

Experiment 6 – Lenses and the Human Eye

Introduction

Lenses focus or spread light. An understanding of the lenses in Experiment 7 will help you to understand more complicated structures with multiple lenses, such as telescopes and microscopes.

1 Physics

1.1 Review of Reflection and Refraction

When a parallel beam of light is incident on a plane surface of a material that is transparent (e.g. glass or water) two beams are formed as shown in Figure 1. The following laws describe the reflected and refracted beams with respect to the normal to the surface:

- 1) The incident, reflected and refracted rays lie in the same plane as the normal to the surface.
- 2) The angle of reflection is equal to the angle of incidence for all wavelengths and any pair of substances.
- 3) For monochromatic (only one wavelength present) light, the ratio of sine of the angle of incidence to the sine of the angle of refraction is a constant called the refractive index.

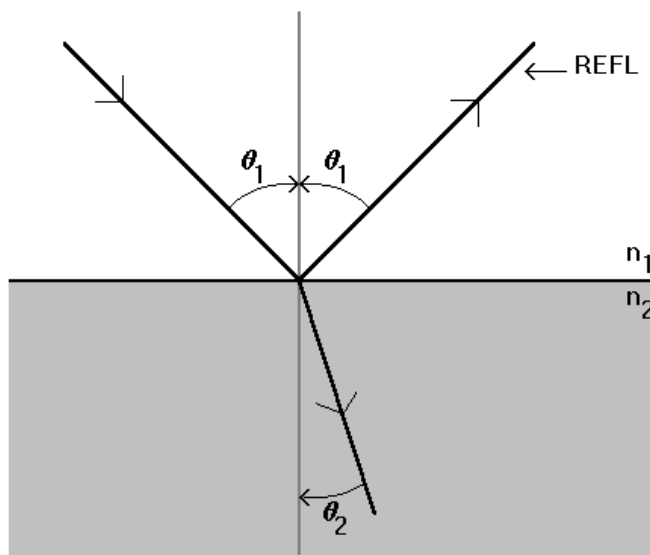


Figure 1 Reflection and Refraction at a Plane Surface

These basic properties of light (and all electromagnetic radiation) enable the formation of images by refraction from curved surfaces. In the first part of this Experiment, you will study the properties of thin convergent and divergent lenses and their combinations. In the latter part of the Experiment you will make some studies of the human eye and the way its optical performance can be modified by the use of lenses.

1.2 Summary of Thin Lens Theory – Convergent Lenses

A convergent lens has two foci, F_1 and F_2 . For a double convex lens with symmetric lens concavity, such as the one in the picture (and the ones you will use in the laboratory) F_1 and F_2 are the same distance from the center of the lens. In general, $f_1 \neq f_2$ but in this case $f_1 = f_2$ and $f \equiv f_1 = f_2$ will be referred to as the focal length (f). In order to construct the following picture 3 principal rays must be identified.

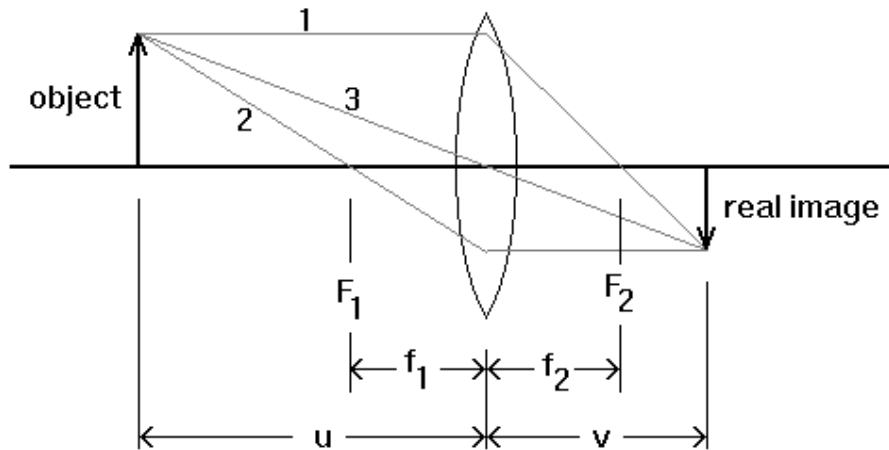


Figure 2 Real Image formation by a Convergent Lens

Parallel Ray (1): A ray parallel to the axis on the incident side passes through the focus on the other side.

Focal Ray (2): A ray through the focus on the incident side emerges parallel.

Center Ray (3): A ray directed towards the center of the lens on the incident side emerges undeflected.

These three rules allow geometrical construction of the image formed of any object. We assume the object to be on the left of the lens and denote its distance from the lens by u , which is greater than zero. The distance of the image from the lens is denoted by v . v is taken as positive if the image is on the opposite side of the lens as the object and negative if it is on the same side. One can then prove the so-called lens formula connecting u , v , and f :

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (f > 0) \quad (1)$$

The magnification, i.e. ratio of image to object size, is

$$m = \frac{-v}{u} \quad (2)$$

Here a negative m denotes an inverted image, a positive m an erect one.

If the rays originating in an object point actually converge on an image point, so that they could be received on a screen, the image is called real (Figure 2). If the rays do not actually converge but appear to come from the image point, the image is called virtual (Figure 3).

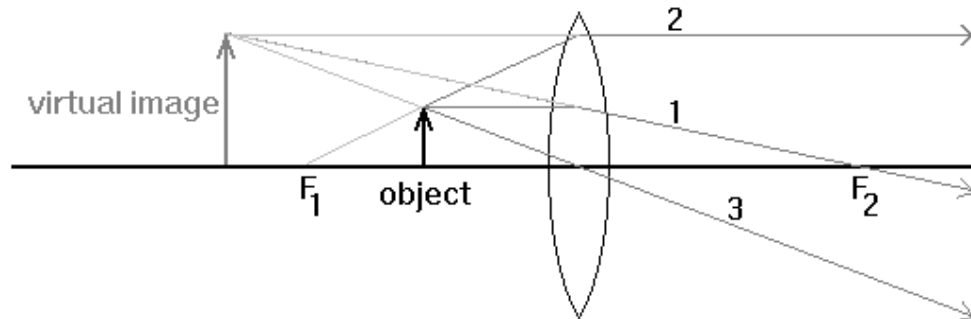


Figure 3 *Virtual Image Formation by Convergent Lens*

Question 7.1

What is the distance to the object (u) with uncertainty if the focal length is $f = 12.5 \pm 0.3\text{cm}$ and the distance to the image is $v = 50 \pm 5\text{cm}$?

1.3 Summary of Thin Lens Theory – Divergent Lenses

A divergent lens has two foci, F_1 and F_2 , located on its axis. They have a common distance from its center. The focal length f is defined as the negative of this distance. The three principal rays are as follows (Figure 4):

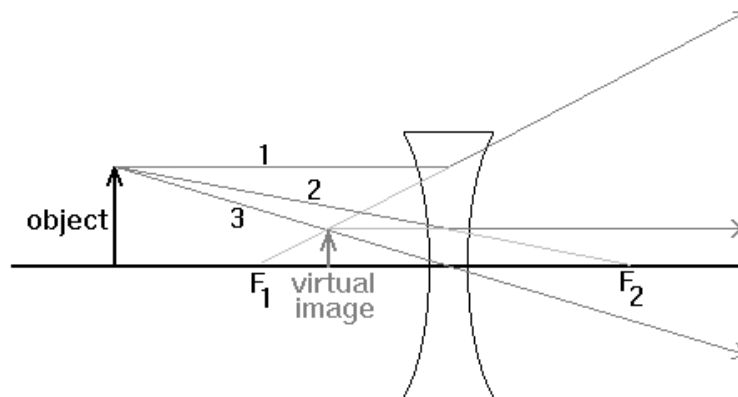


Figure 4 *Virtual Image formation by Divergent Lens*

Parallel Ray (1): A ray parallel to the axis incident from the left side emerges as though it was coming from the focus F_1 .

Focal Ray (2): A ray incident from the left heading for focus F_2 emerges parallel.

Central Ray (3): A ray incident towards the center emerges undeflected.

The lens formula is again Equation 1,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (f < 0) \quad (3)$$

but with f negative. Equation 2 and the discussion of magnification m of real and virtual images remain unchanged.

1.4 Summary of Thin Lenses Theory – Combination of Lenses and Power

When two thin lenses, of focal lengths f_1 and f_2 , are put close together, the combination acts like a single lens of focal length f , given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad (4)$$

In Equation 4, all focal lengths are signed quantities.

The shorter the focal length of a lens, the greater the deflection of incident rays. Optometrists, therefore, work with the so-called power of a lens, defined as

$$P = \frac{1}{f[m]} \quad (5)$$

The unit is called the diopter ($= [m]^{-1}$). $f[m]$ denotes that the focal length has units of meters.

Question 7.2

A divergent lens has focal length $f = -50 \pm 5 \text{ cm}$. What is the power in diopters with uncertainty?

Equation 4 can be rewritten simply in terms of power as:

$$P = P_1 + P_2 \quad (6)$$

1.5 The Human Eye

For the purpose of this Experiment it is sufficient to consider the model of the eye shown in Figure 5. For a normal eye, when the eye muscles are relaxed, the focal length f of the lens L is slightly less than its diameter, which is $D = 2.4 \text{ cm}$. An idealized eye will focus an object at infinity on the retina which is located at distance, D , behind the lens of the eye (see Figures 5 and 6a below). When the eye views a closer object, the eye muscles produce a shortening of f (so-called accommodation) down to a minimum value f_{\min} . The closest point on which the eye can focus is called the **near point**. Its distance from L , u_{near} , is about 25 cm for the normal eye.

The retina, on which are situated the light-sensitive elements (rods and cones), extends roughly over the posterior hemisphere of the eye. There are two special spots on it: the **fovea**, about 0.025 cm in diameter, where visual sensitivity is especially acute; and the **blind spot**, the area where the optic nerve is attached and which is devoid of rods and cones.

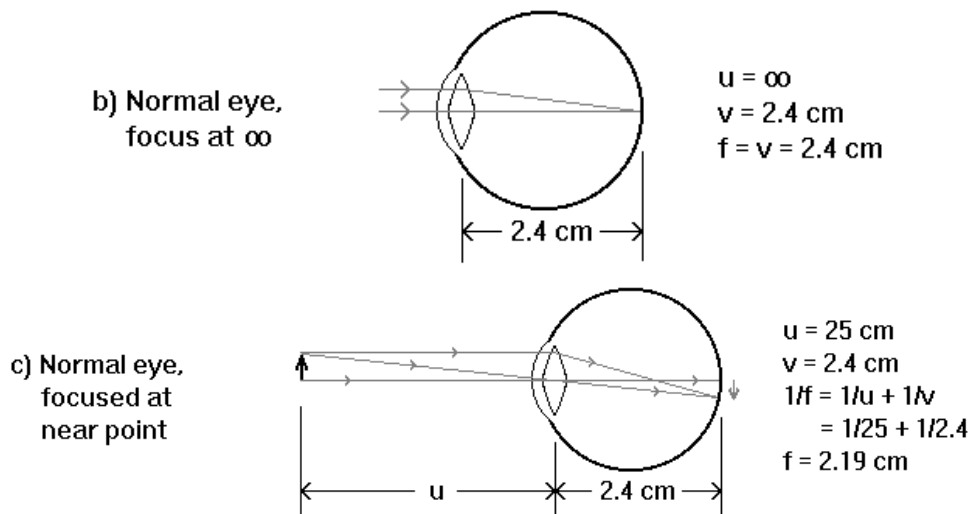
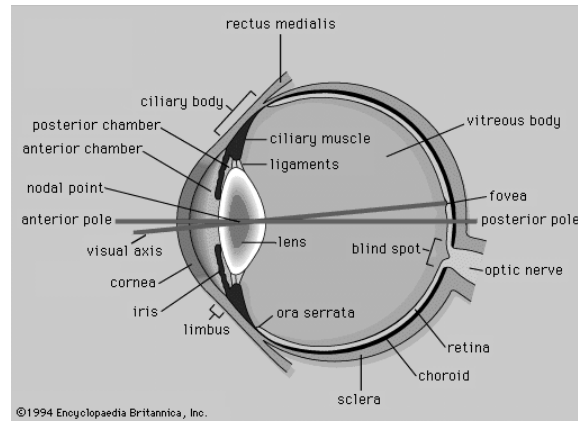


Figure 5 View of the human eye and adjustments for focus at long distances or near points.

For a **shortsighted (myopic)** eye, the focal length f of the lens is too short. Hence, when the eye is relaxed, an object at infinity is focused in front of the retina (Figure 6b). The distance of the farthest point that can be found, u_{far} , is finite and $u_{near} < 25 \text{ cm}$. A divergent lens is used for correction.

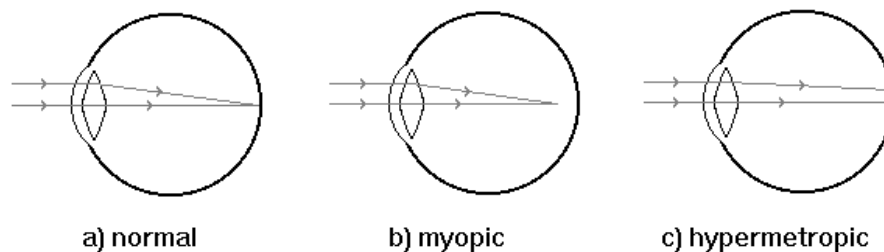


Figure 6 Normal, shortsighted and farsighted eyes in relaxed state ($f = f_{\max}$).

For a **far-sighted (hypermetropic)** eye, the focal length, f , is too long. Hence, when the eye is relaxed, an object at infinity is focused behind the retina (Figure 6c). The farsighted eye can focus on objects with $u_{far} < u < \infty$ where $u_{near} > 25$ cm. A convergent lens is used for correction.

There are two other common deficiencies of the eye. One is **reduced power of accommodation**, i.e. reduced variability of the focal length, f . This occurs with advancing age. The other is **astigmatism** in which the curvature of the front surface of the eye (the cornea) is different in different planes.

2 The Experiment

2.1 Equipment

Optical bench – 2

Light source – Lamp

Screen

Lens holders – 3

Convex and concave lenses - L_1 , L_2 , L_3 , and possibly L_4 .

Ruler

Make sure that your materials are still on the table after your Experiment.

2.2 Finding Focal Lengths

Take one of the convergent lenses, L_1 , and mount it in a lens holder. Mount the screen at the end of the optical bench. Using the lamp as your light source, vary the positions of the screen and lens until the image is crisp on the screen. Calculate the focal length of L_1 . Record f_1 and power P_1 , including an error estimate. Note whether the image is erect or inverted, real or virtual. Repeat with the second converging lens, L_2 . Next, use L_1 and L_2 , next to each other, placed as close together as you can and determine the focal length f_{12} and power P_{12} of the combination, reckoning distance from the midpoint of the lenses.

Next, take the concave lens, L_3 , and verify that, by itself it cannot produce a real image of a distant object. Now combine it with the stronger (higher powered, smaller focal length) convex lens, and measure the focal length f and power P of the combination. If neither L_1 nor L_2 are strong enough to produce a convergent image, then an additional convergent lens, L_4 , will be provided. Use a strong enough lens in combination with L_3 to deduce f_3 and P_3 of the concave lens.

Place the light source at the end of the optical bench. Place the screen 110 cm from the light source. Put the stronger convergent lens (of L_1 and L_2) between the screen and light source. Vary the position of the lens until the image is crisp on the screen. The distance to the screen is the image distance, v . Repeat this measurement lowering the distance between the light source and the screen 10 cm until the lens no longer produces a convergent image on the screen.

Make sure to record uncertainty in u and v for each measurement. The uncertainty in u is the uncertainty in measuring the distance. The uncertainty in v is half the distance over which the image on

the screen is somewhat convergent. To find this uncertainty place the screen just past the position where the image is crisp on either side. Now, divide the distance between the two unclear positions by two.

Using ORIGIN plot $\frac{1}{u}$ vs. $\frac{1}{v}$. See *Question 7.5* for a plotting strategy.

Question 7.3

Let $y = \frac{1}{v}$ and $x = \frac{1}{u}$. What is the uncertainty of x and y ?

Question 7.4

Replace u and v with x and y from Question 7.4 in Equation 1. If you are fitting the Equation $y = mx + b$, what are m and b ? When creating the plot for section 2.2, do not use m and b , but rather what you have found them to represent.

2.3 The Eye as a Variable Lens

In this section Experiments will be done in which the eye acts as a lens, L , of variable focal length. At the same time the retina acts as a screen at a fixed distance $v = D$ behind L . The far and near points of the eye and their modification by lenses will be studied. Use $D = 2.4$ cm.

Far Point

In this section you will find the focal length of your relaxed eye (with glasses or contact lenses, if you wear them). First place one of the weaker converging lenses close to your eye so that you can use the thin lens approximation for addition of focal lengths (Equation 4). Then bring the screen toward your eye until you can barely see it clearly. Have your partner measure the distance between the lens and the object; this is your far point*. Using the lens addition formula (Equation 4) and the lens formula (Equations 1 and 3) you will solve the following Equation for the maximum focal length of your eye.

$$\frac{1}{f_{\text{combination}}} = \frac{1}{f_{\text{eye, max}}} + \frac{1}{f_{\text{lens}}} = \frac{1}{u} + \frac{1}{v} \quad (7)$$

In your analysis, you will calculate how far you can see with your relaxed eye using this focal length and the diameter of your eye.

Near Point

Using the bench screen as object, measure approximately the distance u_{near} of the near point where you can still just manage to focus clearly on the scale (with glasses or contact lenses, if you wear them). Using $v = 2.4$ cm calculate f_{min} from u_{near} , and P_{max} from f_{min} . Also, determine P_{min} from f_{max} , and finally calculate the accommodating power of your eye in diopters:

* If you looked for your actually far point, you would either need to make very accurate, very large distance measurements, which would be larger than the size of the room, or you would need to assume that this value is infinite.

$$P_{acc} = P_{max} - P_{min} \quad (8)$$

Age	Mean Value	Usual Upper Limit
8	13.8	15.4
12	13.1	14.7
16	12.4	13.9
20	11.5	13.0
25	10.2	11.8
30	8.9	10.4
35	7.3	8.9
40	5.9	7.2
50	2.0	3.0
60+	1.1	1.5

Table 1 Values of the Accommodative Power at Ages from 8 to 68 (Duane 1912) Measure in diopters.

Compare the value for the accommodative power for your eye with the table.

2.4 Finding the Blind Spot

In this Experiment, you will map out the blind spot. Abnormally shaped blind spots occur in glaucoma patients. Additional blind spots (scars) can be created by excessive exposure to the sun or other strong light sources such as welding arcs.

Your partner (the subject) sits down 200 cm from a blackboard and covers his left eye. You will draw a small asterisk directly in front of the subject's right eye. The subject must now focus firmly and persistently on the asterisk. You will very slowly move a marker along the blackboard, starting from the asterisk and moving horizontally to the subject's right. At a certain point the marker will become invisible to the subject; further on it will again become visible. Both points should be recorded on the board (Figure 7). Starting from the midpoint P between these two points, the partner now moves the marker in various directions and records the points of reappearance. In this way, a map of the blind spot is produced on the board.

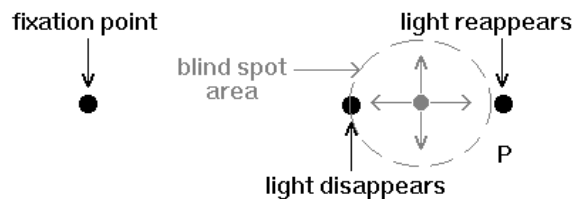


Figure 7 Mapping the Blind Spot

Record the dimensions and shape of the blind spot on the board and its position relative to the asterisk. Make a scale drawing in your report.

Using Equation 2, the known distance of the object, $u = 200$ cm, and the known distance of the retinal image, $v = D = 2.4$ cm, calculate the actual distance from the fovea to the blind spot on the retina,

and the actual vertical and horizontal diameters of the blind spot. (The eye automatically forms the image of the point of fixation [the asterisk] on the fovea.) This calculation is an approximation that ignores effects caused by the medium in the eye not being air (it is called vitreous humor and has a refractive index similar to water). This modifies the results slightly - your calculations will overestimate the real values by about 20%.

Analysis

Compute the following:

- Experimental focal length of L_1 ;
- Experimental focal length of L_2 ;
- The theoretical combination of focal length of L_1 and L_2 ;
- Experimental power of L_1 and L_2 combination;
- Experimental focal length of L_3 ;
- The focal range of your eye;
- The accommodating power of your eye;
- The distance to your blind spot;
- The dimensions of your blind spot.

Calculate t -values for the following:

- Combination of focal length of L_1 and L_2 ;
- The focal length from the ORIGIN plot.

Conclusions

Highlight the themes of the lab and the physics the experiment verifies. You should discuss the errors you encounter in the lab and how you could improve the lab if you had to repeat it. If your results are unexpected or your t -values are high, you should identify possible explanations.

Hints on reports

- Print (preferably type) neatly—if your TA cannot read it, you could lose points.
- Be organized—if your TA cannot find it, you could lose points.
- Report your data, including plots—if your data is not in your report, your TA does know you did it.
- Record uncertainty.
- Propagate uncertainty.
- Write your final answers with proper significant figures.