# Experimental Studies of Oscillations and Damping in RLC Circuits

John Smith

*Warren College, University of California, San Diego, La Jolla, CA 92093*

(February 20, 2002)

## Abstract

An electrical circuit consisting of a resistor $R$, capacitor $C$, and inductor $L$ in series was driven by the square-wave generator. An oscilloscope was used to explore oscillations and damping. The experimental values for the oscillation frequency and the $Q$-factor are in agreement with theoretical estimates based on the known values of $R, L$ and $C$. The value of $Q$ was determined both by counting the oscillations before decay to $1/e$ of the original strength and by fitting the measured peak amplitude to an exponential decay function. Finally, the additional resistance need for "critical damping" was determined experimentally and was found to be in accord with the theoretical prediction.

## I. INTRODUCTION

Oscillations and damping are observed in diverse physical situations ranging from a child on a swing to a plucked guitar string. In general, oscillations occur when an object is displaced from its equilibrium position. A restoring force accelerates the object back towards equilibrium, but inertia causes it to "overshoot" so that the object proceeds to the other side of equilibrium. The process then repeats resulting in oscillations, unless there is some frictional "drag" which causes the motion to decay away.

Oscillations occurring in a series RLC circuit are analogous, even though one cannot directly "see" these oscillations. Oscillating currents transfer energy back and forth from capacitor to inductor, until dissipation in the resistor causes the excitation to decay away.

In this experiment we verified that the second-order differential equations derived in the Physics 2CL lab manual[1] do indeed quantitatively describe the oscillations and decay of current in an RLC circuit. The circuit was excited by a "step" voltage from a signal generator and the resulting currents were measured by monitoring the potential difference across the resistor using an oscilloscope.

The currents revealed oscillatory character with the decaying amplitude. The frequency of the of the oscillations as well as the decay rate were readily obtained. As expected, the oscillations were well-characterized by a natural oscillation frequency $f$ and a quality factor $Q$. Further, it was shown that any desired damping rate can be readily obtained by varying the value of the resistance $R$. One goal of the experiment was to design a circuit with "critical damping". Under this critical regime the excitation is damped away faster than the system completes one full cycle of the oscillations. A second goal was to determine the inductance of the "unknown inductor".

## II. THEORY

Consider the series RLC circuit shown in Fig. 1. The voltage drop across the resistor $V_R = iR$, capacitor $V_C = \frac{1}{C}\int i dt$ and inductor $V_L = L\frac{di}{dt}$ follows the Kirchhoff's law[2]:

$$V_S = V_R + V_C + V_L \tag{1}$$

where $V_S$ is the applied voltage from the generator. Differentiation of Eq. 1 leads to an exponential form for the current:

$$i = i_0 e^{\Box t} \tag{2}$$

with □ given by the following equation:

$$\Box = -\frac{R}{2L} \pm \frac{1}{2}\sqrt{(R^2/L^2) - (4/LC)} \tag{3}$$

The real part of □ stands for damping of the current whereas the imaginary part describes the oscillatory character of the current. Thus it is customary to write Eq. 3 in the following form:

$$i = i_0 e^{-t/\Box_d} sin(\omega t). \tag{4}$$

In the Eq. 4 the frequency $\omega$ can be expressed through $R$, $L$ and $C$ as:

$$\omega^2 = (R^2/L^2) - (4/LC) = \omega_0^2 - R^2/4L^2. \tag{5}$$

A plot of Eq. 3 is presented in Fig. 2. Notably, for the case of reasonably weak damping $\omega^2 \simeq \omega_0^2$.

## III. EXPERIMENTAL TECHNIQUES

Experiments on the RLC circuit were performed using a standard signal generator and oscilloscope as shown in Fig. 1. The resistances of the resistor $R$ and of the inductor $R_L$ were measured. We estimated the internal resistance of the signal generator $R_S$ to be close to 50- . We used labeled values of the resistance $R = 501 \pm 1$- , capacitance $C = 0.1\Box F \pm 10\%$, and inductance $L = 1mH \pm \%$ with $R_L = 12.0 \pm 0.1$- . The signal generator produced the square wave excitations of amplitude $V_S = 5.0 \pm 0.1V$ at a frequency of about 100 Hz. Note that the frequency of these excitations is irrelevant since we are merely observing the oscillations induced by the leading edge of the square wave pulses.

In our experiment we recorded the voltage across the resistor using the oscilloscope as a non-perturbative measurement tool. The scope does not significantly alter the circuit because its input impedance is about 1 M- , which is much greater than the resistance of the circuit. We found it convenient to trigger the oscilloscope synchronously with the square wave drive by connecting the TTL output of the generator to the TRIG IN of the oscilloscope.

## IV. RESULTS

Our first task was to determine the oscillation frequency. For this, we measured the time between 5 oscillation peaks on the screen of the oscilloscope $t_5 = (1.2 \pm 0.1)$ ms yielding the frequency of 12.5±1 kHz. This value compares well with the theoretical estimate $f_{th} = (11.8 \pm 0.2)$ determined from Eq. 5. The predicted and measured values differ by 0.7 kHz which is less than the combined theoretical and experimental uncertainty of □ = 1.5 kHz. We obtain a ratio of t=0.7 indicating that our measurements are within one standard deviation of the predicted theoretical value. This good agreement suggests that the theory is adequate. The error in the prediction is largely due to 10% uncertainty in the labeled component values.

We determined $Q$ by the simple method of counting oscillations before the signal decays down to 1/e of its original strength. We obtained N=... leading to $Q$ = ....[1]. We then performed a more accurate measurement of $Q$ based on the determination of the amplitudes and times of the "peaks" in the current oscillations. The results are presented in Table 1 and are plotted in Fig. 3. We carried out the exponential fit $y = V_0 e^{-x/t_D}$ of our results using the "Origin" data analysis package. The fitted dependence is also depicted in Fig. 3. From the fit we obtained the following values of $V_0 = \ldots$ and $t_D = \ldots$

## V. DISCUSSION

We find that the oscillations and damping of currents in a simple RLC circuit are well described by the easily understood from Eq. 2-5. These equations are easily derived from the basic conservation laws. Furthermore, oscillations can be readily displayed on the screen of the oscilloscope. While the theory predicts that the dissipation of energy in the oscillating circuit is related to heating of the resistor, no significant heating was detected. A quick estimate of the dissipated energy suggested that the amount of energy lost is miniscule and can be observed only with accurate temperature sensors. Finally, we note that *all* resistors in a series circuit are responsible for the current decay. Therefore, it is important to take into account of "hidden" resistors for the quantitative estimates.

The author would like to acknowledge valuable discussions with the teaching assistants.

Table 1. Oscillations peaks heights at different times.

| time (ms) | $V_{peak}$ (Volts) |
|---|---|
| 1.1 | $5.7\pm 0.1$ |
| 2.3 | $3.2 \pm 0.1$ |
| 3.5 | $1.7 \pm 0.1$ |
| 4.7 | $0.5 \pm 0.1$ |

## VI. REFERENCES

[1] *Laboratory Manual for Physics 2CL*, University of California, San Diego, 1998.

[2] D.Halliday, R.Resnick, J.Walker, *Fundamental of Physics, 5ed*, 1997.

## VII. FIGURE CAPTIONS

Figure 1: Electrical Circuit Studied.

Figure 2: Expected current oscillations and "envelope" function.

Figure 3. Amplitude of voltage peaks as a function of time. The dotted line shows exponential fit to the data.

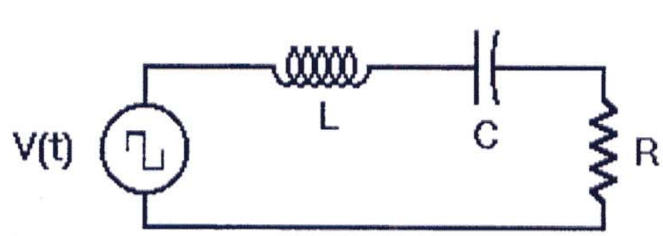


Fig.1

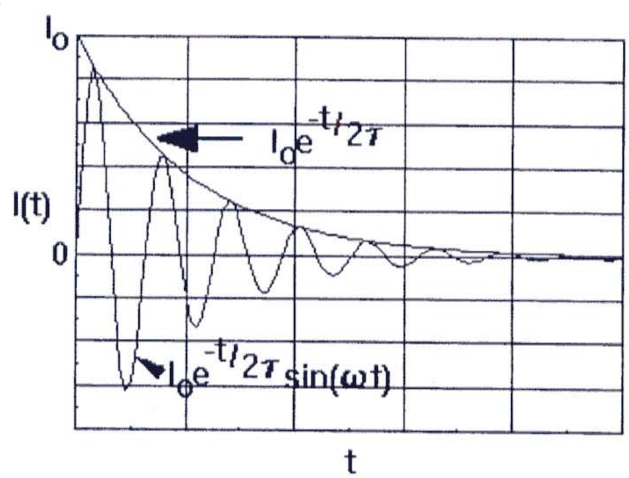


Fig.2

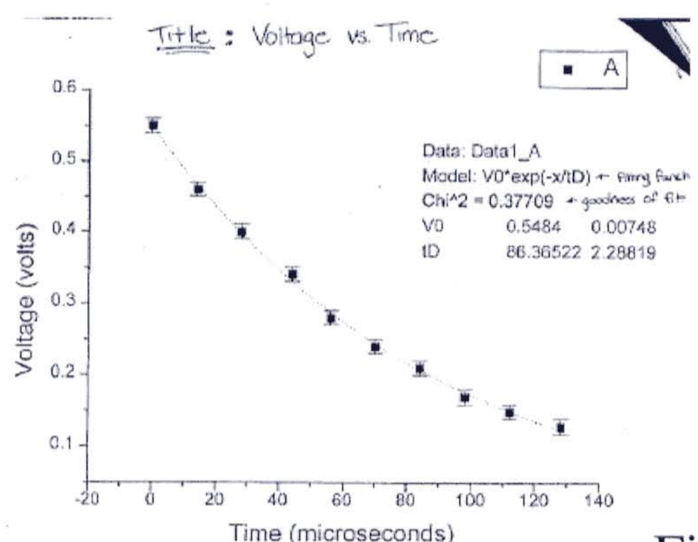


Fig.3