Uncertainty, Measurement, and Models

Lecture 2

Physics 2CL

Summer Session 2010

Outline

- Brief overview of lab write-ups
- What uncertainty (error) analysis can for you
- Issues with measurement and observation
- What does a model do?
- Brief overview of circuit analysis

Preparing for Lab

- Write-up due at the end of the last meeting
- Prepare by doing Taylor homework and prelab questions

Lab Write-ups

- Begin with lab number & title, date and you and your partners name
- Start with Taylor homework and prelab questions
- State briefly the objective
- Record all data with units and uncertainties
- Brief description of procedure
- Make clear labeled diagrams of setups
- Use graphs to present data, label axes, plot error bars - Origin

Lab Write-up continued

- Include and justify functional fit of data
- Show calculations of final derived quantities, include uncertainty analysis
- State results and comment on the agreement with expectations (or not)
  - Be quantitative (within uncertainty, t-value)

What is uncertainty (error)?

- Uncertainty (or error) in a measurement is not the same as a mistake
- Uncertainty results from:
  - Limits of instruments
    - finite spacing of markings on ruler
  - Design of measurement
    - using stopwatch instead of photogate
  - Less-well defined quantities
    - composition of materials

Understanding uncertainty is important

- for comparing values
- for distinguishing between models
- for designing to specifications/planning

Measurements are less useful (often useless) without a statement of their uncertainty

An example

Batteries

rated for 9 V potential difference across terminals

in reality…

Utility of uncertainty analysis

- Evaluating uncertainty in a measurement and calculated quantities
- Propagating errors – ability to extend results to other measurements
- Analyzing a distribution of values
- Quantifying relationships between measured values

Evaluating error in measurements

- To measure height of building, drop rock and measure time to fall: $d = \frac{1}{2} g t^2$
- Measure times

  2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s
- What is the “best” value
- How certain are we of it?

Calculate “best” value of the time

- Calculate average value (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
  - $\bar{t} = \sum_{i=1}^{n} t_i/n$
  - $\bar{t} = 2.516666666666666666666666$ s
- Is this reasonable?

Uncertainty in time

- Measured values - (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
- By inspection can say uncertainty < 0.3 s
- Calculate **standard deviation**

$$\sigma = \sqrt{\sum (t_i - \overline{t})^2/(n-1)}$$

$\sigma = 0.1949976$ s

$\sigma = 0.2$ s (But what does this mean???)

How to quote best value

- Now what is best quote of average value
  - $\bar{t} = 2.516666666666666666666666$ s
  - $\bar{t} = 2.52$ s
- What is uncertainty
  - Introduce **standard deviation of the mean**

    $\sigma_{\bar{t}} = \sigma/\sqrt{n} = 0.08$ s
- Best value is
  - $\bar{t} = 2.52 \pm 0.08$ s

Propagation of error

- Same experiment, continued…
- From best estimate of time, get best estimate of distance: 30.6 meters
- Know uncertainty in time, what about uncertainty in distance?
- From error analysis tells us how errors propagate through mathematical function

$$\varepsilon(d) = 2 * \varepsilon(t)$$

8% uncertainty or ± 2.4m

Drawing Conclusions: The t - value

- Does value agree or not with accepted value of 30.7m?
- How different is it from the accepted? Introduce “t-value” (*t*)

$$t = |d_{meas} - d_{accept}|/\sigma = 0.1/2.4 = .04$$

- If value difference $\leq$ 1 then they agree
- Later we will learn what this means in a more quantitative way

Expected uncertainty in a calculated sum $a = b + c$

- Each value has an uncertainty
  - $b = \bar{b} \pm \delta b$
  - $c = \bar{c} \pm \delta c$
- Uncertainty for a (δa) is **at most** the sum of the uncertainties
  - $\delta a = \delta b + \delta c$
- Better value for δa is
  - $\delta a = \sqrt{(\delta b^2 + \delta c^2)}$
- Best value is
  - $a = \bar{a} \pm \delta a$

Expected uncertainty in a calculated product $a = b*c$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
- Relative uncertainty for a (εa) is **at most** the sum of the RELATIVE uncertainties
  - $\varepsilon a = \delta a/a = \varepsilon b + \varepsilon c$
- Better value for δa is
  - $\varepsilon a = \sqrt{(\varepsilon b^2 + \varepsilon c^2)}$
- Best value is
  - $a = \overline{a} \pm \varepsilon a$ (fractional uncertainty)

What about powers in a product

$$a = b * c^2$$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
  - $\varepsilon a = \delta a / a$ (relative uncertainty)
- powers become a prefactor (weighting) in the error propagation
  - $\varepsilon a = \sqrt{(\varepsilon b^2 + (2 * \varepsilon c)^2)}$

How does uncertainty in t effect the calculated parameter d ?

- $d = \frac{1}{2} g t^2$

$\varepsilon d = \sqrt{(2 * \varepsilon t)^2} = 2 * \varepsilon t$

$\varepsilon d = 2 * (.09/2.52) = 0.071$

$\delta d = .071 * 31 \text{ m} = 2.2 \text{ m} = 2 \text{ m}$

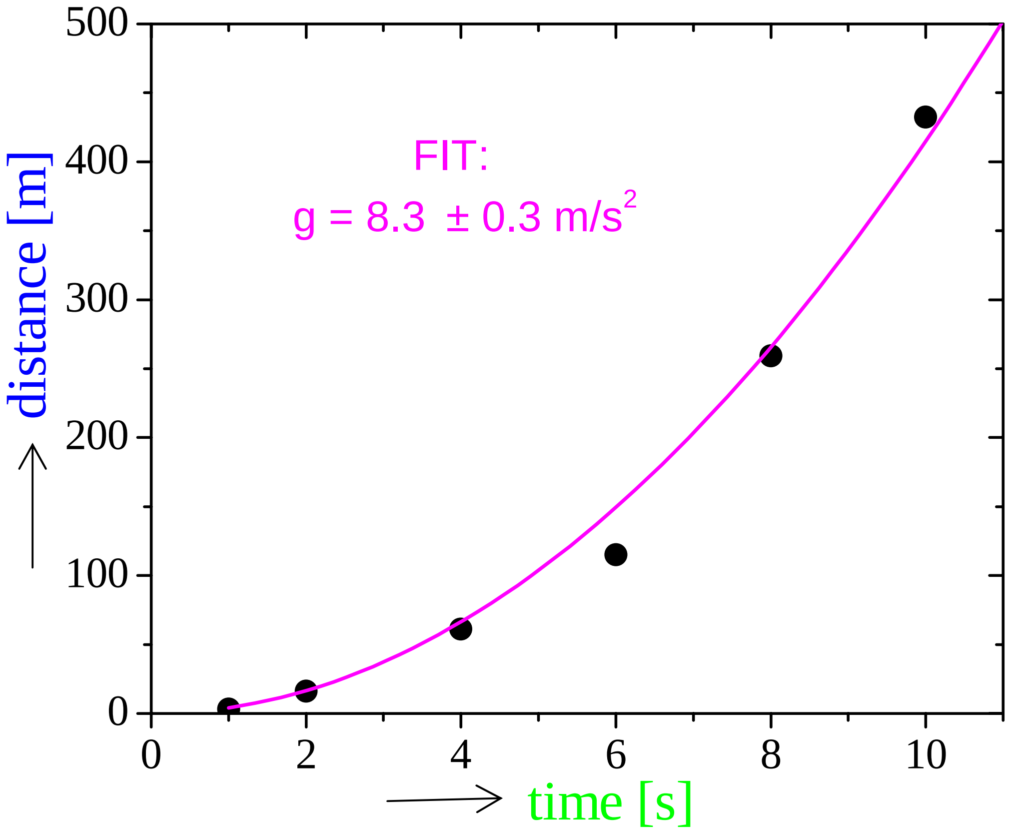
Statistical error

Relationships

- Know there is a functional relation between d and t $d = \frac{1}{2} g t^2$
- d is directly proportional to t^2
- Related through a constant ½ g
- Can measure time of drop (t) at different heights (d)
- plot d versus t to obtain constant

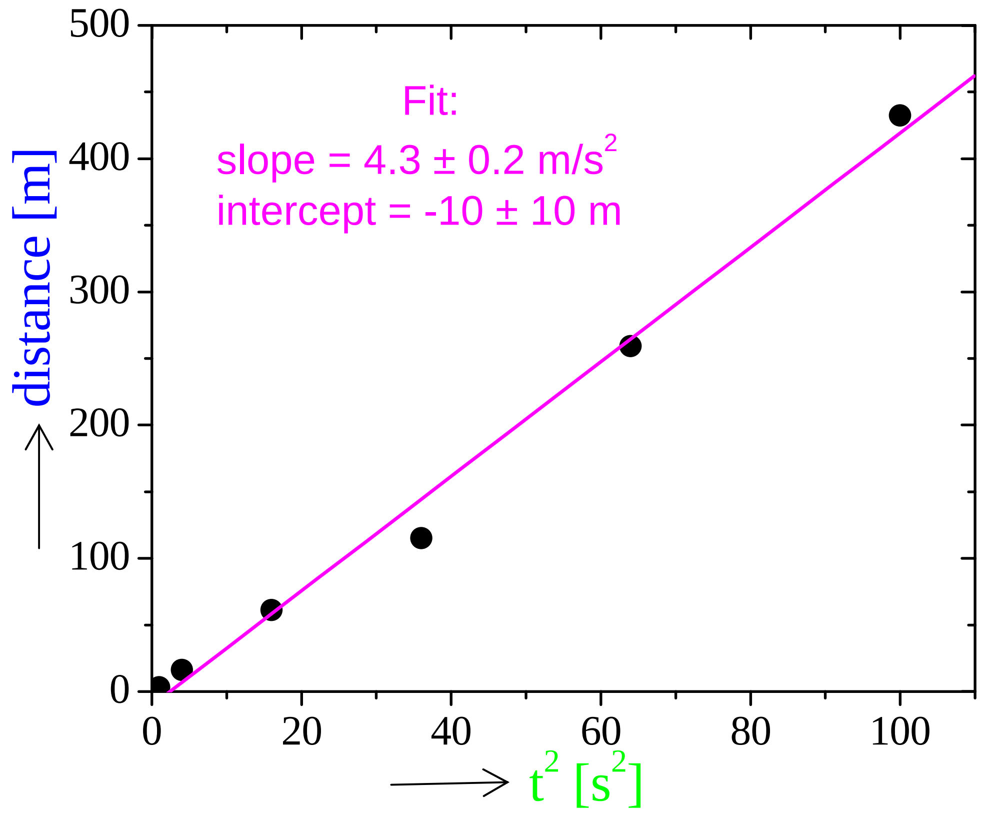
Quantifying relationships

$$d = \tfrac{1}{2} g t^2$$



Different way to plot

$$d = \frac{1}{2} g(t^2)$$



$$\text{slope} = \frac{1}{2} g$$

Compare analysis of SAME data

- From a fit of the curve d versus t obtained
  - $g = 8.3 \pm 0.3$ m/s^2
- From the fit of d versus t^2 obtained
  - $g = 8.6 \pm 0.4$ m/s^2
- Do the two values agree?
- Which is the better value?

Measurement and Observation

- Measurement: deciding the amount of a given property by observation
- Empirical
- Not logical deduction
- Not all measurements are created equal…

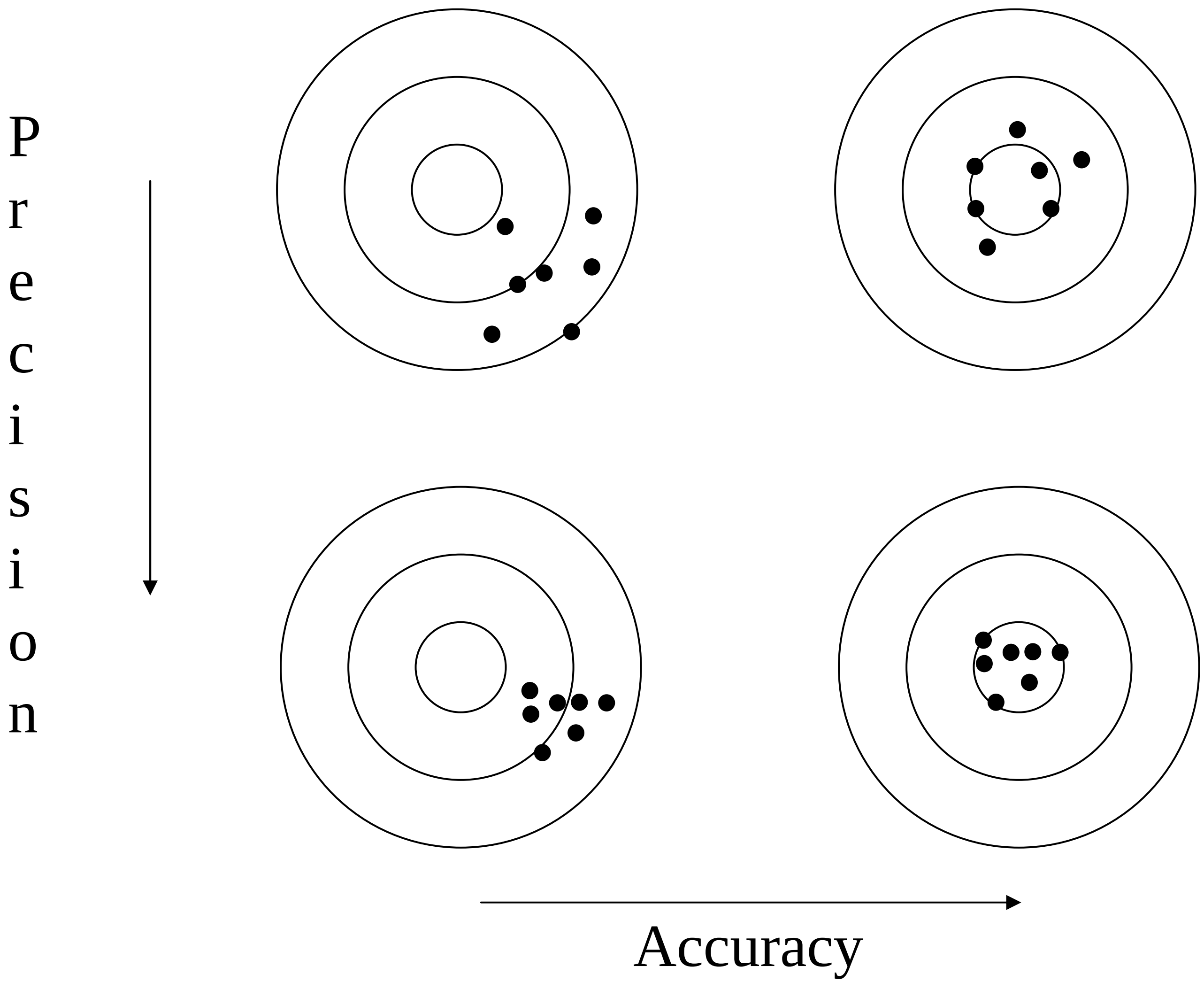
Reproducibility

- Same results under similar circumstances
  - Reliable/precise
- ‘Similar’ - a slippery thing
  - Measure resistance of metal
    - need same sample purity for repeatable measurement
    - need same people in room?
    - same potential difference?
  - Measure outcome of treatment on patients
    - Can’t repeat on same patient
    - Patients not the same

Precision and Accuracy

- Precise - reproducible
- Accurate - close to true value
- Example - temperature measurement
  - thermometer with
    - fine divisions
    - or with coarse divisions
  - and that reads
    - 0 C in ice water
    - or 5 C in ice water

Accuracy vs. Precision



Random and Systematic Errors

- Accuracy and precision are related to types of errors
  - random (thermometer with coarse scale)
    - can be reduced with repeated measurements, careful design
  - systematic (calibration error)
    - difficult to detect with error analysis
    - compare to independent measurement

Observations in Practice

- Does a measurement measure what you think it does? Validity
- Are scope of observations appropriate?
  - Incidental circumstances
  - Sample selection bias
- Depends on model

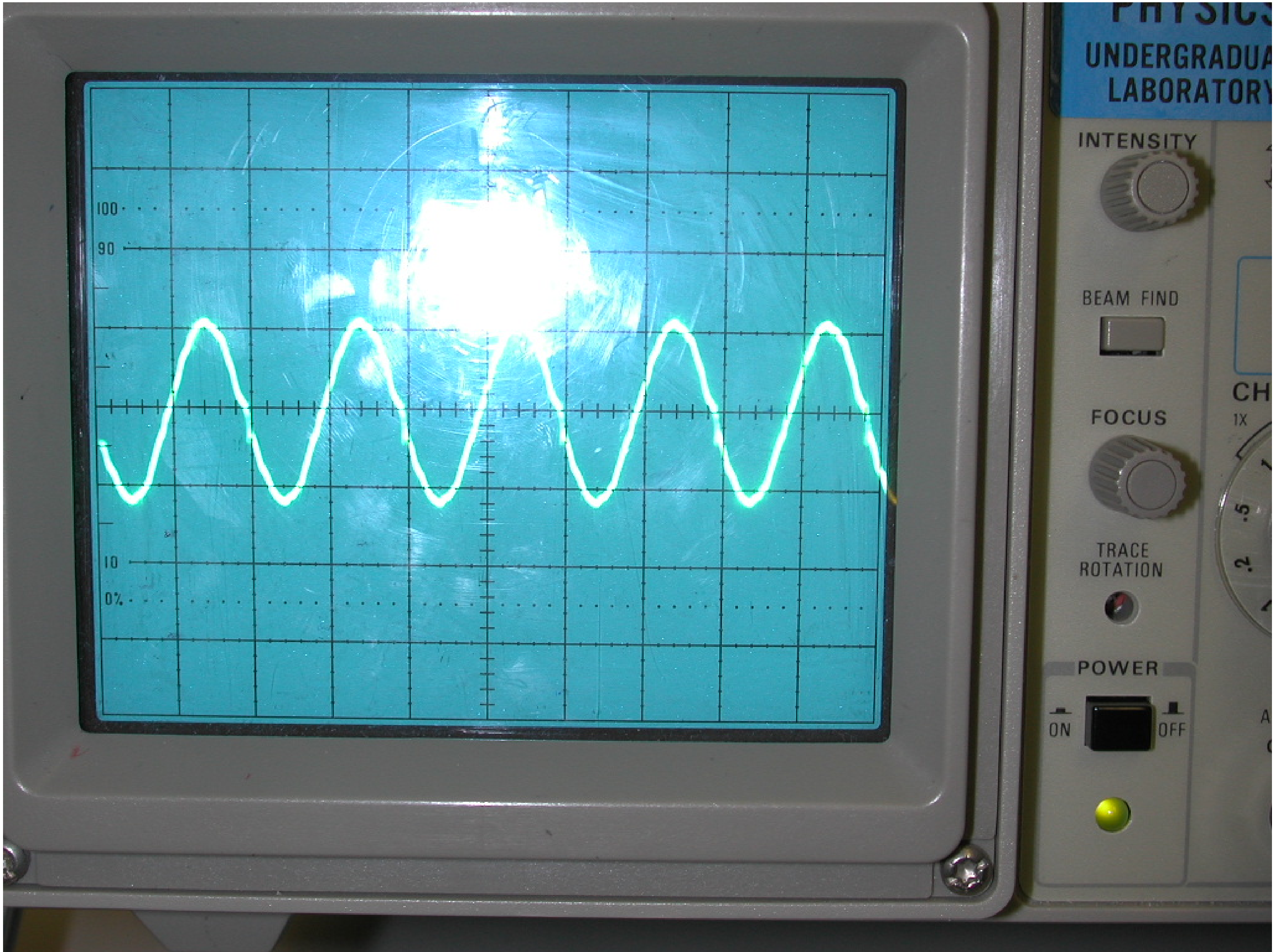
Models

- Model is a construction that represents a subject or imitates a system
- Used to predict other behaviors (extrapolation)
- Provides context for measurements and design of experiments
  - guide to features of significance during observation

Testing model

- Models must be consistent with data
- Decide between competing models
  - elaboration: extend model to region of disagreement
  - precision: prefer model that is more precise
  - simplicity: Ockham’s razor

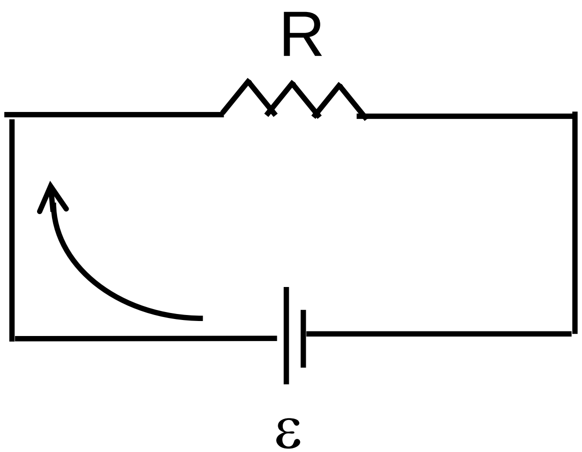
Oscilloscope – screen



Oscilloscope – voltage scale

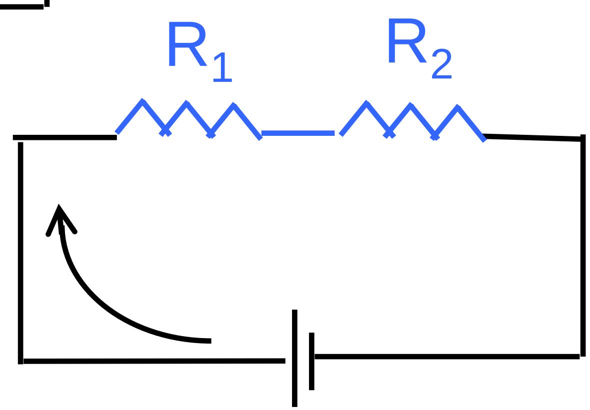
Oscilloscope – time scale

Series and Parallel

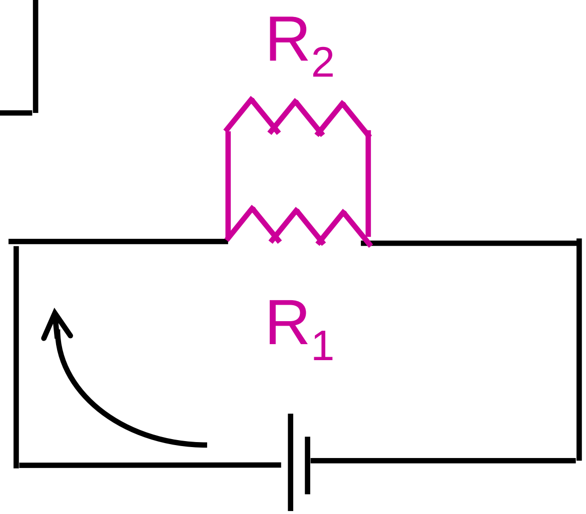


Simple circuit

series

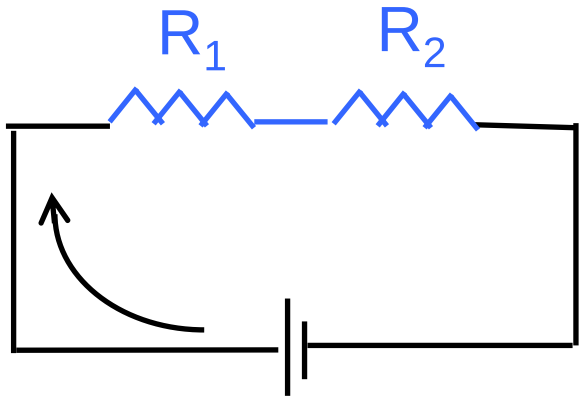


parallel



Uncertainties - Series and Parallel

series

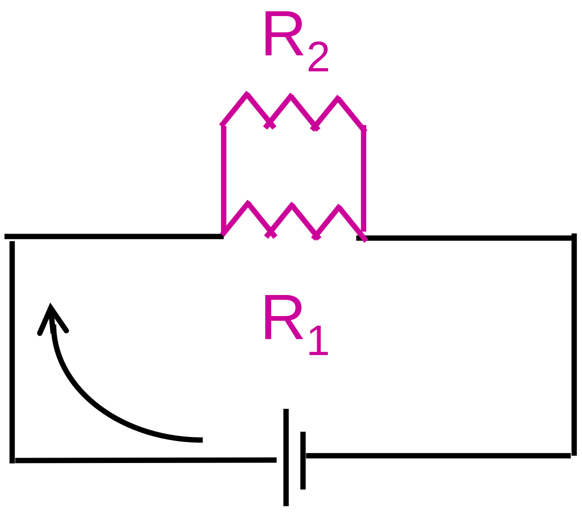


$$R_{TOTAL} = R_1 + R_2$$

$$\delta R_{TOTAL} = ??$$

$$\delta R_{TOTAL} = \sqrt{\delta R_1^2 + \delta R_2^2}$$

parallel



$$1/R_{TOTAL} = 1/R_1 + 1/R_2$$

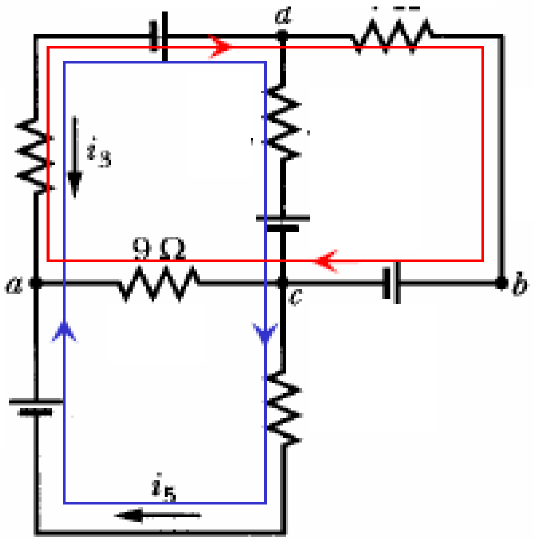
$$R_{TOTAL} = R_2 * R_1/(R_1 + R_2)$$

$$\varepsilon R_{TOTAL} = ??$$

$$\varepsilon R_{TOTAL} = \sqrt{\varepsilon(R_2 * R_1)^2 + \varepsilon(R_2 + R_1)^2}$$

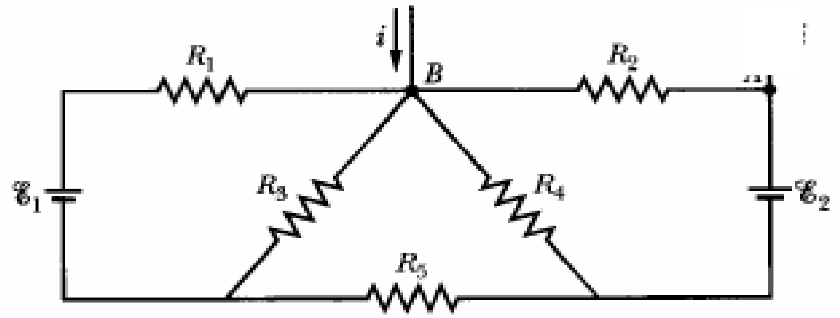
Circuit Analysis

Loop rule



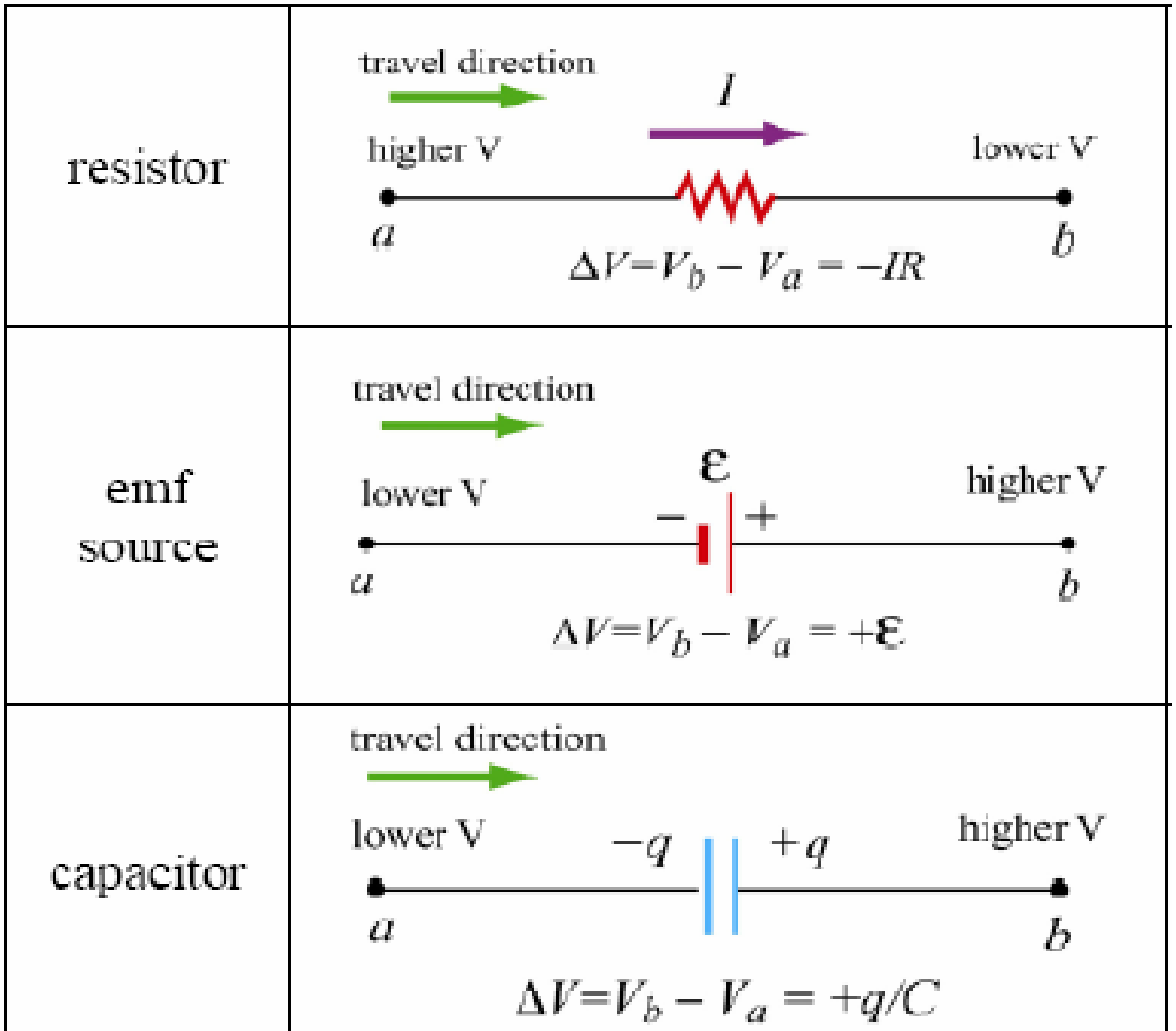
$$\sum_k \Delta V_k = \sum_j E_j$$

Junction rule

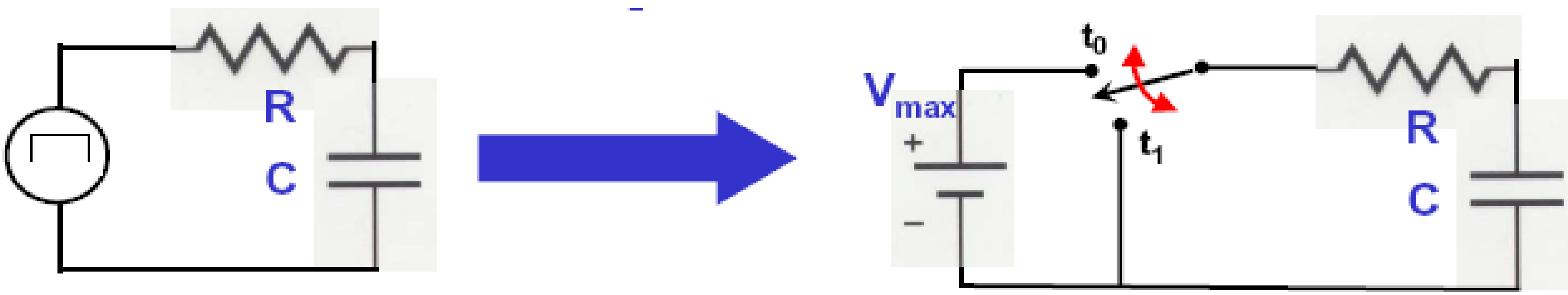


$$\sum_k i_k = 0$$

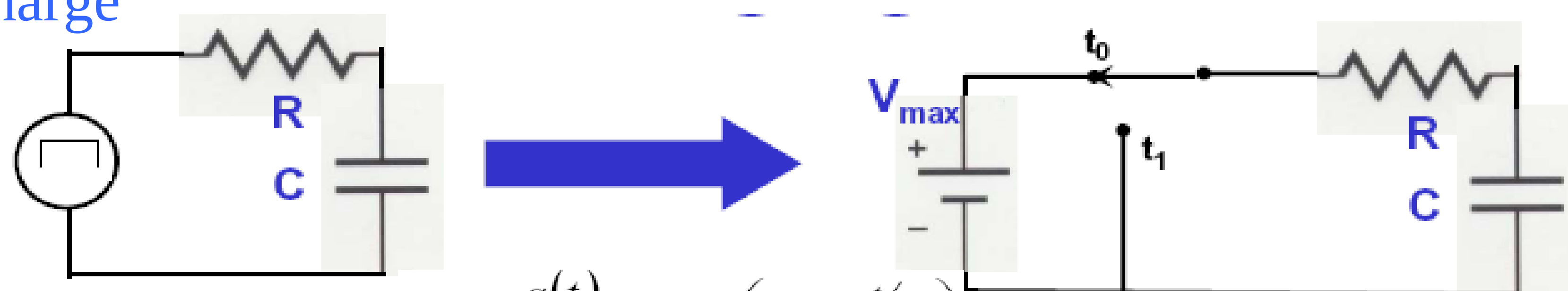
Kirchoff's Rules



Circuit Exp. 1

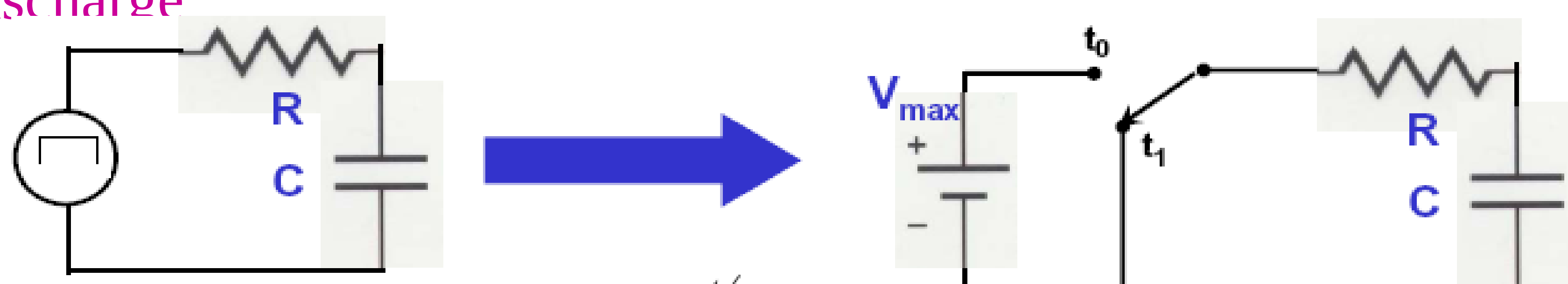


Charge



$$V_c = \frac{q(t)}{C} = V_{max}\left(1 - e^{-t/RC}\right)$$

Discharge



$$V_C(t) = V_{max} e^{-t/RC}$$

Reminder

- perform lab #0
- Read and Prepare for lab # 1 on Tuesday/Wednesday
- Read Taylor through chapter 3
- Do assigned homework – Taylor problems 3.7, 3.36, 3.41