Problem Solutions

20.1 The magnetic flux through the area enclosed by the loop is

$$\Phi_B = BA\cos\theta = B\left(\pi r^2\right)\cos 0° = \left(0.30\ \text{T}\right)\left[\pi\left(0.25\ \text{m}\right)^2\right] = \boxed{5.9\times10^{-2}\ \text{T}\cdot\text{m}^2}$$

20.2 The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$ where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = \left(5.00\times10^{-5}\ \text{T}\right)\left[20.0\ \text{cm}^2\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)^2\right]\cos\theta = \left(1.00\times10^{-7}\ \text{T}\cdot\text{m}^2\right)\cos\theta$$

(a) When $\vec{\mathbf{B}}$ is perpendicular to the plane of the loop, $\theta = 0°$ and

$\Phi_B = \boxed{1.00\times10^{-7}\ \text{T}\cdot\text{m}^2}$

(b) If $\theta = 30.0°$, then $\Phi_B = \left(1.00\times10^{-7}\ \text{T}\cdot\text{m}^2\right)\cos 30.0° = \boxed{8.66\times10^{-8}\ \text{T}\cdot\text{m}^2}$

(c) If $\theta = 90.0°$, then $\Phi_B = \left(1.00\times10^{-7}\ \text{T}\cdot\text{m}^2\right)\cos 90.0° = \boxed{0}$

20.3 The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$ where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = \left(0.300\ \text{T}\right)\left(2.00\ \text{m}\right)^2\cos 50.0° = \boxed{7.71\times10^{-1}\ \text{T}\cdot\text{m}^2}$$

20.6 The magnetic field generated by the current in the solenoid is

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{250}{0.200\ \text{m}}\right)\left(15.0\ \text{A}\right) = 2.36\times10^{-2}\ \text{T}$$

and the flux through each turn on the solenoid is

$$\Phi_B = BA\cos\theta$$

$$= \left(2.36\times10^{-2}\ \text{T}\right)\left[\frac{\pi\left(4.00\times10^{-2}\ \text{m}\right)^2}{4}\right]\cos 0° = \boxed{2.96\times10^{-5}\ \text{T}\cdot\text{m}^2}$$

20.8 $$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{(\Delta B)A\cos\theta}{\Delta t} = \frac{(1.5\text{ T}-0)\left[\pi\left(1.6\times10^{-3}\text{ m}\right)^2\right]\cos0^\circ}{120\times10^{-3}\text{ s}} = 1.0\times10^{-4}\text{ V} = \boxed{0.10\text{ mV}}$$

20.10 $$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{(0.15\text{ T})\left[\pi(0.12\text{ m})^2 - 0\right]\cos0^\circ}{0.20\text{ s}} = 3.4\times10^{-2}\text{ V} = \boxed{34\text{ mV}}$$

20.13 The required induced emf is $|\varepsilon| = IR = (0.10\text{ A})(8.0\ \Omega) = 0.80\text{ V}$.

From $|\varepsilon| = \dfrac{\Delta\Phi_B}{\Delta t} = \left(\dfrac{\Delta B}{\Delta t}\right)NA\cos\theta$

$$\frac{\Delta B}{\Delta t} = \frac{|\varepsilon|}{NA\cos\theta} = \frac{0.80\text{ V}}{(75)\left[(0.050\text{ m})(0.080\text{ m})\right]\cos0^\circ} = \boxed{2.7\text{ T/s}}$$

20.14 The initial magnetic field inside the solenoid is

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(\frac{100}{0.200\text{ m}}\right)(3.00\text{ A}) = 1.88\times10^{-3}\text{ T}$$

(a) $\Phi_B = BA\cos\theta = \left(1.88\times10^{-3}\text{ T}\right)\left(1.00\times10^{-2}\text{ m}\right)^2\cos0^\circ$

$$= \boxed{1.88\times10^{-7}\text{ T}\cdot\text{m}^2}$$

(b) When the current is zero, the flux through the loop is $\Phi_B = 0$ and the average induced emf has been

$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{1.88\times10^{-7}\text{ T}\cdot\text{m}^2 - 0}{3.00\text{ s}} = \boxed{6.28\times10^{-8}\text{ V}}$$

20.17 If the magnetic field makes an angle of 28.0° with the plane of the coil, the angle it makes with the normal to the plane of the coil is $\theta = 62.0°$. Thus,

$$|\varepsilon| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NB(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{200(50.0\times10^{-6}\text{ T})\left[(39.0\text{ cm}^2)(1\text{ m}^2/10^4\text{ cm}^2)\right]\cos 62.0°}{1.80\text{ s}} = 1.02\times10^{-5}\text{ V} = \boxed{10.2\ \mu\text{V}}$$

20.18 From $\varepsilon = B\ell v$, the required speed is

$$v = \frac{\varepsilon}{B\ell} = \frac{IR}{B\ell} = \frac{(0.500\text{ A})(6.00\ \Omega)}{(2.50\text{ T})(1.20\text{ m})} = \boxed{1.00\text{ m/s}}$$

20.21 (a) Observe that only the horizontal component, B_h, of Earth's magnetic field is effective in exerting a vertical force on charged particles in the antenna. For the magnetic force, $F_m = qvB_h\sin\theta$, on positive charges in the antenna to be directed upward and have maximum magnitude (when θ =90°), the car should move $\boxed{\text{toward the east}}$ through the northward horizontal component of the magnetic field.

(b) $\varepsilon = B_h\ell v$, where B_h is the horizontal component of the magnetic field.

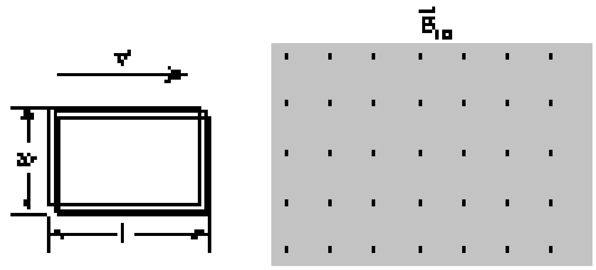
$$\varepsilon = \left[(50.0\times10^{-6}\text{ T})\cos 65.0°\right](1.20\text{ m})\left[\left(65.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right)\right]$$

$$= \boxed{4.58\times10^{-4}\text{ V}}$$

20.23 (a) To oppose the motion of the magnet, the magnetic field generated by the induced current should be directed to the right along the axis of the coil. The current must then be $\boxed{\text{left to right}}$ through the resistor.

(b) The magnetic field produced by the current should be directed to the left along the axis of the coil, so the current must be $\boxed{\text{right to left}}$ through the resistor.

20.25 (a) After the right end of the coil has entered the field, but the left end has not, the flux through the area enclosed by the coil is directed into the page and is increasing in magnitude. This increasing flux induces an emf of magnitude



$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NB(\Delta A)}{\Delta t} = NBwv$$

in the loop. Note that in the above equation, ΔA is the area enclosed by the coil that enters the field in time Δt. This emf produces a counterclockwise current in the loop to oppose the increasing inward flux. The magnitude of this current is $I = \mathcal{E}/R = NBwv/R$. The right end of the loop is now a conductor, of length Nw, carrying a current toward the top of the page through a field directed into the page. The field exerts a magnetic force of magnitude

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

on this conductor, and hence, on the loop.

(b) When the loop is entirely within the magnetic field, the flux through the area enclosed by the loop is constant. Hence, there is no induced emf or current in the loop, and the field exerts $\boxed{\text{zero}}$ force on the loop.

(c) After the right end of the loop emerges from the field, and before the left end emerges, the flux through the loop is directed into the page and decreasing. This decreasing flux induces an emf of magnitude $|\mathcal{E}| = NBwv$ in the loop, which produces an induced current directed clockwise around the loop so as to oppose the decreasing flux. The current has magnitude $I = \mathcal{E}/R = NBwv/R$. This current flowing upward, through conductors of total length Nw, in the left end of the loop, experiences a magnetic force given by

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

20.29 When the switch is closed, the current from the battery produces a magnetic field directed toward the left along the axis of both coils.

(a) As the current from the battery, and the leftward field it produces, increase in magnitude, the induced current in the leftmost coil opposes the increased leftward field by flowing $\boxed{\text{right to left}}$ through R and producing a field directed toward the right along the axis.

(b) As the variable resistance is decreased, the battery current and the leftward field generated by it increase in magnitude. To oppose this, the induced current is $\boxed{\text{right to left}}$ through R, producing a field directed toward the right along the axis.

(c) Moving the circuit containing R to the left decreases the leftward field (due to the battery current) along its axis. To oppose this decrease, the induced current is $\boxed{\text{left to right}}$ through R, producing an additional field directed toward the left along the axis.

(d) As the switch is opened, the battery current and the leftward field it produces decrease rapidly in magnitude. To oppose this decrease, the induced current is $\boxed{\text{left to right}}$ through R, generating additional magnetic field directed toward the left along the axis.

20.30 $$\varepsilon_{\text{max}} = NB_{horizontal}A\omega = 100\left(2.0\times10^{-5}\text{ T}\right)\left(0.20\text{ m}\right)^2\left[\left(1500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 1.3\times10^{-2}\text{ V} = \boxed{13\text{ mV}}$$

20.33 (a) When a coil having N turns and enclosing area A rotates at angular frequency ω in a constant magnetic field, the emf induced in the coil is

$$\mathcal{E} = \mathcal{E}_{\text{max}}\sin\omega t \quad \text{where} \quad \mathcal{E}_{\text{max}} = NB_\perp A\omega$$

Here, $B_\perp$ is the magnitude of the magnetic field perpendicular to the rotation axis of the coil. In the given case, $B_\perp = 55.0\ \mu\text{T}$; $A = \pi ab$ where $a = (10.0\text{ cm})/2$ and $b = (4.00\text{ cm})/2$; and

$$\omega = 2\pi f = 2\pi\left(100\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = 10.5\text{ rad/s}$$

Thus, $$\mathcal{E}_{\text{max}} = (10.0)\left(55.0\times10^{-6}\text{ T}\right)\left[\frac{\pi}{4}(0.100\text{ m})(0.0400\text{ m})\right](10.5\text{ rad/s})$$

or $$\mathcal{E}_{\text{max}} = 1.81\times10^{-5}\text{ V} = \boxed{18.1\ \mu\text{V}}$$

(b) When the rotation axis is parallel to the field, then $B_\perp = 0$ giving $\mathcal{E}_{max} = \boxed{0}$

It is easily understood that the induced emf is always zero in this case if you recognize that the magnetic field lines are always parallel to the plane of the coil, and the flux through the coil has a constant value of zero.

20.34 (a) Using $\varepsilon_{max} = NBA\omega$,

$$\varepsilon_{max} = 1\,000(0.20\text{ T})(0.10\text{ m}^2)\left[\left(60\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\right] = 7.5\times10^3 = \boxed{7.5\text{ kV}}$$

(b) ε_{max} occurs when the flux through the loop is changing the most rapidly. This is when the plane of the loop is $\boxed{\text{parallel to the magnetic field}}$.

20.39 (a) $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(400)^2\left[\pi(2.5\times10^{-2}\text{ m})^2\right]}{0.20\text{ m}}$

$$= 2.0\times10^{-3}\text{ H} = \boxed{2.0\text{ mH}}$$

(b) From $|\varepsilon| = L(\Delta I/\Delta t)$, $\dfrac{\Delta I}{\Delta t} = \dfrac{|\varepsilon|}{L} = \dfrac{75\times10^{-3}\text{ V}}{2.0\times10^{-3}\text{ H}} = \boxed{38\text{ A/s}}$

20.42 (a) The time constant of the *RL* circuit is $\tau = L/R$, and that of the *RC* circuit is $\tau = RC$. If the two time constants have the same value, then

$$RC = \frac{L}{R},\ \text{or}\ R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1.00\times10^3\ \Omega = \boxed{1.00\text{ k}\Omega}$$

(b) The common value of the two time constants is

$$\tau = \frac{L}{R} = \frac{3.00\text{ H}}{1.00\times10^3\ \Omega} = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$$

20.46 (a) $\tau = \dfrac{L}{R} = \dfrac{8.00\text{ mH}}{4.00\ \Omega} = \boxed{2.00\text{ ms}}$

(b) $I=\dfrac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right)=\left(\dfrac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1-e^{-250\times10^{-6}\text{ s}/2.00\times10^{-3}\text{ s}}\right)=\boxed{0.176\text{ A}}$

(c) $I_{\max}=\dfrac{\mathcal{E}}{R}=\dfrac{6.00\text{ V}}{4.00\ \Omega}=\boxed{1.50\text{ A}}$

(d) $I=I_{\max}\left(1-e^{-t/\tau}\right)$ yields $e^{-t/\tau}=1-I/I_{\max}$,

and $t=-\tau\ln\left(1-I/I_{\max}\right)=-\left(2.00\text{ ms}\right)\ln\left(1-0.800\right)=\boxed{3.22\text{ ms}}$

20.49 The current in the circuit at time t is $I=\dfrac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right)$, and the energy stored in the inductor is $PE_L=\dfrac{1}{2}LI^2$

(a) As $t\to\infty$, $I\to I_{\max}=\dfrac{\mathcal{E}}{R}=\dfrac{24\text{ V}}{8.0\ \Omega}=3.0\text{ A}$, and

$$PE_L\to\frac{1}{2}LI_{\max}^2=\frac{1}{2}\left(4.0\text{ H}\right)\left(3.0\text{ A}\right)^2=\boxed{18\text{ J}}$$

(b) At $t=\tau$, $I=I_{\max}\left(1-e^{-1}\right)=\left(3.0\text{ A}\right)\left(1-0.368\right)=1.9\text{ A}$

and $PE_L=\dfrac{1}{2}\left(4.0\text{ H}\right)\left(1.9\text{ A}\right)^2=\boxed{7.2\text{ J}}$

$$Q=\frac{\left(15.0\times10^{-6}\text{ T}\right)\left[\left(0.200\text{ m}\right)^2-0\right]}{0.500\ \Omega}=1.20\times10^{-6}\text{ C}=\boxed{1.20\ \mu\text{C}}$$

20.56 (a) $PE_L=\dfrac{1}{2}LI^2=\dfrac{1}{2}\left(50.0\text{ H}\right)\left(50.0\times10^3\text{ A}\right)^2=\boxed{6.25\times10^{10}\text{ J}}$

(b) $\dfrac{F}{\ell}=\dfrac{\mu_0 I_1 I_2}{2\pi d}=\dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(50.0\times10^3\text{ A}\right)^2}{2\pi\left(0.250\text{ m}\right)}$

$$=2.00\times10^3\ \frac{\text{N}}{\text{m}}=\boxed{2.00\ \frac{\text{kN}}{\text{m}}}$$

20.61 (a) $$\left|\mathcal{E}_{av}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B(\Delta A)}{\Delta t} = \frac{B\left[\left(\pi d^2/4\right) - 0\right]}{\Delta t}$$

$$= \frac{(25.0\text{ mT})\pi\left(2.00\times10^{-2}\text{ m}\right)^2}{4\left(50.0\times10^{-3}\text{ s}\right)} = \boxed{0.157\text{ mV}}$$

As the inward directed flux through the loop decreases, the induced current goes clockwise around the loop in an attempt to create additional inward flux through the enclosed area. With positive charges accumulating at *B*,

| point *B* is at a higher potential than *A* |

(b) $$\left|\mathcal{E}_{av}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A}{\Delta t} = \frac{\left[(100 - 25.0)\text{ mT}\right]\pi\left(2.00\times10^{-2}\text{ m}\right)^2}{4\left(4.00\times10^{-3}\text{ s}\right)} = \boxed{5.89\text{ mV}}$$

As the inward directed flux through the enclosed area increases, the induced current goes counterclockwise around the loop in an attempt to create flux directed outward through the enclosed area.

With positive charges now accumulating at *A*,

| point *A* is at a higher potential than *B* |