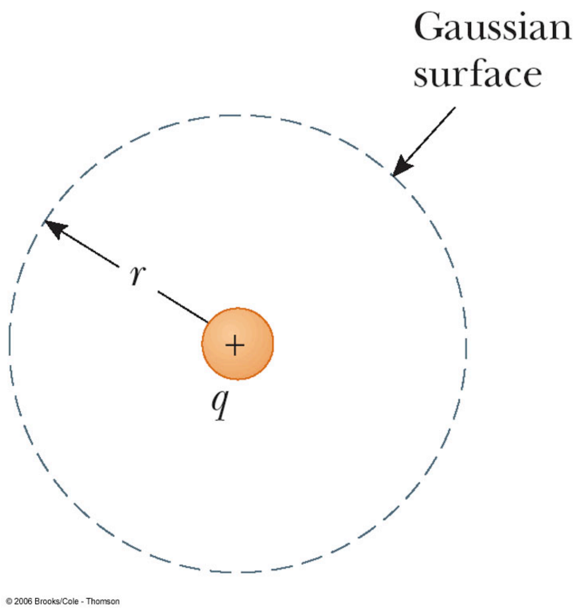
Gauss' Law



At radius r: $E = \frac{k_e q}{r^2}$

$\Phi_E = E \times Area = \frac{k_e q}{r^2} \times (4\pi r^2)$

Define $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

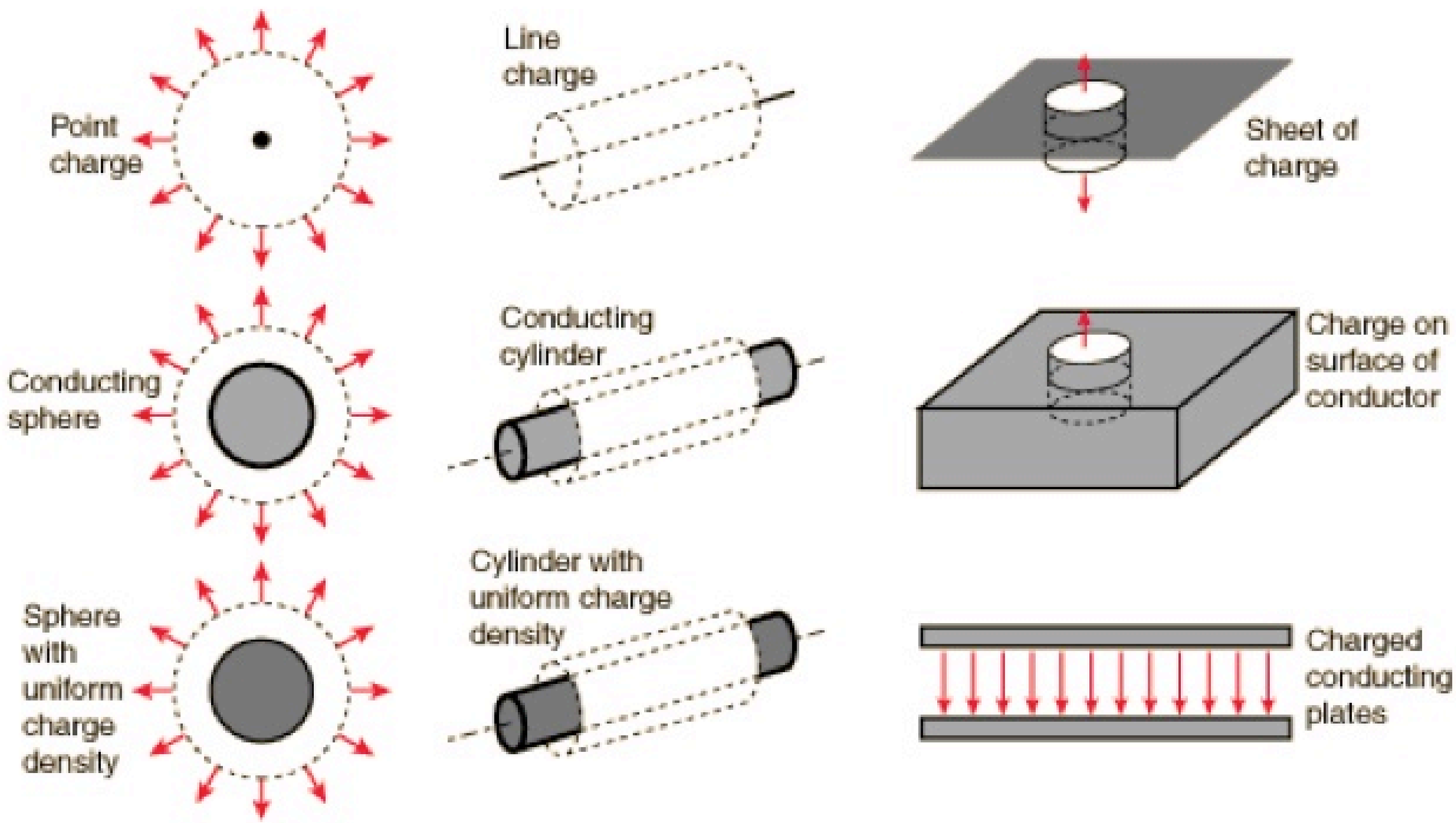
$\epsilon_0 =$ permittivity of free space

$\Phi_E = Q_{encl} / \varepsilon_0$

Φ_E through any closed surface is equal to the net charge enclosed, Q_{encl}, div. by ε_0

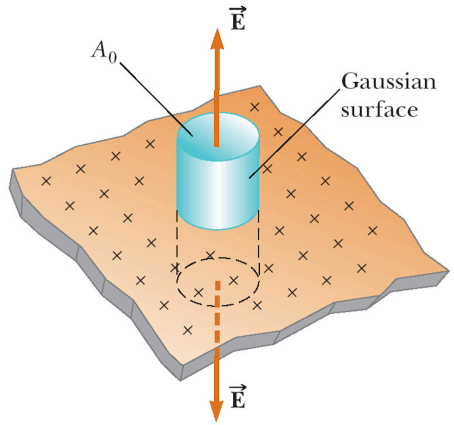
Sample Gaussian surfaces

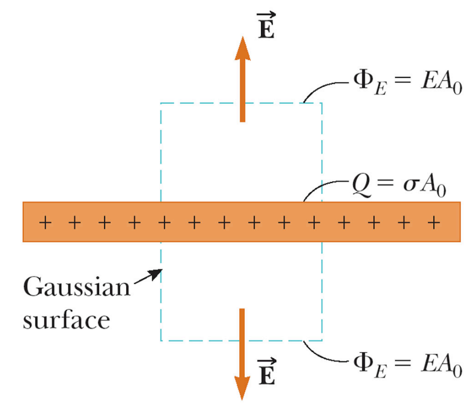
Hint: Choose surfaces such that $\vec{E}$ is ⊥ or || to surface!



Gauss' Law: A sheet of charge

Define σ = charge per unit area





$$\Phi_E = EA = Q_{encl}/\epsilon_0$$

A = area of top + bottom surfaces = 2 A_0

$$Q_{encl} = \sigma A_0$$

$$EA = \frac{\sigma A_0}{\epsilon_0}$$

$$E = \frac{\sigma A_0}{2A_0\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

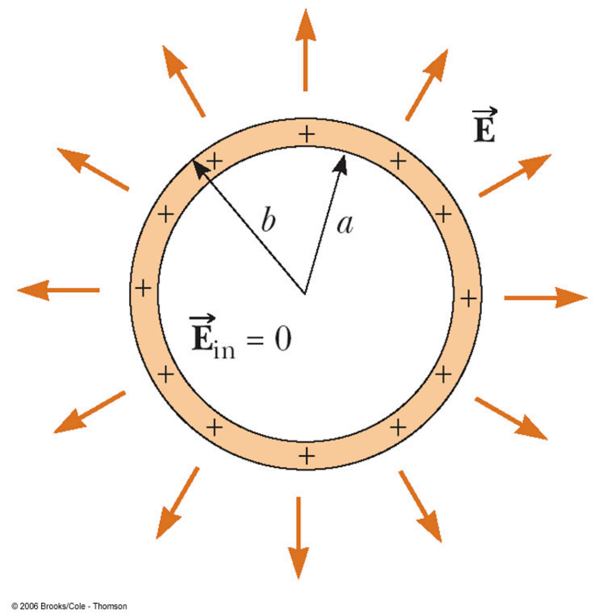
This is the magnitude of $\vec{E}$.

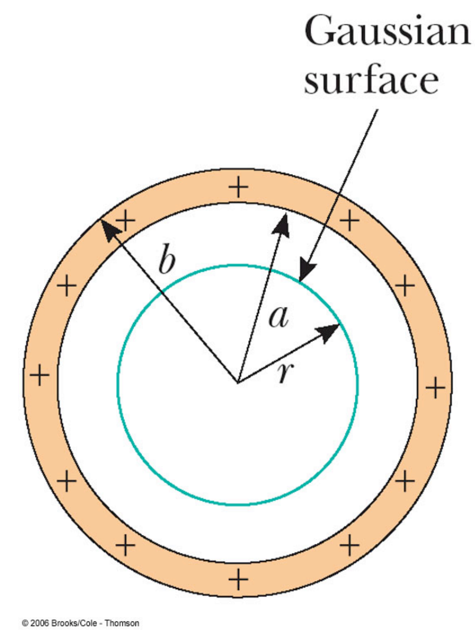
$\vec{E}$ points away from the the plane.

$\vec{E} = +\frac{\sigma}{2\epsilon_0}$ above the plane

$\vec{E} = -\frac{\sigma}{2\epsilon_0}$ below the plane

Gauss' Law: Charged Spherical Shell

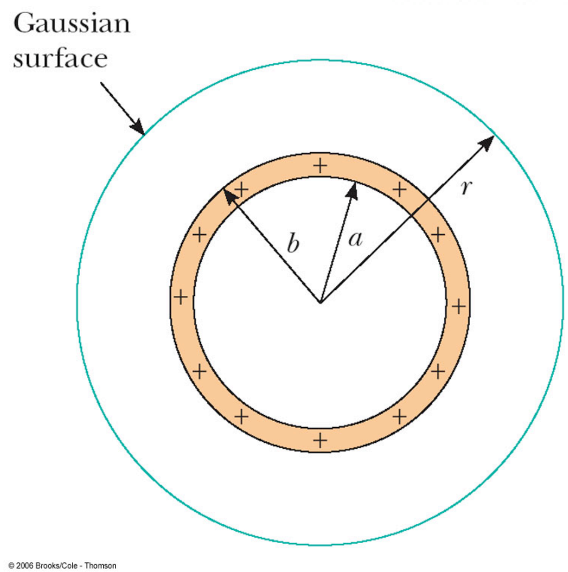




At r < a: $\vec{E} = 0$.

Gauss' Law: Charged Spherical Shell

At r > b, $\Phi_E = EA = 4E\pi r^2 = Q_{encl}/\epsilon_0$

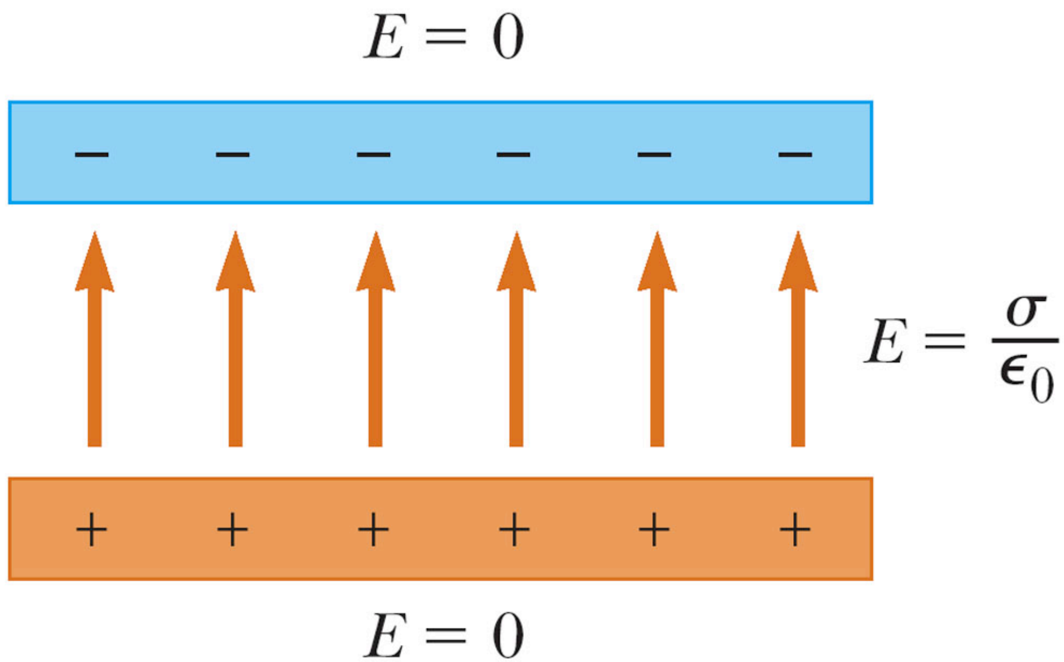


Divide both sides by area:

$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

At r > b, $\vec{E}$ looks like that from a single point charge Q

Gauss' Law: 2 planes with opposing charges



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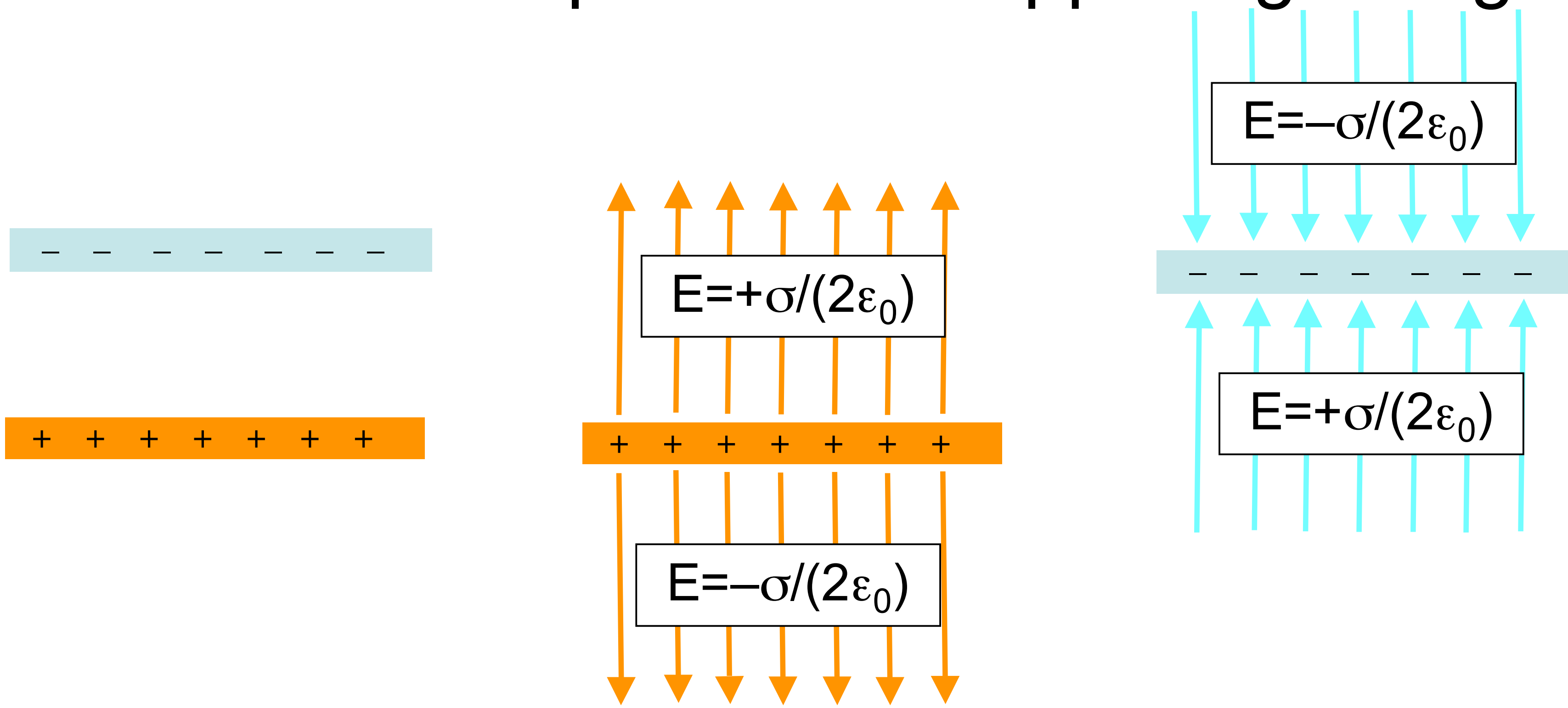
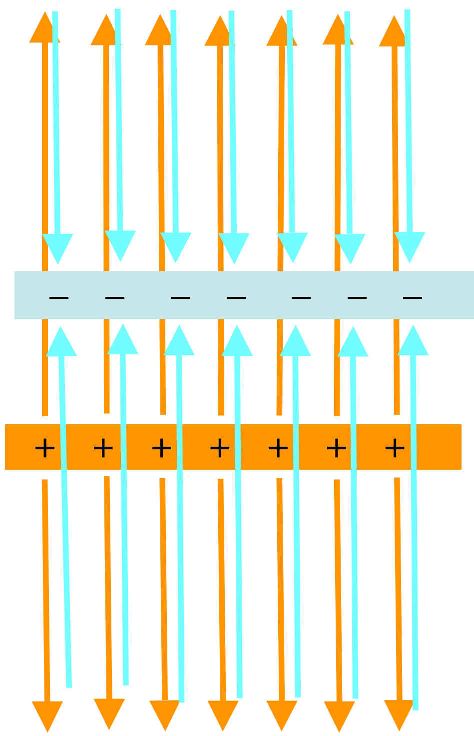


Gauss' Law: 2 planes with opposing charges

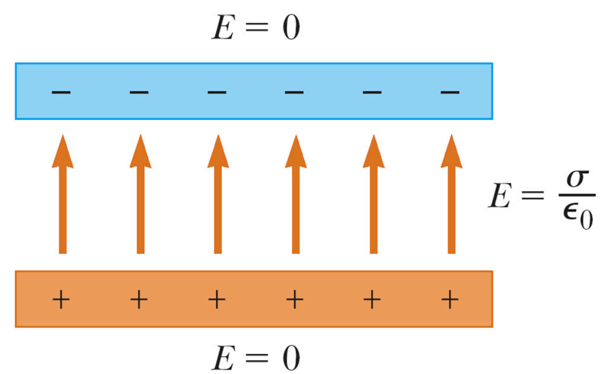


E=0 outside

Inside:

$E = +\sigma/(2\varepsilon_0) + \sigma/(2\varepsilon_0)$
$= \sigma/\varepsilon_0$

E=0 outside



Ch 16: Electric Energy, Potential & Capacitance

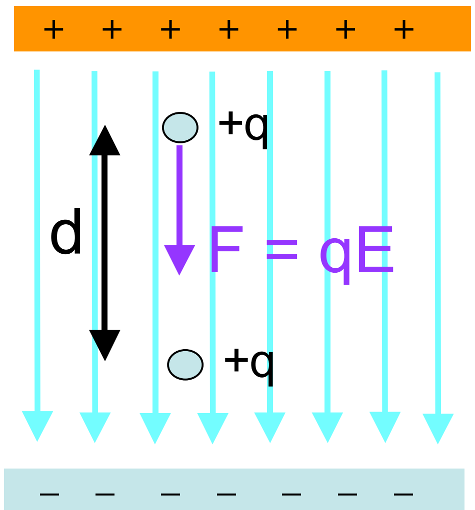
Electrical potential energy corresponding to Coulomb force (e.g., assoc. with distributions of charges)

Total Energy = K.E. + P.E.

Electric Potential = P.E. per unit charge

Circuit Elements: Capacitors: devices for storing electrical energy

Potential Energy of a system of charges



Potential Energy PE (scalar):

ΔPE = – Work done by the Electric field

$$\Delta PE = -W = -Fd = -qEd$$

(units = J)

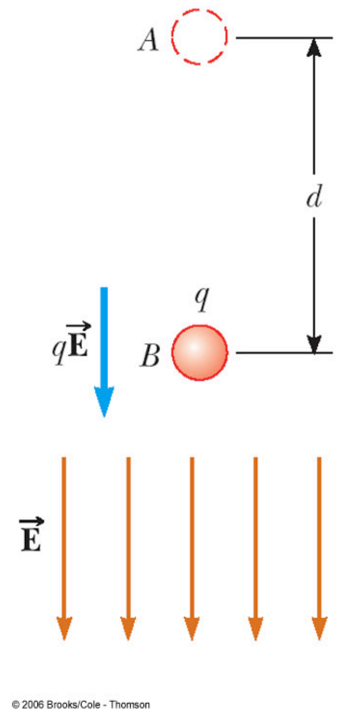
Work done by the E-field (to move the +q closer to the negative plate) REDUCES the P.E. of the system

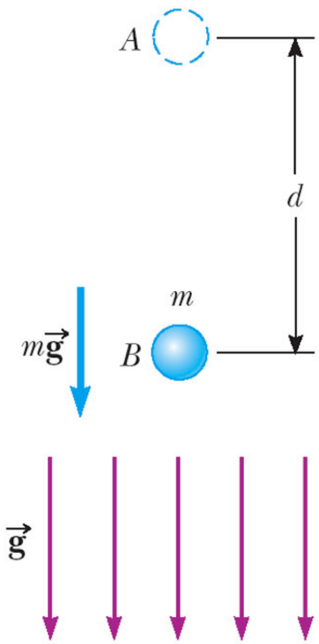
If a positive charge is moved AGAINST an E-field (which points from + to -), the charge-field system gains Pot. Energy. If a negative charge is moved against an E-field, the system loses potential energy

Comparing Electric and Gravitational fields

Higher P.E.➜

Lower P.E.➜





$\Delta PE = -qEd$

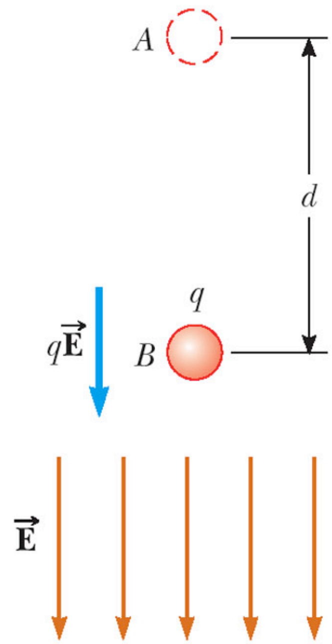
$\Delta PE = -mgd$

Comparing Electric and Gravitational fields

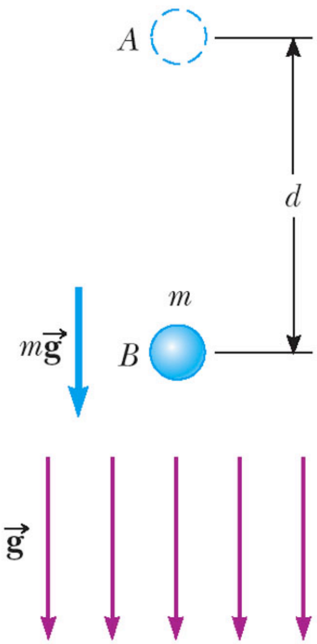
Higher P.E.➜

Lower P.E.➜

If released from rest (K.E.=0) at point A, K.E. when it reaches point B will be -ΔPE

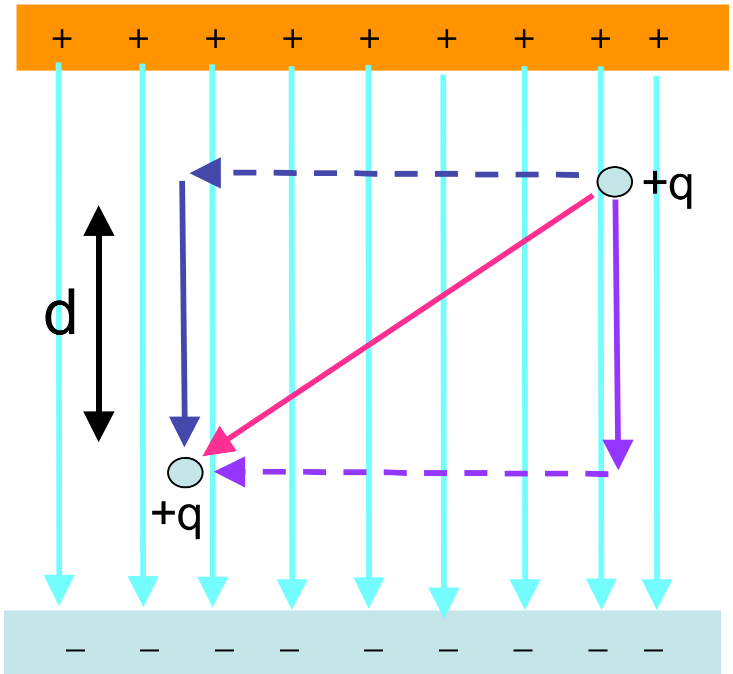


$\Delta PE = -qEd$



$\Delta PE = -mgd$

Electric Force is conservative



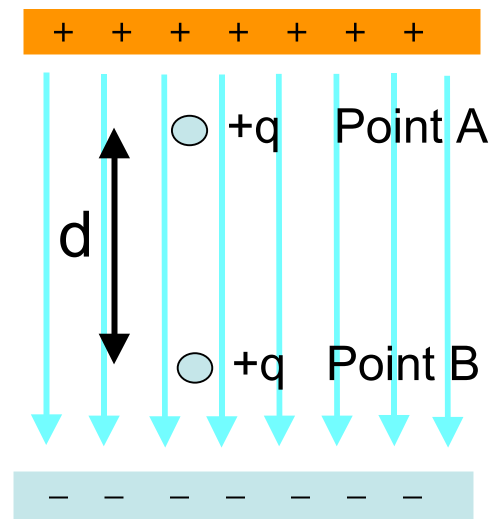
$\Delta PE = -qEd =$ independent of path chosen (depends only on end points)

Electric Potential Difference, ΔV

$\Delta V = V_B - V_A = \Delta PE / q$

Units: Joule/Coulomb = VOLT

Scalar quantity



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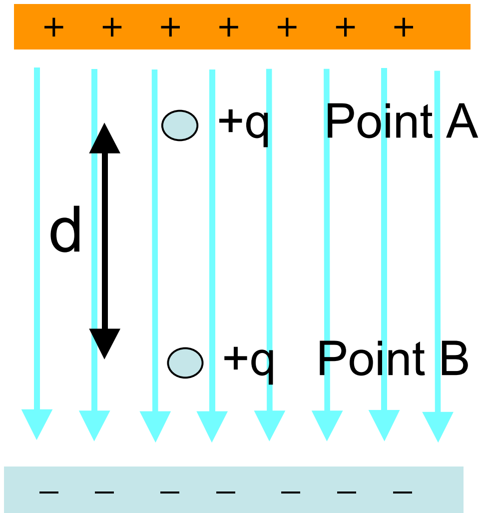


Relation between ΔV and E:

$\Delta V = Ed$

E has units of V/m = N/C

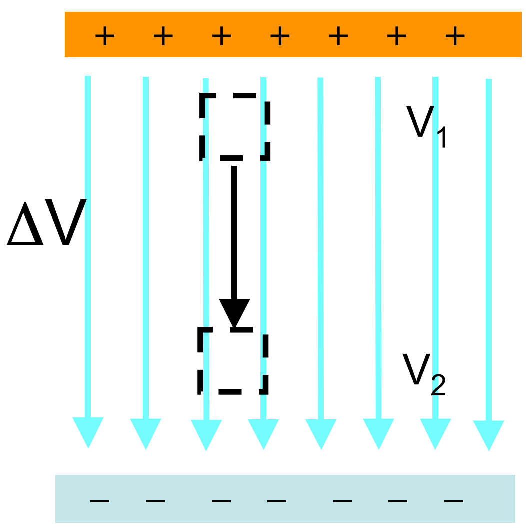
(V / m = J / Cm = Nm / Cm = N / C)

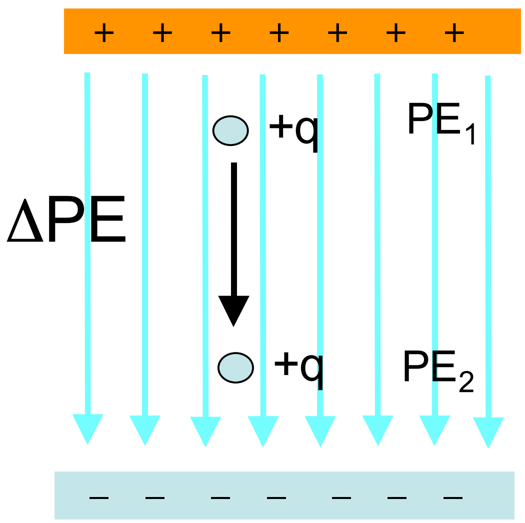
Potential vs. Potential Energy

POTENTIAL: Property of space due to charges; depends only on location

Positive charges will accelerate towards regions of low potential.

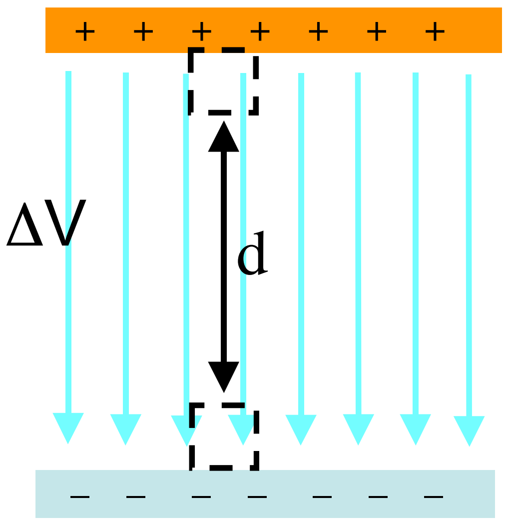
POTENTIAL ENERGY: due to the interaction between the charge and the electric field





Example of Potential Difference

A parallel plate capacitor has a constant electric field of 500 N/C; the plates are separated by a distance of 2 cm. Find the potential difference between the two plates.

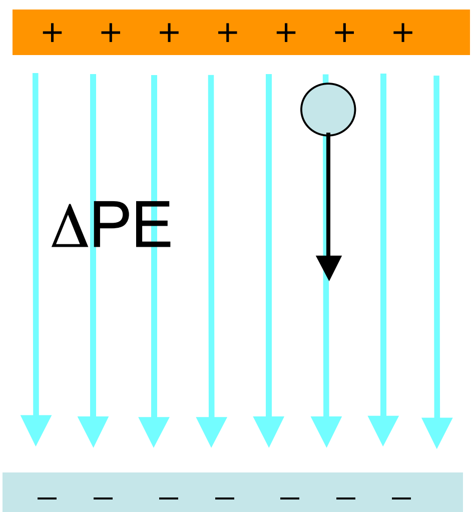


E-field is uniform, so we can use
$\Delta V = Ed = (500V/m)(0.02m) = 10V$

Remember: potential difference ΔV does not depend on the presence of any test charge in the E-field!

Example of Potential Difference

Now that we've found the potential difference ΔV, let's take a molecular ion, CO_2^+ (mass = 7.3×10^{-26} kg), and release it from rest at the anode (positive plate). What's the ion's final velocity when it reaches the cathode (negative plate)?



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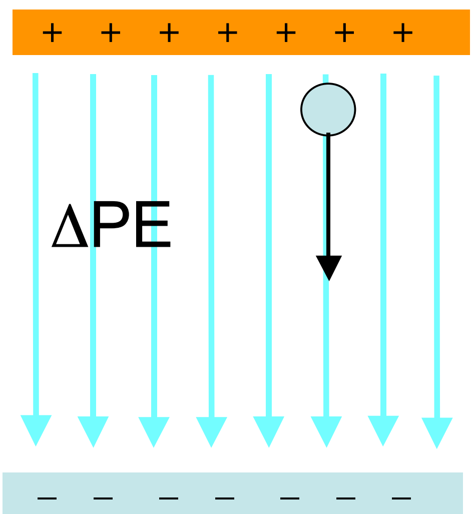

Solution: Use conservation of energy: $\Delta PE = \Delta KE$

$\Delta PE = \Delta V\, q$
$\Delta KE = 1/2\, m\, v_{final}^2 - 1/2\, m\, v_{init}^2$

$\Delta V\, q = 1/2\, m\, v_{final}^2$

$v_{final}^2 = 2\Delta V q/m = (2)(10V)(1.6\times10^{-19}C)/7.3\times10^{-26}$ kg
$v_{final} = 6.6\times10^{3}$ m/s

Thunderstorms:

From ground to cloud base:
$\Delta V = 10^8$ V, $E \sim 10^{4\text{-}5}$ V/m

Lightning: $E = 3\times10^6$ V/m is electric field strength at which air becomes ionized enough to act as a conductor.

Fair weather: $E \sim 10^2$ V/m