Physics 1B: Electricity & Magnetism

Summer Session I, 2010
Quiz #4, July 22

This is version **A**!

Useful coefficients:
Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg; Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Unit of elementary charge $e = 1.6 \times 10^{-19}$ C
Coulomb's constant, $k_e = 8.99 \times 10^9$ N m^2 C^{-2}
Permittivity of free space $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$
1 eV = 1.6 $\times 10^{-19}$ J
$g = 9.8$ m s^{-2}
1 T = 10^4 G

1. Three resistors, each with a resistance of 4Ω, are connected in parallel to a 12-V battery; what's the current through any one resistor?
a. 0.33 A
b. 1.0 A
c. 3.0 A
d. 12.0 A
e. 9.0 A

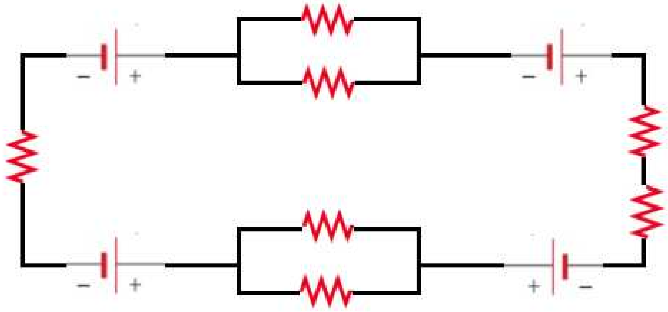
Solution: Calculate the equivalent resistance: $\frac{1}{R_{eq}} = \frac{1}{4\Omega} + \frac{1}{4\Omega} + \frac{1}{4\Omega} = \frac{3}{4\Omega}$
So $R_{eq} = \frac{4}{3}\Omega$
Let's label the current flowing through each resistor as I_1, I_2 and I_3, so that the current flowing through the battery = $I_{tot} = I_1 + I_2 + I_3$.
I_{tot} = EMF / R_{eq} = 12V/1.333Ω = 9.0 A.
Since each resistor has equal resistance, the currents flowing through each of them must be equal: $I_1 = I_2 = I_3 = I_{tot}/3 = 3.0$ A

2. A light bulb with a resistance of 220Ω, a battery with EMF=10 V, and a capacitor with C=4 milliFarads are all connected in series to form a charging RC circuit. What is the value of the voltage drop across the capacitor at a time 2.64 seconds after the switch is closed?
a. 3.68 V
b. 2.64 V
c. 36.8 V
d. 6.32 V
e. 9.50 V

Solution: First, we calculate the time constant τ = RC = (220Ω)(4×10^{-4} F) = 0.88 sec
When a capacitor is charging, the voltage drop across the plates ΔV_C goes from 0 to its maximum value, ϵ = EMF, following $\Delta V_C(t) = \epsilon(1 - e^{-t/\tau})$

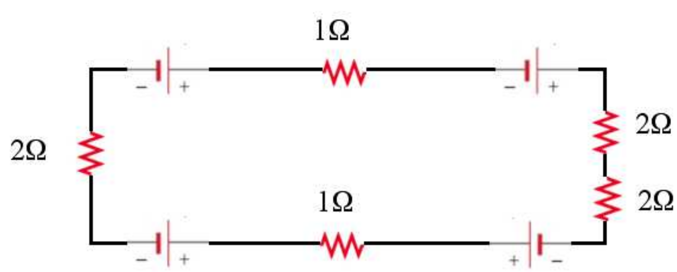
At t=2.64 seconds (note that this is 3 time constants), we have $\Delta V_C(t) = 10V(1-e^{-2.64s/0.88s})$ $= 10V(1-e^{-3}) = 10V(1 - 0.050) = 10V(0.95) = 9.50$ V, i.e., the capacitor has reached 95% of its maximum charge stored at this point.

3. In this circuit, each individual resistor has R=2Ω, and each individual battery supplies



an EMF of 4 V. What is the magnitude and direction of the resulting current?
a. 0.5 A, counter-clockwise
b. 2.0 A, counter-clockwise
c. 0.25 A, counter-clockwise
d. 1.0 A, clockwise
e. 0.5 A, clockwise

Solution: Use the Parallel Law to make the circuit a bit more simple: When two 2Ω resistors are in series, $1/R_{eq} = 1/(2\Omega) + 1/(2\Omega) = 1/(1\Omega)$, or $R_{eq} = 1\Omega$.
So our circuit becomes:



Now we have a circuit where everything's connected in series. Note that three of the batteries want to push the current in a clockwise direction, while one battery is trying to push current counter-clockwise.
Let's "guess" that the current will flow clockwise. Start at the lower-right hand corner, and follow the current and apply Kirchhoff's Loop rule:
$+4\text{ V} - (1\Omega)I - 4V - (2\Omega)I + 4V - (1\Omega)I + 4V - (2\Omega)I - (2\Omega)I = 0$

Note that the second battery has its sign opposite to the others.
$+8V - I(8\Omega) = 0$
$8V = I(8\Omega)$

I = 1.0 V.
Note that if we had guess that the current flowed in the counter-clockwise direction, then starting at the lower-right hand corner and going counter-clockwise, our loop rule equation would be:
$-(2\Omega)I - (2\Omega) - 4V - (1\Omega)I - 4V - (2\Omega)I + 4V - (1\Omega)I - 4V = 0$
$-I(8\Omega) - 8V = 0$
$-I(8\Omega) = 8V$
I = –1.0 A ... the negative sign means the direction for the current we chose is wrong, and the current is actually clockwise.
Final answer = 1.0 A, clockwise

4. What is the maximum number of hair dryers (1000 W) you can connect in parallel in a 120-V home circuit without tripping the home's 45-A circuit breaker?
a. 3
b. 5
c. 6
d. 8
e. 375

Solution: Each hair dryer must draw $I = P/\Delta V = 1000W/120V = 8.33$ A. In accordance with the junction rule, the total current needed to operate N hair dryers in parallel is $N \times 8.33$A. 5 hair dryers draws 41.67 A, but 6 would draw 50 A and would trip the circuit breaker.

5. A capacitor is being discharged through a resistor in series (no battery present). The capacitor is discharged to 50% of its initial value in 1.0 sec. If the resistor's resistance is 14.43 Ω, what is the value of the capacitance? (Hint: try to derive τ)
a. 0.10 F
b. 0.48 F
c. 0.23 F
d. 0.048 F

The expression for the charge as a function of time, Q(t), is $Q_{max}(e^{-t/\tau})$.
$Q/Q_{max} = 0.50 = e^{-t/\tau}$
$0.50 = e^{-t/\tau}$
Take the natural log of both sides:
$-0.693 = -t/\tau$
$\tau = t/0.693$
For t = 1.0 sec, $\tau = 1.443$ sec.
$\tau = RC$ can be rearranged to solve for $C = \tau/R = 1.443\text{sec} / 14.43\Omega = 0.10$ F.

6. A soft magnetic material has which property?
a. It attracts only slowly moving charges.
b. It cannot be magnetized.
c. It is hard to magnetize.
d. It is easy to magnetize.

Solution: Easy to magnetize.

7. The magnetic field of the Earth is believed to be responsible for which of the following?
a. deflection of both charged and uncharged solar wind particles
b. deflection of charged solar wind particles
c. ozone in the upper atmosphere
d. solar flares

Solution: The answer is B; Earth's magnetic field will not affect unchanged particles.

8. A hydroxyl molecule, OH^-, is moving at 6000 m/s parallel to the Earth's surface, in a northeasterly direction, crossing the Earth's magnetic equator. Here, the magnetic field points north, is parallel to the surface, and has a magnitude of 0.5 Gauss. What is the force (magnitude and direction) acting on the molecule at this instant?
a. 4.8×10^{-20} N, downward (towards the Earth's surface)
b. 4.8×10^{-16} N, upward (away from the Earth's surface)
c. 3.4×10^{-20} N, downward (towards the Earth's surface)
d. 3.4×10^{-16} N, in a northwest direction
e. 3.4×10^{-20} N, upward (away from the Earth's surface)

Solution: Use F_B = q v B sin(θ).
For a molecule with a charge of PLUS 1e, we can calculate the magntiude of F_B as (1.6×10^{-19} C)(6000 m/s)(0.5 $\times10^{-4}$ T) sin(45°) = 3.4 $\times10^{-20}$ N.
For a POSITIVELY-charged molecule, we use the right-hand rule: a positively-charged molecule would experience a force upwards, away from the Earth's surface.
The magnitude of the magnetic force on a charge of −1e will be the same as this, but the direction will be reversed: downwards, towards the Earth's surface.

9. At the Fermilab accelerator in Illinois, singly-charged ions with momentum 4.8×10^{-16} kg m s^{-1} are held in a circular orbit of radius 1.0 km by an upward magnetic field. Assuming q = 1.6×10^{-19} C, what magnetic field strength must be used to maintain this orbit?

a. 1.0 T
b. 0.003 T
c. 9.0 T
d. 0.001 T
e. 3.0 T

Since the momentum is mv, we can use r = (mv)/(qB).
Rearranging to solve for B, we get B = (mv)/(rq) = (4.8×10^{-16} kg m s^{-1}) / (1.6×10^{-19} C $\times$ 1000 m) = 3.0 T

10. A wire with a length of 2.0 m carries a current from west to east, and the local magnetic field points north and has a strength of 0.5 Gauss. The wire has a cross-sectional area of 1.0×10^{-5} m^2 and is composed of a conductor whose mass density is 9.0×10^3 kg/m^3. Calculate the (very large) current needed to balance the gravitational and magnetic forces acting on the wire.
a. 8820 A
b. 35280 A
c. 2205 A
d. 17640 A
e. 4410 A

Solution: The total mass of the wire is its volume (area $\times$ length) times the mass density:
m = (1.0×10^{-5} m^2)(2.0m)(9.0×10^3 kg/m^3) = 0.18 kg.
The total gravitational force on the wire is
F_g = mg = (0.18 kg)(9.8 m s^{-2}) = 1.76 N
The magnetic force will act in an upwards direction (following the right-hand rule).
We set the grav. & magn. forces to be equal:
$F_g = F_B = B\ I\ l \sin(\theta)$
And solve for I:
I = (F_g)/(B l sin(90°)) = 1.76 N / (0.5×10^{-4}T $\times$ 2.0m) = 17640 A.