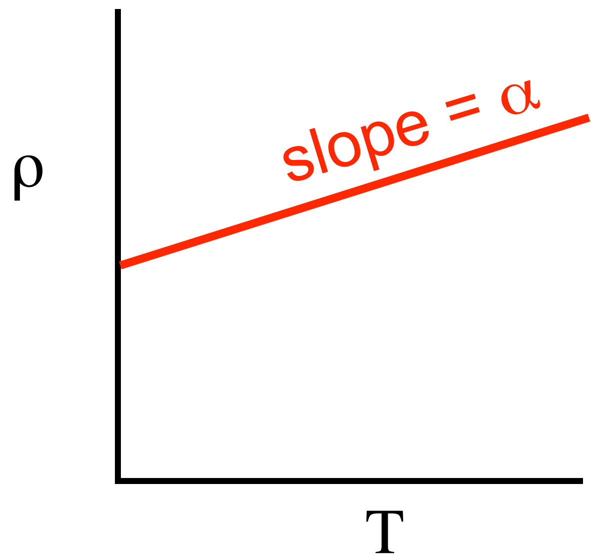
Temperature coefficient of resistivity



$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

$$R = R_o[1 + \alpha(T - T_o)]$$

T_0 = reference temperature

α = temperature coefficient of resistivity, units of $(^oC)^{-1}$

For Ag, Cu, Au, Al, W, Fe, Pt, Pb: values of α are ~ $3\text{-}5\times10^{-3}\ (^oC)^{-1}$

Please note -- I edited some values here compared to the slide I presented in lecture!

Typical tungsten filament: ~1 m long, but 0.05mm in radius.

Calculate typical R.

$A = \pi(5\text{x}10^{-5}\text{m})^2 = 7.9\text{x}10^{-9}\ \text{m}^2$

$\rho = 5.6\text{x}10^{-8}\ \Omega\text{m}$ (Table 17.1)

$R = \rho L/A = (5.6\text{x}10^{-8}\ \Omega\text{m})\ (1\text{m})/\ 7.9\text{x}10^{-9}\ \text{m}^2 = 7.1\ \ \Omega$

Note: As per section 17.6, the resistivity value used above is valid only at a temperature of 20°C, so this derived value of R holds only for T=20°C.

Calculate ρ at T=4000°C, assuming a linear ρ-T relation:

For tungsten, $\alpha = 4.5 \times 10^{-3}$/°C

$\rho = \rho_0[1+\alpha(T-T_0)] = 8.3 \times 10^{-7}\ \Omega m$

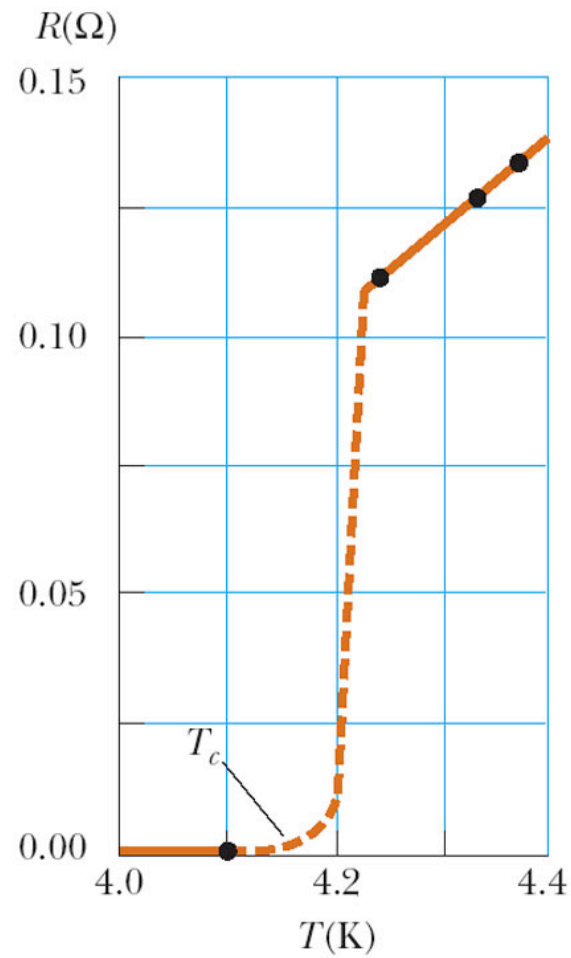
$R = \rho L/A = 106\ \Omega$.

(note-- this is still less than the estimate of >200 Ω we'll derive in class in a few minutes… I suspect the ρ-T relation in reality may not be strictly linear over such a wide range of temperature; my guess would be that the above value of α may only be valid for temperatures of tens to hundreds of °C)

Superconductors

For some materials, as temperature drops, resistance suddenly plummets to 0 below some T_c.

Once a current is set up, it can persist without any applied voltage because $R \rightarrow 0$!



Superconductors

Applications:

- Energy storage at power plants
- Superconducting magnets with much stronger magnetic fields than normal electromagnets
- Super conducting distribution power lines could eliminate resistive losses

More recently: As the field has advanced, materials with higher values of T_c get discovered

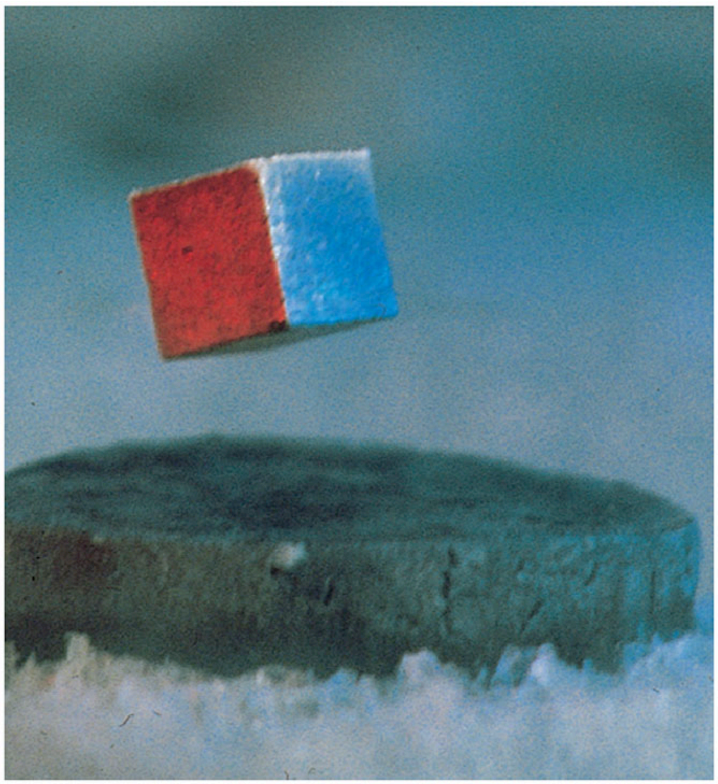


TABLE 17.2

Critical Temperatures for Various Superconductors

Material	T_c **(K)**
Zn	0.88
Al	1.19
Sn	3.72
Hg	4.15
Pb	7.18
Nb	9.46
Nb_3Sn	18.05
Nb_3Ge	23.2
$YBa_2Cu_3O_7$	90
Bi–Sr–Ca–Cu–O	105
Tl–Ba–Ca–Cu–O	125
$HgBa_2Ca_2Cu_3O_8$	134

Electrical Energy and Power

Power dissipated in a R is due to collisions of charge carriers with the lattice. Electrical potential energy is converted to thermal energy in the resistor--
a light bulb filament thus glows
or toaster filaments give off heat (and turn orange)

Power dissipated in a resistor

Power = work / time = $q\Delta V/\Delta t$

$P = I * \Delta V$

$P = I^2 R$

$P = \Delta V^2 / R$

UNITS:

P = I V = Amp * Volt = C/s * J/C = J/s = WATT

Example: A typical household incandescent lightbulb is connected to a 120V outlet. The power output is 60 Watts. What's the current through the bulb? What’s R of the filament?

Example: A typical household incandescent lightbulb is connected to a 120V outlet. The power output is 60 Watts. What's the current through the bulb? What's R of the filament?

$\Delta V = 120$ V (rel. to ground)
$P = I\Delta V \rightarrow I = P/\Delta V = 60W/120V = 0.5$ A

$P = \Delta V^2 / R \rightarrow$ ----> $R = \Delta V^2 / P = (120V)^2 / 60$ W = 240 Ω

Note -- a few slides earlier, we'd estimated the typical resistance of a tungsten light bulb filament at 4000ºC -- that estimate of ~106 Ω assumed for simplicity a constant coefficient of resistivity α from 20ºC to 4000ºC, which might not be the case in reality. If the actual value of α increases as T increases, then the dependence of ρ on T will also be non-linear

Electric Range

A heating element in an electric range is rated at 2000 W. Find the current required if the voltage is 240 V. Find the resistance of the heating element.

$P = I\Delta V \rightarrow I = P/\Delta V = 2000W/240V = 8.3\ A$

$R = \Delta V^2 / P = (240V)^2/2000W = 28.8\ \Omega$

Cost of electrical power

1 kilowatt-hour = 1000 W * 1 hour = 1000 J/s (3600s) = 3.6e6 J.

1kWh costs about $0.13, typically

How much does it cost to keep a single 100W light bulb on for 24 hours?
(100W)*24hrs = 2400 W-hr = 2.4kWh
2.4kWh*$0.13 = $0.31

So how much does it cost per week to keep the ~40 fluorescent lights in this classroom on for 40 hours per week?
(assume P=20W, since fluor. bulbs are ~4x as efficient as producing visible light as incandescent light bulbs).
40x20W*40hr = 32000 W-hr = 32kWh
32kWh*$0.13 = $4.16
How many rooms are there on campus?

Power Transmission

Transmitting electrical power is done much more efficiently at higher voltages due to the desire to minimize (I^2R) losses.

Consider power transmission to a small community which is 100 mi from the power plant and which consumes power at a rate of 10 MW.

In other words, the generating station needs to supply whatever power it takes such that P_{req} =10 MW arrives at the end user (compensating for I^2R losses): $P_{generated} = P_{loss} + P_{req}$

Consider three cases:

A: V=2000 V; I=5000 A ($P_{req} = IV = 10^7$ W)

B: V=20000 V; I=500 A ($P_{req} = IV = 10^7$ W)

C: V=200000 V; I=50 A ($P_{req} = IV = 10^7$ W)

Power Transmission

Resistance/length = 0.0001 Ω / foot.

Length of transmission line = 100 mile = 528000 feet.

Total R = 52.8 Ω.

A: $P_{loss} = I^2R = (5000A)^2(52.8\Omega) = 1.33\text{x}10^3$ MW

$P_{generated} = P_{loss} + P_{req} = 1.33\text{x}10^3$ MW + 10 MW = $1.34\text{x}10^3$ MW

Efficiency of transmission = $P_{req} / P_{generated}$ = 0.75%

B: $P_{loss} = I^2R = (500A)^2(52.8\Omega) = 13.3$ MW

$P_{generated} = P_{loss} + P_{req}$ = 13.3 MW + 10 MW = 23.3 MW

Efficiency of transmission = $P_{req} / P_{generated}$ = 43%

C: $P_{loss} = I^2R = (50A)^2(52.8\Omega) = 0.133$ MW

$P_{generated} = P_{loss} + P_{req}$ = 0.133 MW + 10 MW = 10.133 MW

Efficiency of transmission = $P_{req} / P_{generated}$ = 98.7% (most reasonable)

Lower current during transmission yields a reduction in P_{loss}!

You can do the same exercise for local distribution lines (assume P_{req} = 0.1 MW), which are usually a few miles long (so the value of R is ~ a few) and need to distribute power from substations to local neighborhoods at a voltage of at least a few thousand volts (keeping currents under ~30A, roughly) to have a transmission efficiency above ~90%.

Ch 18: Direct-Current Circuits

EMF

Resistors in Series & in Parallel

Kirchoff’s Junction & Loop Rules for complex circuits

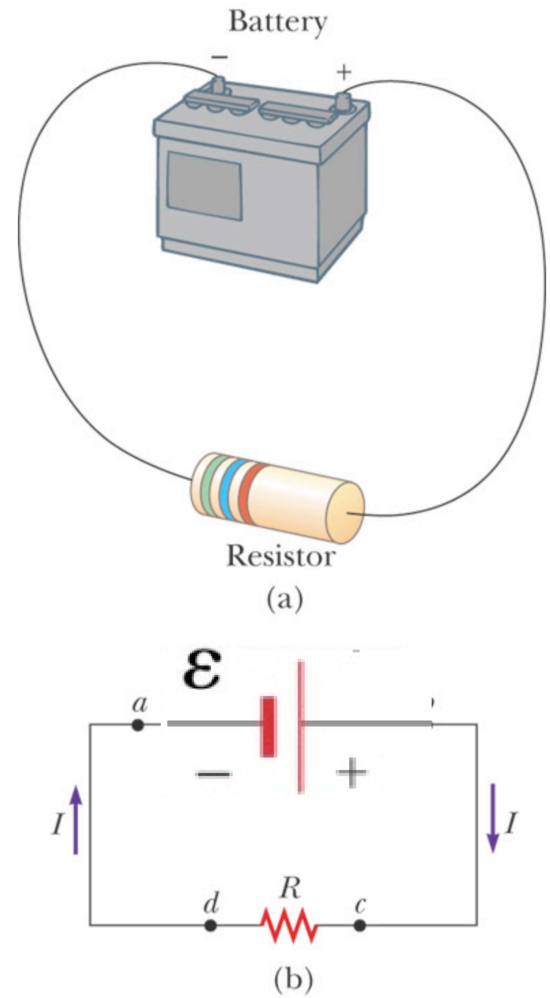
RC Circuits

Household circuits & Electrical Safety

Sources of EMF

In a closed circuit, the source of EMF is what drives and sustains the current.

EMF = work done per charge:
Joule / Coulomb = Volt



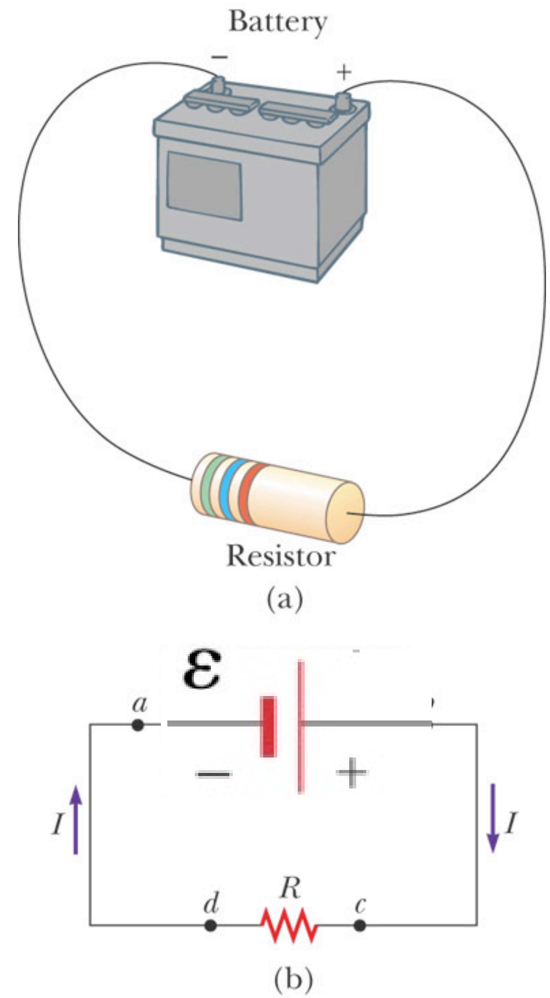
Sources of EMF

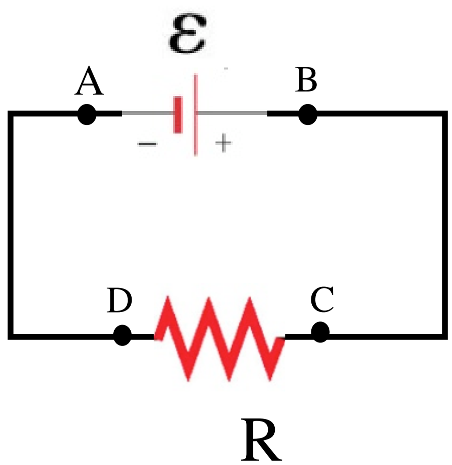
In a closed circuit, the source of EMF is what drives and sustains the current.

EMF = work done per charge: Joule / Coulomb = Volt

Assume internal resistance r of battery is negligible.

Here, $\mathcal{E} = IR$

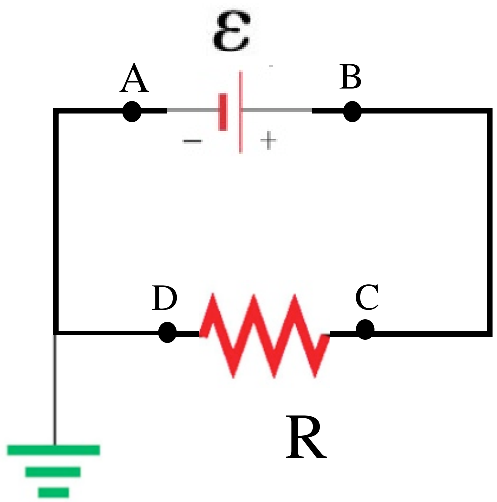




From A to B: Potential increases by $\Delta V = +\varepsilon$

From B to A: Potential decreases by $\Delta V = -\varepsilon$.

From C to D: Potential decreases by $\Delta V = -IR$ $= -\varepsilon$



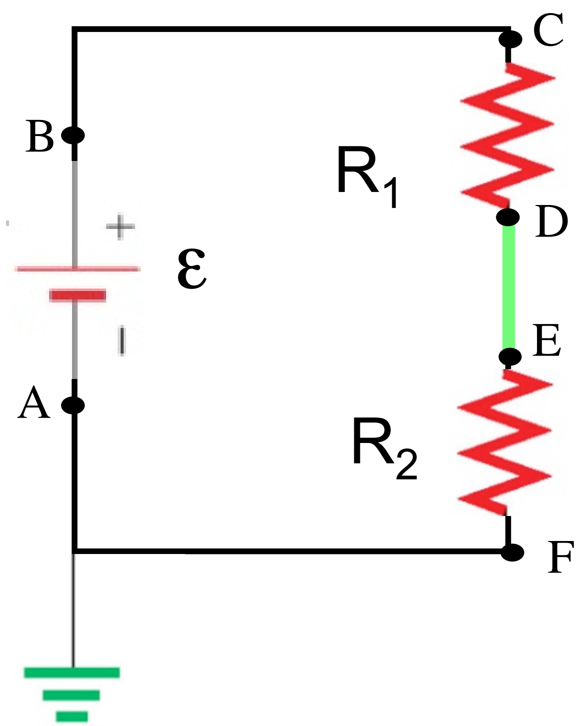
From A to B: Potential increases by $\Delta V = +\varepsilon$

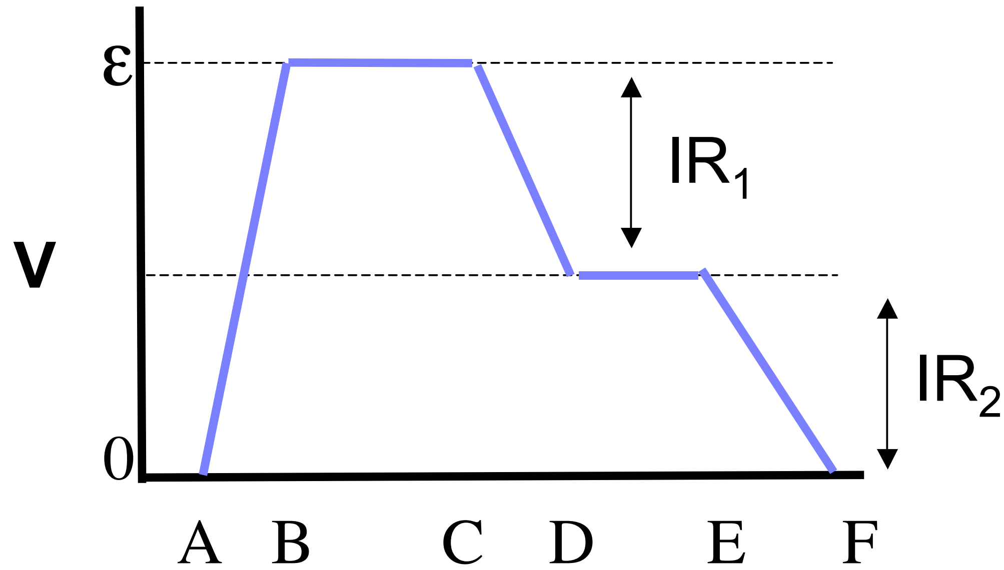
From B to A: Potential decreases by $\Delta V = -\varepsilon$.

From C to D: Potential decreases by $\Delta V = -IR = -\varepsilon$

If circuit is grounded: V at points A & D will be zero.

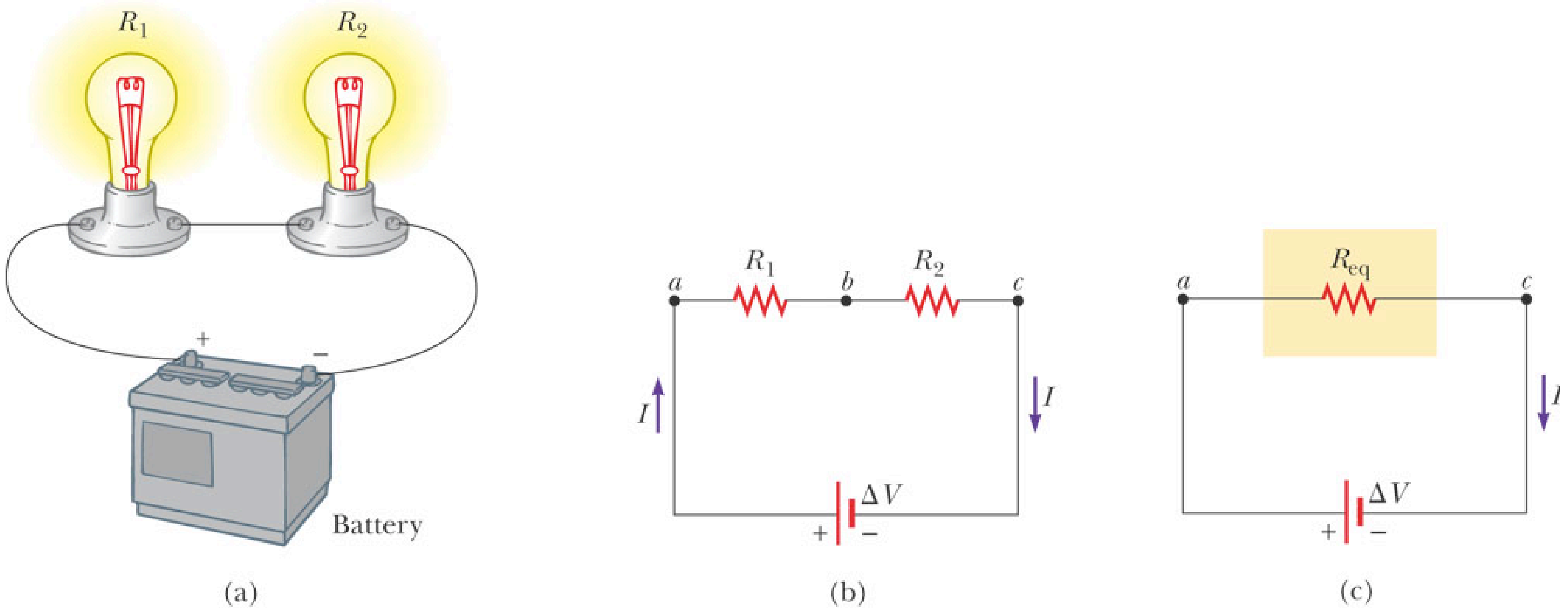
Why is this useful?





The middle voltage can be 'tailored' to any voltage we desire (between 0 and ε) by adjusting R_1 and R_2!

Resistors connected in series



What's R_{eq} in terms of R_1 and R_2?

$$\Delta V = IR_{eq}$$

Resistors connected in series

Note: Current is the same in R_1 and R_2.

$\Delta V_1 = IR_1$

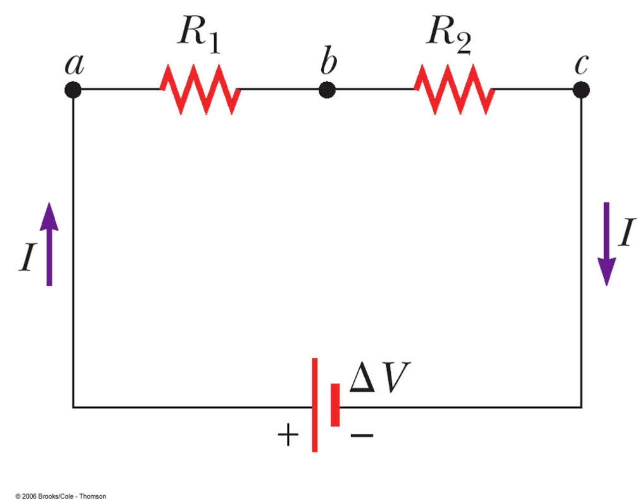
$\Delta V_2 = IR_2$

$\Delta V = \Delta V_1 + \Delta V_2$

$\Delta V = IR_1 + IR_2 = I(R_1+R_2)$

$\Delta V = IR_{eq}$

$R_{eq} = R_1 + R_2$

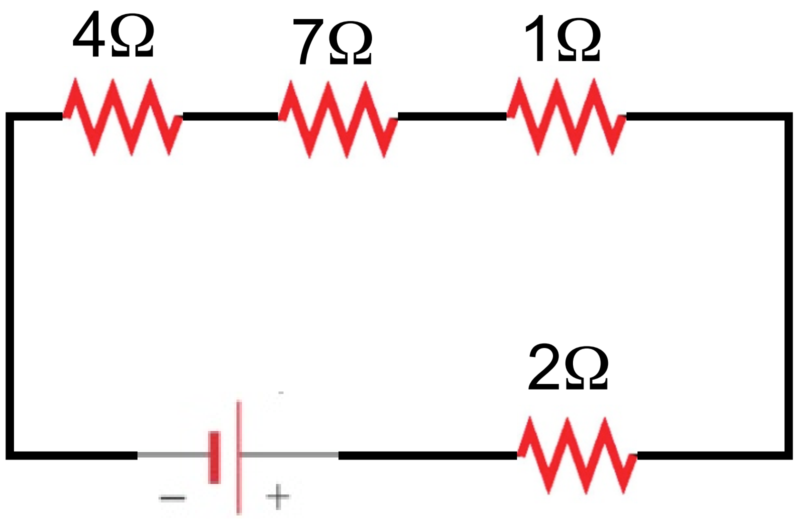


For N resistors in series:

$R_{eq} = R_1 + R_2 + \ldots + R_N$

Note that R_{eq} is larger than any one individual R value

Resistors connected in series



Find R_{eq}:

$$R_{eq} = 4\Omega+7\Omega+1\Omega+2\Omega = 14\Omega$$

Understanding the Series Law

$$R = \rho \frac{L}{A}$$ means R is prop.to L

Total R is prop. to (L_1 + L_2)