Chapter 16

Electrical Energy and Capacitance

Answers to Even Numbered Conceptual Questions

2. Changing the area will change the capacitance and maximum charge but not the maximum voltage. The question does not allow you to increase the plate separation. You can increase the maximum operating voltage by inserting a material with higher dielectric strength between the plates.

4. Electric potential V is a measure of the potential energy per unit charge. Electrical potential energy, $PE = QV$, gives the energy of the total charge Q.

6. A sharp point on a charged conductor would produce a large electric field in the region near the point. An electric discharge could most easily take place at the point.

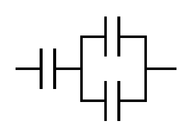
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series - $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$

All three capacitors in parallel - $C_{eq} = C_1 + C_2 + C_3$

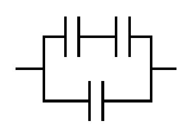
One capacitor in series with a parallel combination of the other two:

$$C_{eq} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2}\right)^{-1},\ C_{eq} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1}\right)^{-1}$$



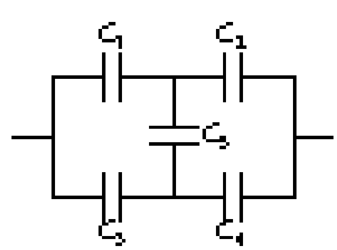
One capacitor in parallel with a series combination of the other two:

$$C_{eq} = \left(\frac{C_1 C_2}{C_1 + C_2}\right) + C_3,\ C_{eq} = \left(\frac{C_3 C_1}{C_3 + C_1}\right) + C_2,\ C_{eq} = \left(\frac{C_2 C_3}{C_2 + C_3}\right) + C_1$$



10. Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, the charge rapidly recombines, leaving the capacitor uncharged.

12. All connections of capacitors are not simple combinations of series and parallel circuits. As an example of such a complex circuit, consider the network of five capacitors C_1, C_2, C_3, C_4, and C_5 shown below.



This combination cannot be reduced to a simple equivalent by the techniques of combining series and parallel capacitors.

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

16. **(a)** i **(b)** ii

18. **(a)** The equation is only valid when the points *A* and *B* are located in a region where the electric field is uniform (that is, constant in both magnitude and direction). **(b)** No. The field due to a point charge is not a uniform field. **(c)** Yes. The field in the region between a pair of parallel plates is uniform.

Problem Solutions

16.1 (a) The work done is $W = F \cdot s\cos\theta = (qE) \cdot s\cos\theta$, or

$$W = \left(1.60\times10^{-19}\ \text{C}\right)\left(200\ \text{N/C}\right)\left(2.00\times10^{-2}\ \text{m}\right)\cos 0^\circ = \boxed{6.40\times10^{-19}\ \text{J}}$$

(b) The change in the electrical potential energy is

$$\Delta PE_e = -W = \boxed{-6.40\times10^{-19}\ \text{J}}$$

(c) The change in the electrical potential is

$$\Delta V = \frac{\Delta PE_e}{q} = \frac{-6.40\times10^{-19}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = \boxed{-4.00\ \text{V}}$$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\,000\ \text{J/C}}{1.5\times10^{-2}\ \text{m}} = \boxed{1.7\times10^{6}\ \text{N/C}}$

16.6 Since potential difference is work per unit charge $\Delta V = \dfrac{W}{q}$, the work done is

$$W = q(\Delta V) = \left(3.6\times10^{5}\ \text{C}\right)\left(+12\ \text{J/C}\right) = \boxed{4.3\times10^{6}\ \text{J}}$$

16.8 From conservation of energy, $\dfrac{1}{2}mv_f^2 - 0 = |q(\Delta V)|$ or $v_f = \sqrt{\dfrac{2|q(\Delta V)|}{m}}$

(a) For the proton, $v_f = \sqrt{\dfrac{2\left|\left(1.60\times10^{-19}\ \text{C}\right)\left(-120\ \text{V}\right)\right|}{1.67\times10^{-27}\ \text{kg}}} = \boxed{1.52\times10^{5}\ \text{m/s}}$

(b) For the electron, $v_f = \sqrt{\dfrac{2\left|\left(-1.60\times10^{-19}\ \text{C}\right)\left(+120\ \text{V}\right)\right|}{9.11\times10^{-31}\ \text{kg}}} = \boxed{6.49\times10^{6}\ \text{m/s}}$

16.13 (a) Calling the $2.00\ \mu\text{C}$ charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{8.00\times 10^{-6}\ \text{C}}{0.060\,0\ \text{m}} + \frac{4.00\times 10^{-6}\ \text{C}}{0.030\,0\ \text{m}} + \frac{2.00\times 10^{-6}\ \text{C}}{\sqrt{(0.060\,0)^2 + (0.030\,0)^2}\ \text{m}}\right)$$

$$V = \boxed{2.67\times 10^6\ \text{V}}$$

(b) Replacing $2.00\times 10^{-6}\ \text{C}$ by $-2.00\times 10^{-6}\ \text{C}$ in part (a) yields

$$V = \boxed{2.13\times 10^6\ \text{V}}$$

16.15 (a) $V = \sum_i \dfrac{k_e\, q_i}{r_i}$

$$= \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}} - \frac{3.00\times 10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$$

(b) $PE = \dfrac{k_e q_1 q_2}{r_{12}}$

$$= \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(5.00\times 10^{-9}\ \text{C}\right)\left(-3.00\times 10^{-9}\ \text{C}\right)}{0.350\ \text{m}} = \boxed{-3.85\times 10^{-7}\ \text{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges (that is, bring them up to a state of zero potential energy).

16.16 The potential at distance $r = 0.300 \text{ m}$ from a charge $Q = +9.00 \times 10^{-9} \text{ C}$ is

$$V = \frac{k_e Q}{r} = \frac{\left(8.99 \times 10^{9} \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(9.00 \times 10^{-9} \text{ C}\right)}{0.300 \text{ m}} = +270 \text{ V}$$

Thus, the work required to carry a charge $q = 3.00 \times 10^{-9} \text{ C}$ from infinity to this location is

$$W = qV = \left(3.00 \times 10^{-9} \text{ C}\right)\left(+270 \text{ V}\right) = \boxed{8.09 \times 10^{-7} \text{ J}}$$

16.21 $V = \dfrac{k_e Q}{r}$ so

$$r = \frac{k_e Q}{V} = \frac{\left(8.99 \times 10^{9} \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(8.00 \times 10^{-9} \text{ C}\right)}{V} = \frac{71.9 \text{ V} \cdot \text{m}}{V}$$

For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.719 \text{ m, } 1.44 \text{ m, and } 2.88 \text{ m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

16.22 (a) $Q = C(\Delta V) = \left(4.00 \times 10^{-6} \text{ F}\right)\left(12.0 \text{ V}\right) = 48.0 \times 10^{-6} \text{ C} = \boxed{48.0 \ \mu\text{C}}$

(b) $Q = C(\Delta V) = \left(4.00 \times 10^{-6} \text{ F}\right)\left(1.50 \text{ V}\right) = 6.00 \times 10^{-6} \text{ C} = \boxed{6.00 \ \mu\text{C}}$

16.23 (a) $C = \epsilon_0 \dfrac{A}{d} = \left(8.85 \times 10^{-12} \ \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \dfrac{\left(1.0 \times 10^{6} \text{ m}^2\right)}{\left(800 \text{ m}\right)} = \boxed{1.1 \times 10^{-8} \text{ F}}$

(b) $Q_{max} = C(\Delta V)_{max} = C\left(E_{max} d\right)$

$$= \left(1.11 \times 10^{-8} \text{ F}\right)\left(3.0 \times 10^{6} \text{ N/C}\right)\left(800 \text{ m}\right) = \boxed{27 \text{ C}}$$

16.25 (a) $E = \dfrac{\Delta V}{d} = \dfrac{20.0 \text{ V}}{1.80 \times 10^{-3} \text{ m}} = 1.11 \times 10^{4} \text{ V/m} = \boxed{11.1 \text{ kV/m}}$ directed toward the negative plate

(b) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\times10^{-4}\ \text{m}^2\right)}{1.80\times10^{-3}\ \text{m}}$

$= 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$

(c) $Q = C(\Delta V) = \left(3.74\times10^{-12}\ \text{F}\right)\left(20.0\ \text{V}\right) = 7.47\times10^{-11}\ \text{C} = \boxed{74.7\ \text{pC}}$ on one plate and $\boxed{-74.7\ \text{pC}}$ on the other plate.

16.30 (a) For parallel connection,

$$C_{eq} = C_1 + C_2 + C_3 = (5.00 + 4.00 + 9.00)\ \mu\text{F} = \boxed{18.0\ \mu\text{F}}$$

(b) For series connection, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$\frac{1}{C_{eq}} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} + \frac{1}{9.00\ \mu\text{F}},\ \text{giving } C_{eq} = \boxed{1.78\ \mu\text{F}}$$

16.31 (a) Using the rules for combining capacitors in series and in parallel, the circuit is reduced in steps as shown below. The equivalent capacitor is shown to be a $\boxed{2.00\ \mu\text{F}}$ capacitor.

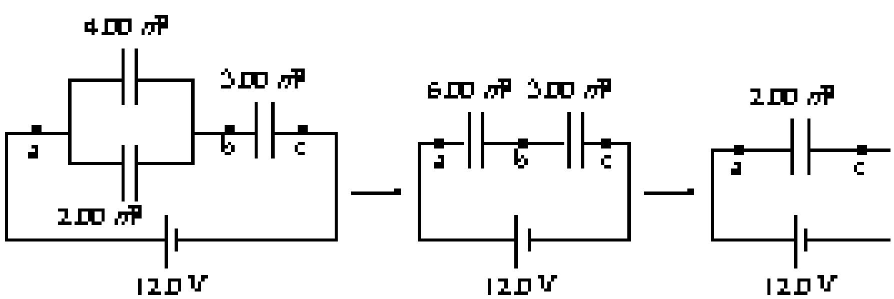


Figure 1 Figure 2 Figure 3

(b) From Figure 3: $Q_{ac} = C_{ac}(\Delta V)_{ac} = (2.00\ \mu\text{F})(12.0\ \text{V}) = 24.0\ \mu\text{C}$

From Figure 2: $Q_{ab} = Q_{bc} = Q_{ac} = 24.0\ \mu\text{C}$

Thus, the charge on the $3.00\ \mu\text{F}$ capacitor is $Q_3 = \boxed{24.0\ \mu\text{C}}$

Continuing to use Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ab}}{C_{ab}} = \dfrac{24.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 4.00\ \text{V}$

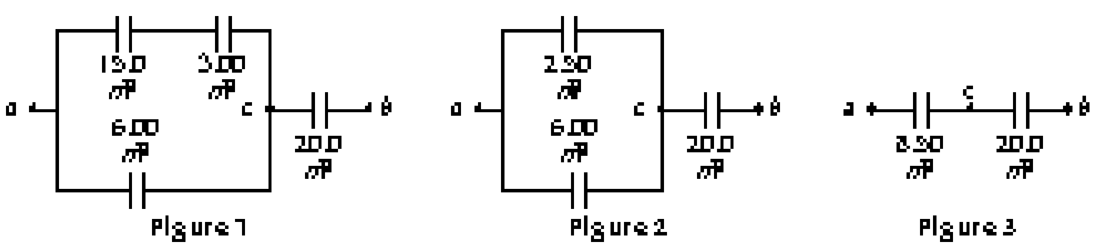
and $(\Delta V)_3 = (\Delta V)_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{8.00\ \text{V}}$

From Figure 1, $(\Delta V)_4 = (\Delta V)_2 = (\Delta V)_{ab} = \boxed{4.00\ \text{V}}$

and $Q_4 = C_4(\Delta V)_4 = (4.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V)_2 = (2.00\ \mu\text{F})(4.00\ \text{V}) = \boxed{8.00\ \mu\text{C}}$

16.33



(a) The equivalent capacitance of the upper branch between points *a* and *c* in Figure 1 is

$$C_s = \frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F} + 3.00\ \mu\text{F}} = 2.50\ \mu\text{F}$$

Then, using Figure 2, the total capacitance between points *a* and *c* is

$$C_{ac} = 2.50\ \mu\text{F} + 6.00\ \mu\text{F} = 8.50\ \mu\text{F}$$

From Figure 3, the total capacitance is

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $$Q_{ab} = Q_{ac} = Q_{cb} = (\Delta V)_{ab} C_{eq}$$

$$= (15.0\ \text{V})(5.96\ \mu\text{F}) = 89.5\ \mu\text{C}$$

Thus, the charge on the $20.0\ \mu\text{C}$ is $Q_{20} = Q_{cb} = \boxed{89.5\ \mu\text{C}}$

$$(\Delta V)_{ac} = (\Delta V)_{ab} - (\Delta V)_{bc} = 15.0\ \text{V} - \left(\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}}\right) = 10.53\ \text{V}$$

Then, $Q_6 = (\Delta V)_{ac}(6.00\ \mu F) = \boxed{63.2\ \mu\text{C}}$ and

$$Q_{15} = Q_3 = (\Delta V)_{ac}(2.50\ \mu F) = \boxed{26.3\ \mu\text{C}}$$

16.34 (a) The combination reduces to an equivalent capacitance of $\boxed{12.0\ \mu\text{F}}$ in stages as shown below.

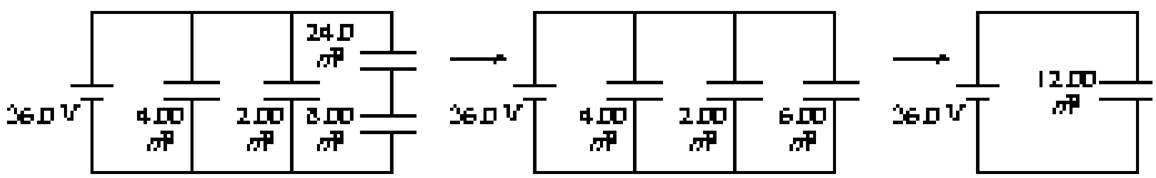


Figure 1 Figure 2 Figure 3

(b) From Figure 2, $Q_4 = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$

$$Q_2 = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72.0\ \mu\text{C}}$$

and $Q_6 = (6.00\ \mu\text{F})(36.0\ \text{V}) = 216\ \mu\text{C}$

Then, from Figure 1, $Q_{24} = Q_8 = Q_6 = \boxed{216\ \mu\text{C}}$

16.37 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25 \times 10^3\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00 \times 10^3\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) When the two capacitors are connected in parallel, the equivalent capacitance is $C_{eq} = C_1 + C_2 = 25.0\ \mu\text{F} + 40.0\ \mu\text{F} = 65.0\ \mu\text{F}$.

Since the negative plate of one was connected to the positive plate of the other, the total charge stored in the parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00 \times 10^3\ \mu\text{C} - 1.25 \times 10^3\ \mu\text{C} = 750\ \mu\text{C}$$

The potential difference across each capacitor of the parallel combination is

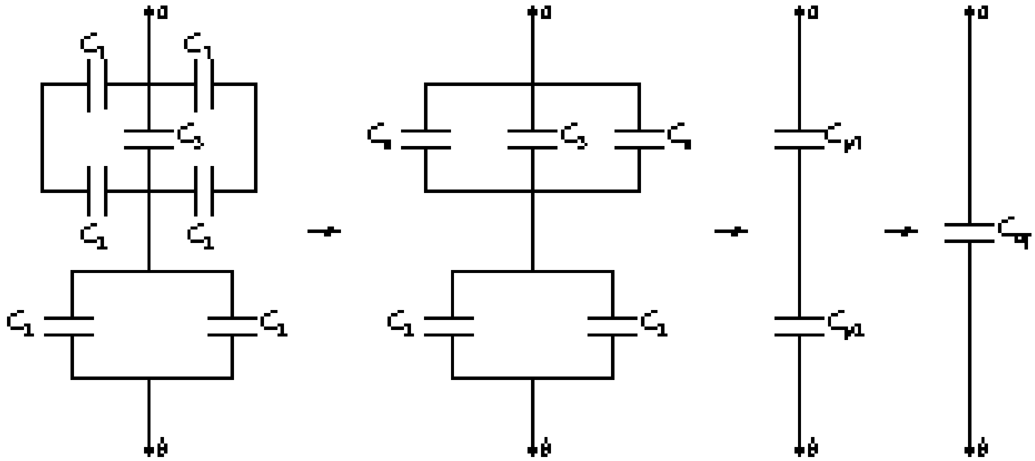
$$\Delta V = \frac{Q}{C_{eq}} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

and the final charge stored in each capacitor is

$$Q'_{25} = C_1(\Delta V) = (25.0\ \mu\text{F})(11.5\ \text{V}) = \boxed{288\ \mu\text{C}}$$

and $\quad Q'_{40} = Q - Q'_{25} = 750\ \mu\text{C} - 288\ \mu\text{C} = \boxed{462\ \mu\text{C}}$

16.40 The original circuit reduces to a single equivalent capacitor in the steps shown below.



$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}} + \frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2\left(3.33\ \mu\text{F}\right) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2\left(10.0\ \mu\text{F}\right) = 20.0\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

16.43 The capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.00\times10^{-4}\ \text{m}^2\right)}{5.00\times10^{-3}\ \text{m}} = 3.54\times10^{-13}\ \text{F}$$

and the stored energy is

$$W = \frac{1}{2}C\left(\Delta V\right)^2 = \frac{1}{2}\left(3.54\times10^{-13}\ \text{F}\right)\left(12.0\ \text{V}\right)^2 = \boxed{2.55\times10^{-11}\ \text{J}}$$

16.45 The capacitance of this parallel plate capacitor is

$$C = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1.0 \times 10^6\ \text{m}^2\right)}{\left(800\ \text{m}\right)} = 1.1 \times 10^{-8}\ \text{F}$$

With an electric field strength of $E = 3.0 \times 10^6\ \text{N/C}$ and a plate separation of $d = 800\ \text{m}$, the potential difference between plates is

$$\Delta V = Ed = \left(3.0 \times 10^6\ \text{V/m}\right)\left(800\ \text{m}\right) = 2.4 \times 10^9\ \text{V}$$

Thus, the energy available for release in a lightning strike is

$$W = \frac{1}{2}C\left(\Delta V\right)^2 = \frac{1}{2}\left(1.1 \times 10^{-8}\ \text{F}\right)\left(2.4 \times 10^9\ \text{V}\right)^2 = \boxed{3.2 \times 10^{10}\ \text{J}}$$

16.47 The initial capacitance (with air between the plates) is $C_i = Q/\left(\Delta V\right)_i$, and the final capacitance (with the dielectric inserted) is $C_f = Q/\left(\Delta V\right)_f$ where Q is the constant quantity of charge stored on the plates.

Thus, the dielectric constant is $\kappa = \dfrac{C_f}{C_i} = \dfrac{\left(\Delta V\right)_i}{\left(\Delta V\right)_f} = \dfrac{100\ \text{V}}{25\ \text{V}} = \boxed{4.0}$

16.49 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{(2.1)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(175 \times 10^{-4}\ \text{m}^2\right)}{0.040\,0 \times 10^{-3}\ \text{m}}$$

$$C = 8.13 \times 10^{-9}\ \text{F} = \boxed{8.13\ \text{nF}}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60.0 \times 10^6\ \text{V/m}$, so the maximum voltage is

$$V_{max} = E_{max}d = \left(60.0 \times 10^6\ \text{V/m}\right)\left(0.040\,0 \times 10^{-3}\ \text{m}\right)$$

$$V_{max} = 2.40 \times 10^3\ \text{V} = \boxed{2.40\ \text{kV}}$$

16.51 (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = \dfrac{4\pi r^3}{3}$, the radius is $r = \left[\dfrac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \in_0 A}{d}$

$$= \frac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^5}$$

16.59 The energy stored in a charged capacitor is $W = \dfrac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(300\ \text{J})}{30.0\times10^{-6}\ \text{F}}} = 4.47\times10^3\ \text{V} = \boxed{4.47\ \text{kV}}$$