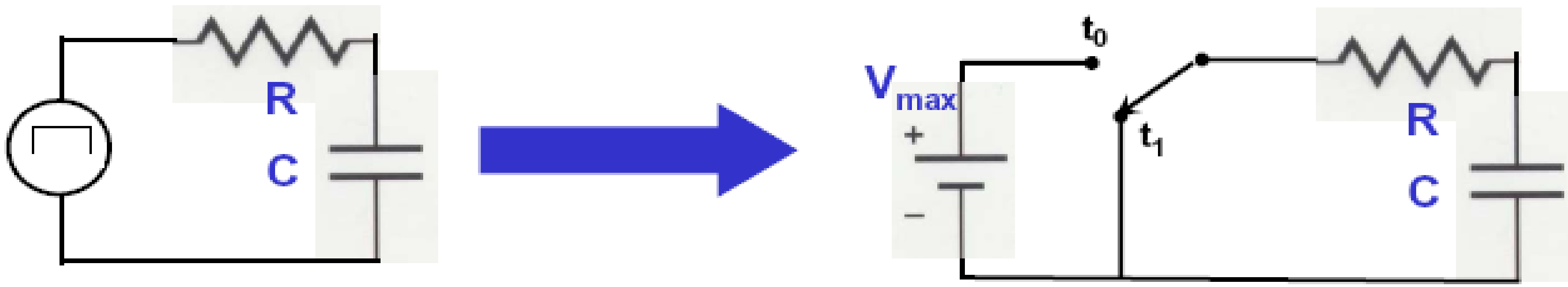


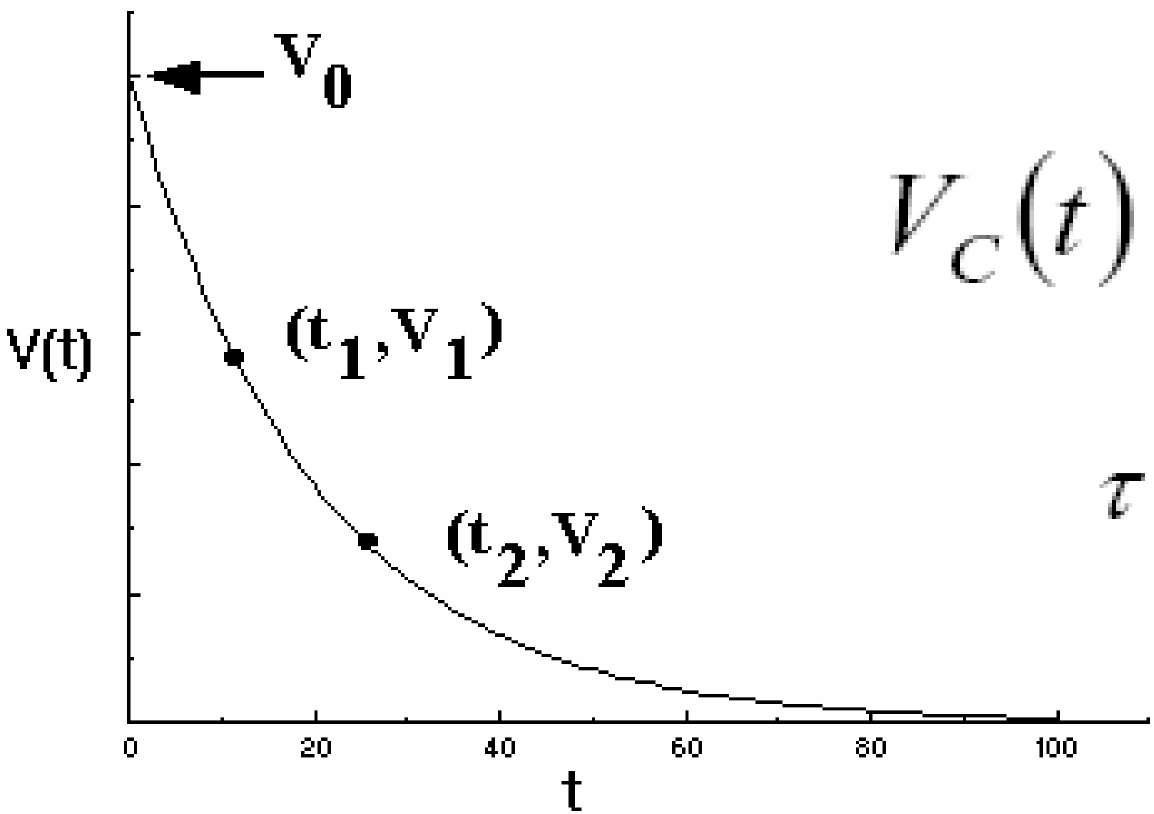
Discharge Capacitor





Discharge Capacitor

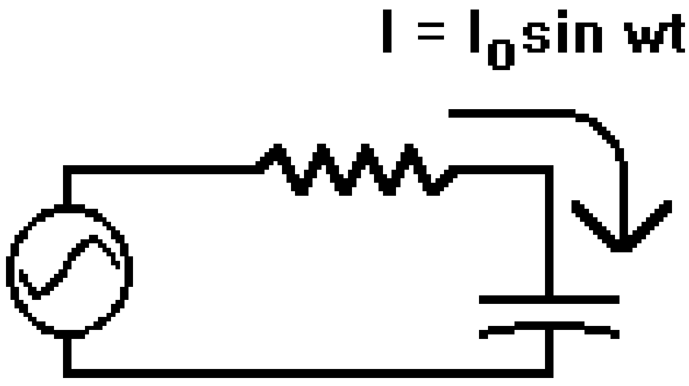




$$V_C(t) = V_{\max} e^{-t/RC}$$

$$\tau = \frac{t_2 - t_1}{\ln(V_1 / V_2)}$$

RC in Frequency Domain



$V = Z*I$ $\qquad Z_R = R$ $\qquad Z_C = 1/i\omega C$

$Z_L = i\omega L$

Remember

- Experiment writeup
- Prelab questions
- Bring Exp. 0 for Origin reference
- Hints in lab manual
- t-values = $x_{meas} - x_{exp} / \sigma$
- Read Taylor through Chapter 4
- Homework for lab 2 (Problems 4.2, 4.18)