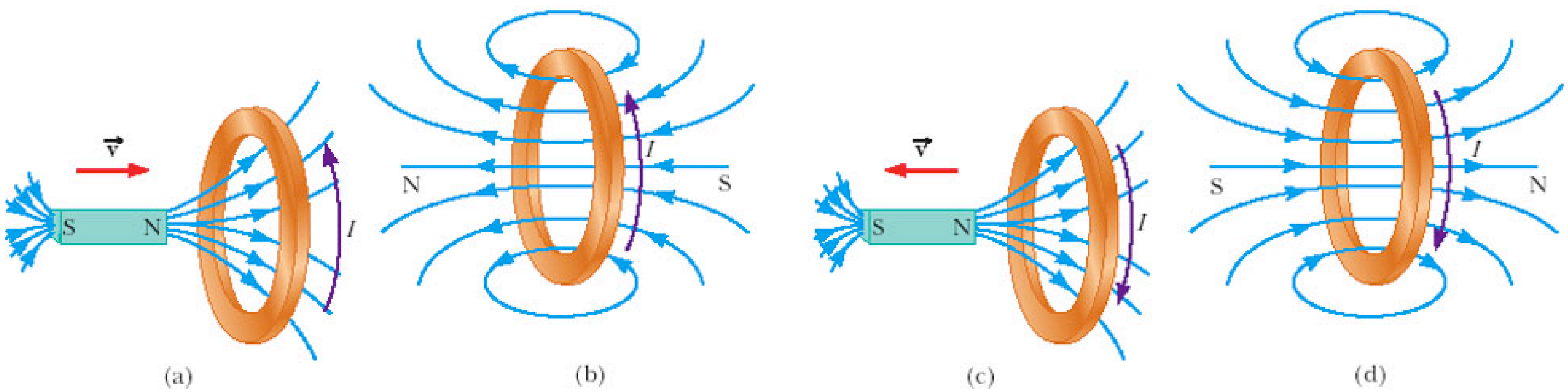
20.4: A closer examination of Lenz's Law

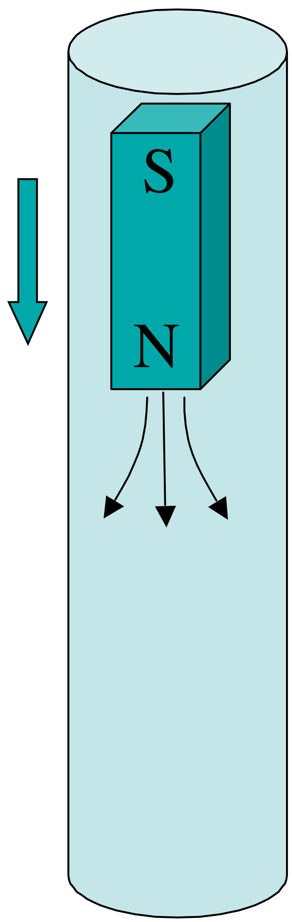
A force is exerted by a magnet on a loop to induce current.
A force must be exerted by the current on the magnet to oppose the change.

The induced current creates a magnetic dipole in the loop.
The magnetic dipole creates a force acting on the magnet.



Eddy Currents

Drop a magnet down a cylindrical conductor, N-pole first.

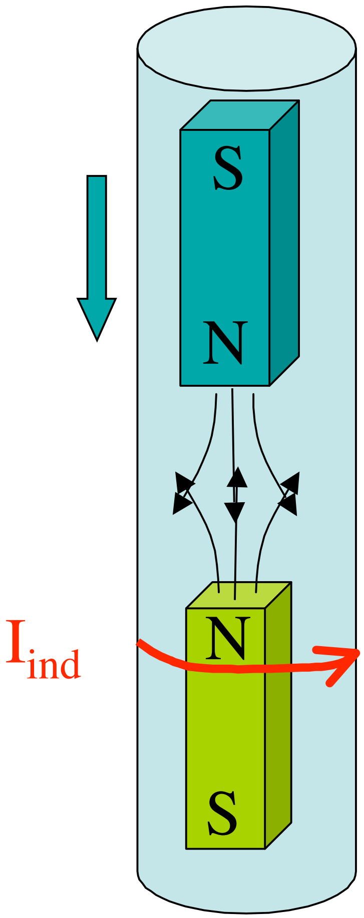


Ahead of (below) the magnet:

B increasing

Eddy Currents

Drop a magnet down a cylindrical conductor, N-pole first. Currents are induced in the conductor to oppose the movement of the magnet:



Ahead of (below) the magnet:

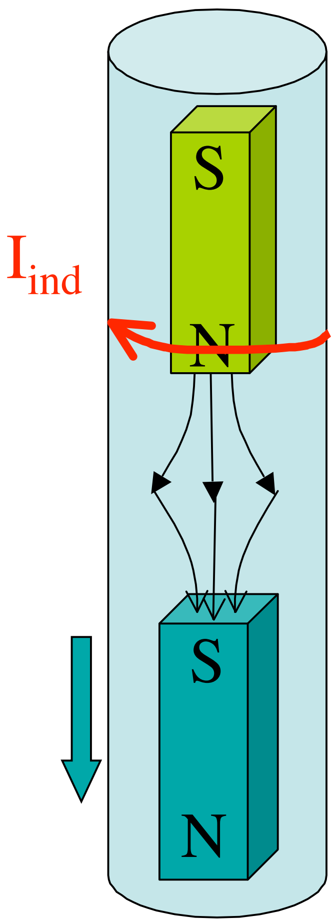
B increasing

induced magnetic dipole

two N poles: repel

Eddy Currents

In the region behind (above) the magnet:



induced magnetic dipole

B decreasing

N & S pole: attract

Eddy Currents

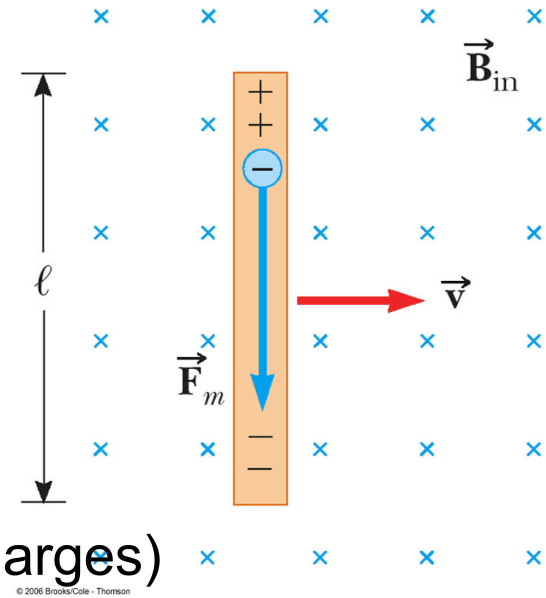
Selected videos of eddy currents:

http://www.youtube.com/watch?v=nrw-i5Ku0mI

http://www.youtube.com/watch?v=pcVG6c_OvYU

http://www.youtube.com/watch?v=37e_OROP9dA

20.3 Motional EMF



ε_{ind} from a conductor which is moving through a B-field

Conducting rod moving at a constant velocity $\vec{v}$

Charges experience $\vec{F}_M = qvB$ upward (for + charges)

Negative charges accumulate at bottom. Net + charge accumulates at top.

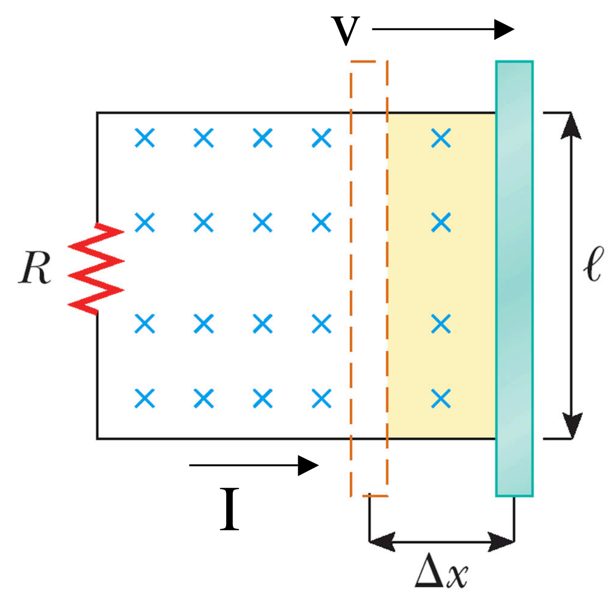
This continues until $\vec{F}_E$ and $\vec{F}_B$ are balanced: $qE = qvB$

Potential difference $\Delta V = E\ell = B\ell v$

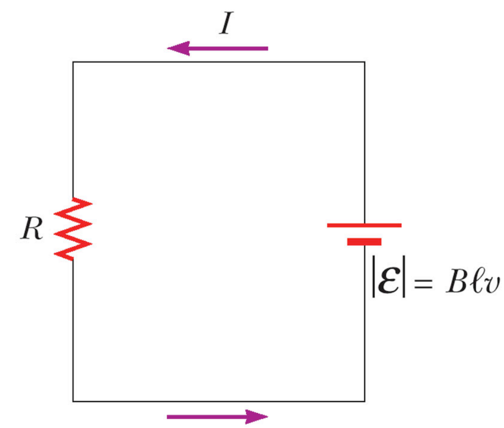
Upper end is at higher potential than lower end.

What if the moving conductor
is part of a closed circuit?

Conducting rod is being pulled with velocity $\vec{v} = \Delta x/\Delta t$



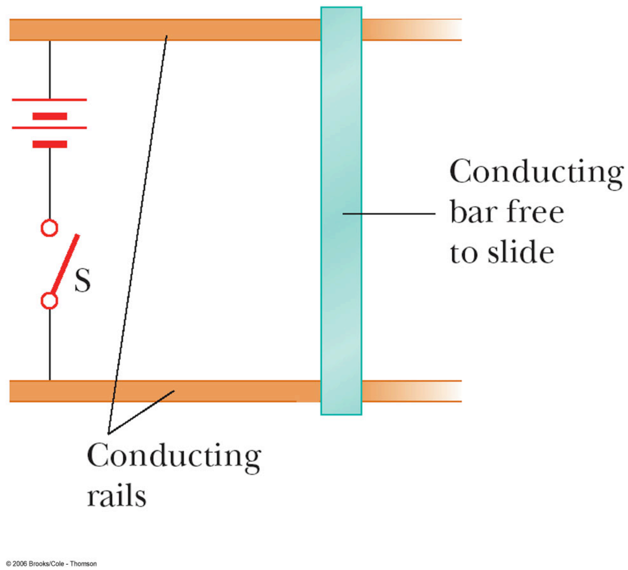
$\vec{B}$ is constant, but the AREA of the circuit (and hence Φ_B) is increasing



$$\Delta V = B\ell v = B\ell \frac{\Delta x}{\Delta t} = \frac{\Delta A}{\Delta t} B = \frac{\Delta \Phi_B}{\Delta t} = |\varepsilon|$$

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$

Rail gun/space catapult



When switch is closed:

Horizontal conducting rods create B-field into page.

Current in bar is downward, and a force $F_m = IB\ell$ pushes the bar to the right causing it to accelerate.

Payloads can be accelerated to several km/s

(U.S. Navy has tested one…)

Generators

Uses mechanical work to generate electrical current. Rotary motion from falling water (hydroelectric) or steam (coal-fired plant) directed against turbine blades.
A coil rotates, Φ_B through the coil changes, and EMF is produced. (similar to a motor in reverse)

EMF in wire BC = $B\ell v_\perp = B\ell v\sin\theta$

EMF in wire AD = $B\ell v_\perp = B\ell v\sin\theta$

Total EMF $\varepsilon = 2B\ell v\sin\theta$

Assume loop rotates with constant angular speed $\omega = \theta/t$. ($\omega = 2\pi f$)

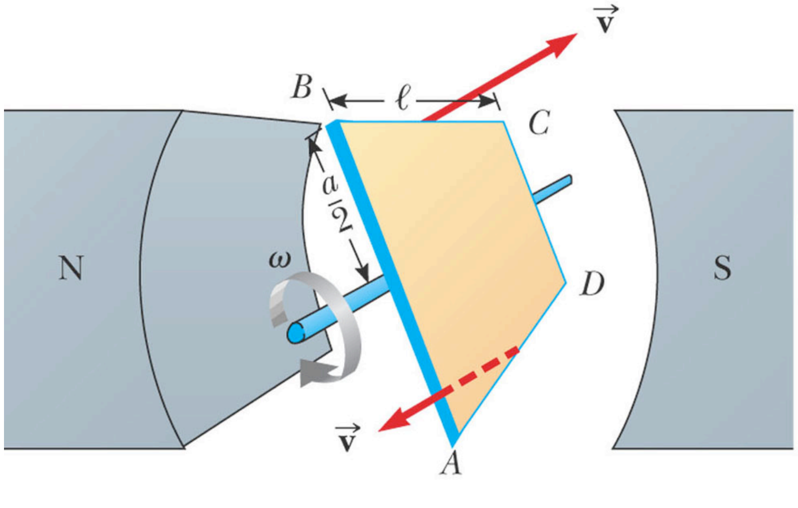
$v = r\omega = (a/2)\omega$

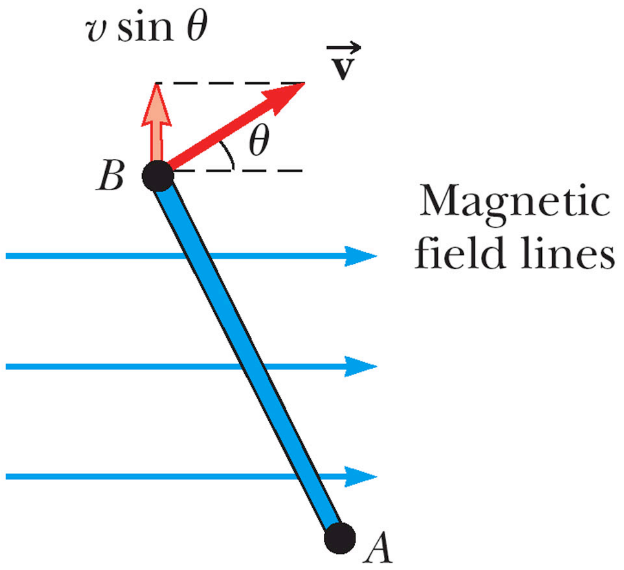
$\varepsilon = 2B\ell\,(a/2)\omega\,\sin\omega t$

$a\ell$ = Area A

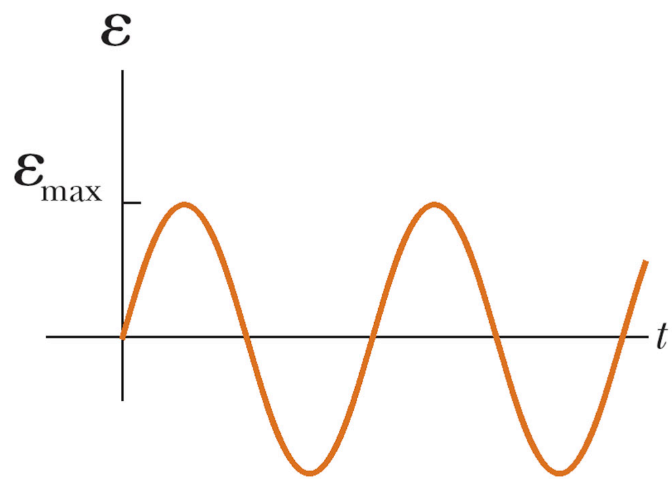
Assume coil has N loops:

$\varepsilon = NBA\omega\,\sin\omega t$

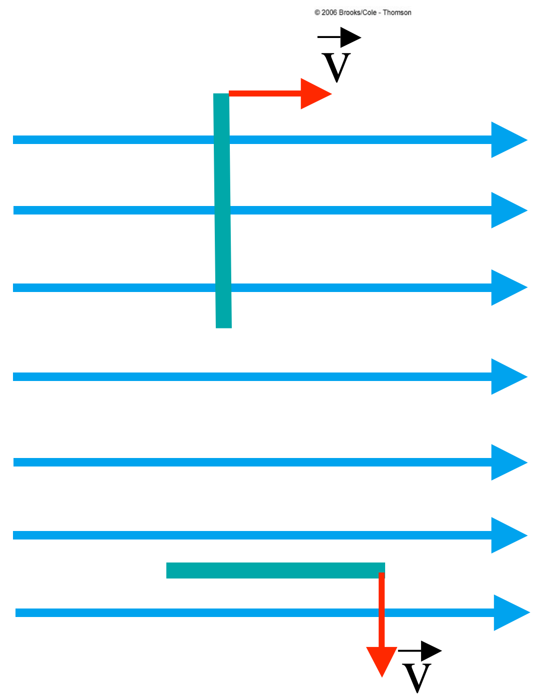


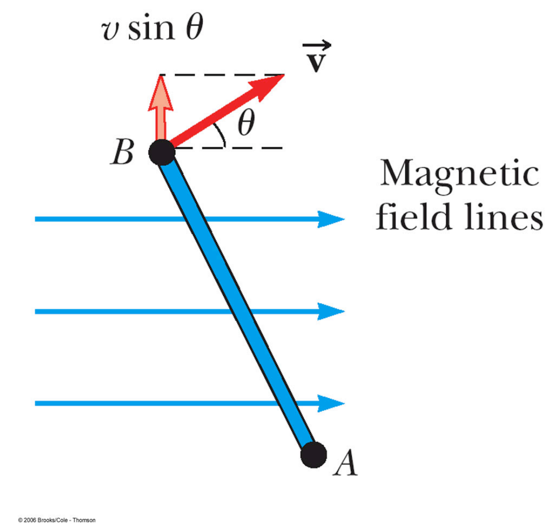


$$\varepsilon_{max} = NBA\omega$$



$\varepsilon = 0$ when $\theta = 0°$ or 180°: when plane of loop is $\perp$ to B-field



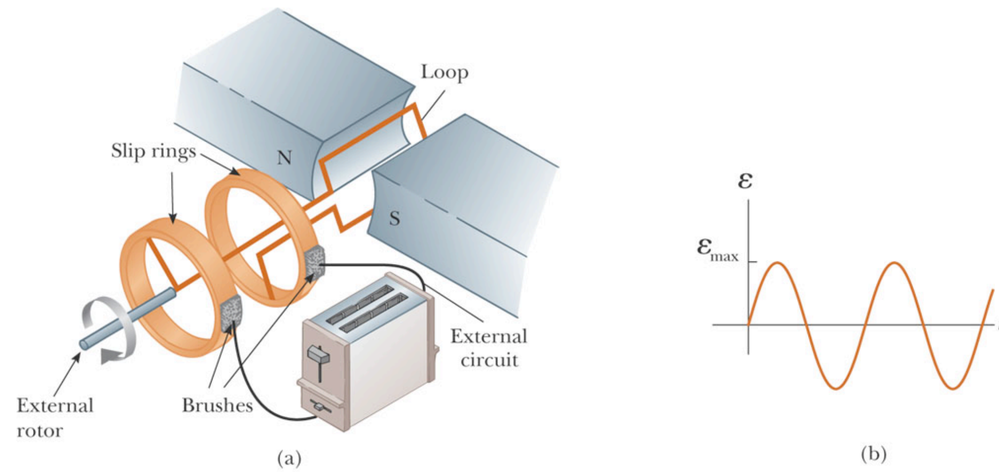


$\varepsilon = \varepsilon_{max}$ when $\theta = 90°$ or 270°: when plane of loop is parallel top B-field lines ($v_{\perp}$ is maximized)

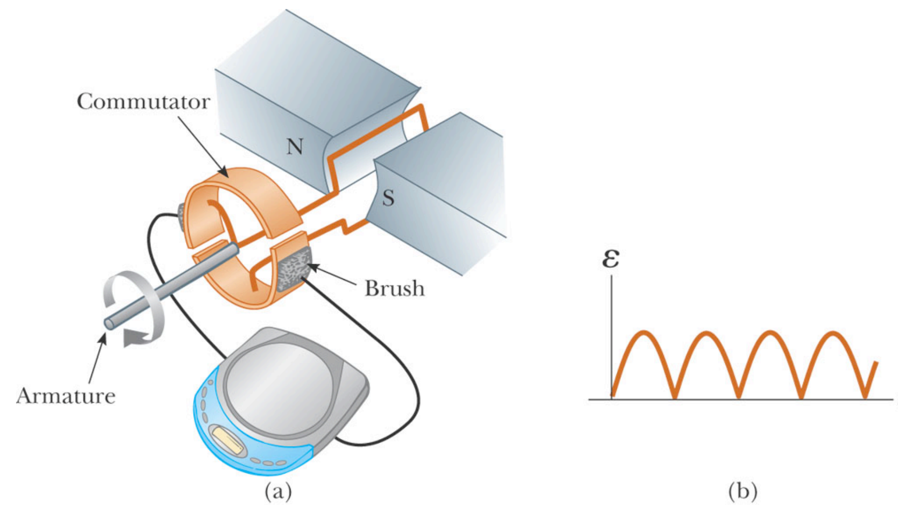
AC generators

f = 60 Hz (U.S. & Canada), 50 Hz for Europe

$\omega = 2\pi f = 377$ rad/s



DC generators: use commutators



To get a DC output with minimal fluctuations: Use many loops & commutators distributed around the axis of rotation so that sinusoidal pulses overlap in phase

Example for an AC generator

In a model AC generator, a 50-turn coil measuring 0.1m x 0.2 m rotates at a frequency of 20 revs/second in a field of 0.5 T. What's the maximum EMF that can be produced?

Soln: Use $\varepsilon_{max} = NBA\omega$

ε_{max} = (50)(0.5T)(0.1m0.2m)(2π*20 Hz) = 62.8 V

The total resistance of the wire is 10 Ω. What's the maximum induced current?

$I_{max} = \varepsilon_{max} / R$ = 62.8 V / 10 Ω = 6.28 A

Remember: ε and I vary sinusoidally as a function of time; ε_{max} and I_{max} are their amplitudes.

20.6: Self-Inductance

A property of a circuit carrying a current

A voltage is induced that opposes the change in current

Used to make devices called inductors

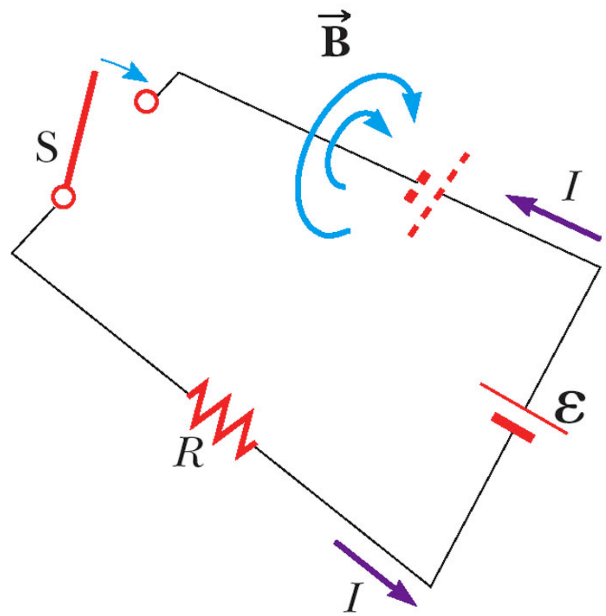
20.6: Self-Inductance

Close switch: current begins to flow.
Φ_B increases.
Faraday's Law + Lenz's Law: ε induced which opposes change in Φ_B

This is self-induced EMF, or back EMF

The effect is called self-induction

As I increases, the RATE of current lessens, and so ε_{ind} lessens.
So current can increase gradually



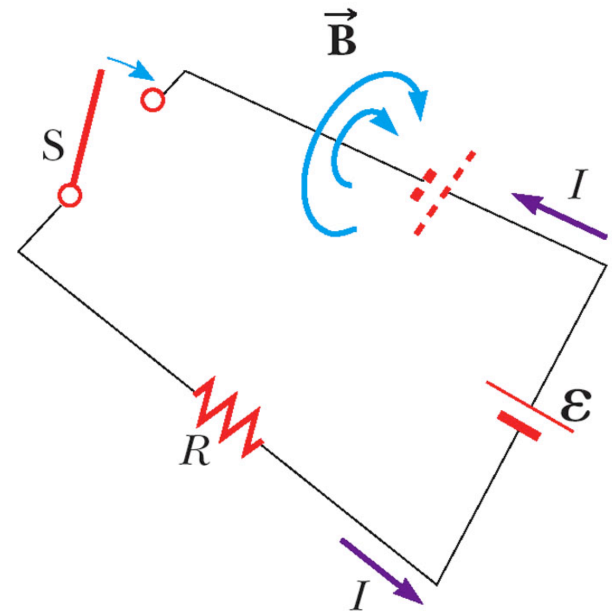
Back EMF in motors

Because the magnetic flux is always changing, an EMF is induced which acts to reduce the current in the coil.

This is “back EMF”

Back EMF = zero when circuit is first turned on

Voltage available to supply current = (voltage of external power source) – (back EMF)

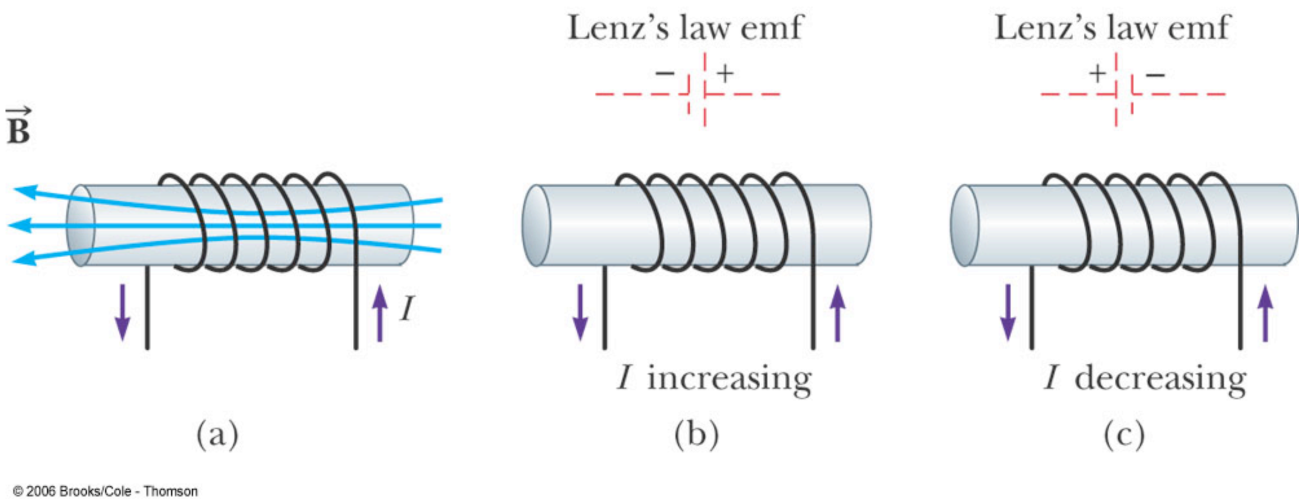


When switch is opened: current doesn't immediately fall to zero.

It tries to fall to zero, but now ε_{ind} is working in same direction as ε_{orig}.
ε_{ind} tries to keep the circuit going

Ex.: removing a plug when a machine is still running:
sparks appear as current attempts to jump gap
Application: spark plugs!

Self-inductance in a solenoid

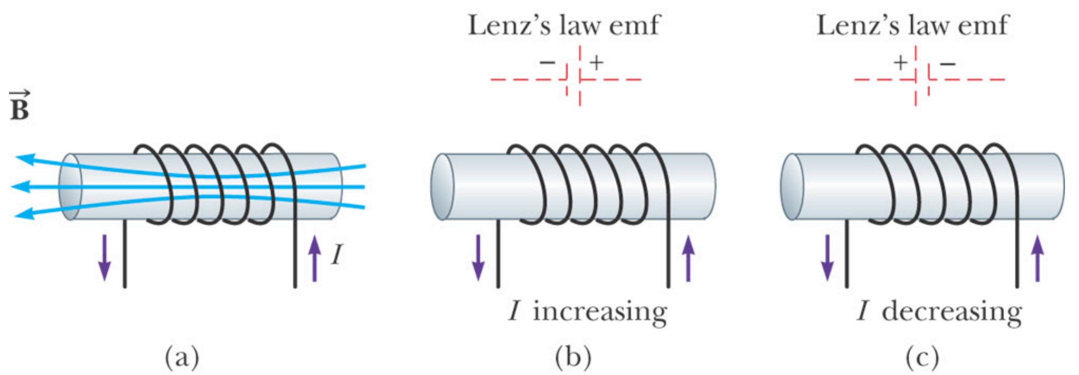


Faraday's Law: $$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

Φ_B is prop. to magnitude of B-field

$|B|$ is prop. to the current

Self-inductance in a solenoid



So induced ε and $\Delta\Phi_B/\Delta t$ must be prop. to ΔI/ΔT

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

L = a proportionality constant called 'inductance'

Units = Henrys (Joseph Henry, U.S.). 1 H = Volt s / Amp

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

If current is increasing: $\Delta I/\Delta T$ is positive, ε is negative, opposing the increase in current

If current is decreasing, $\Delta I/\Delta T$ is negative, ε is positive

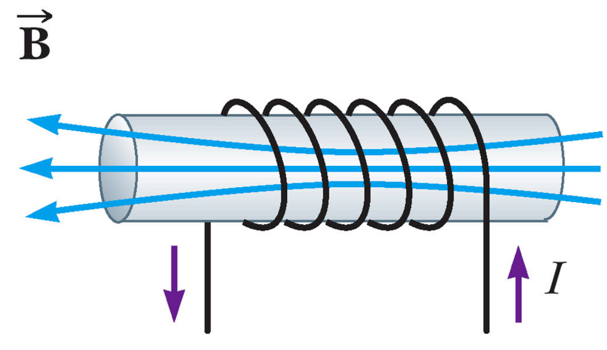
---

Determine an expression for L:

$$N\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t}$$

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}$$

Self-inductance of a solenoid



Total length ℓ. N turns. Air-core.

$$\Phi_B = BA = (\mu_0\,(N/\ell)\,I)\,A$$

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I} = (\mu_0\,(N^2/\ell))\,A$$

Example:

An air-core solenoid with 200 turns has a diameter of 0.5 cm and a total length of 2 cm. Calculate L

$L = (\mu_0 (N^2/\ell))\, A$

$= (4\pi \times 10^{-7}\ \mathrm{Tm/A})(200^2)(0.02\mathrm{m})^{-1}(\pi(0.0025\mathrm{m})^2)$

$= 4.9 \times 10^{-5}\ \mathrm{H} = 49\ \mu\mathrm{H}.$

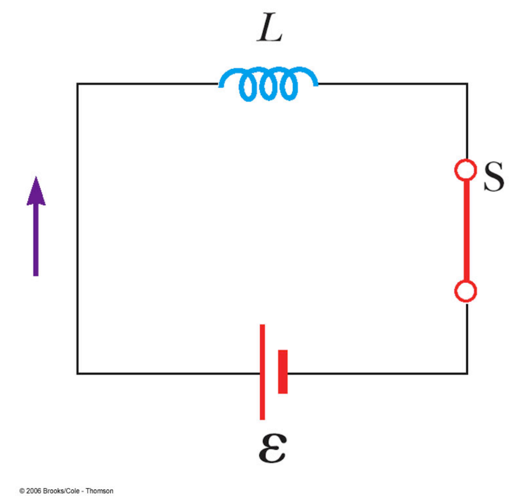
Inductors

Circuit elements that have large inductance

Circuit symbol: 

Prevents current from changing too rapidly. Can be used to produce very high voltages (spark plugs)

Consider the following circuit, just after its switch is closed.



Current tries to begin to flow, but the EMF of the battery equals the back EMF generated in the inductor.

L is a measure of opposition to $\Delta I/\Delta t$

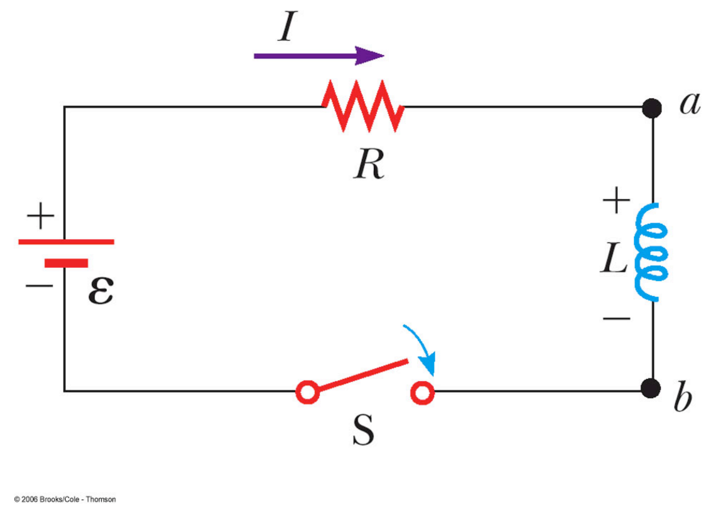
20.7: RL circuits

Let's close the switch and apply Kirchhoff's Loop rule:

$+\varepsilon - \Delta V_R - \Delta V_L = 0$

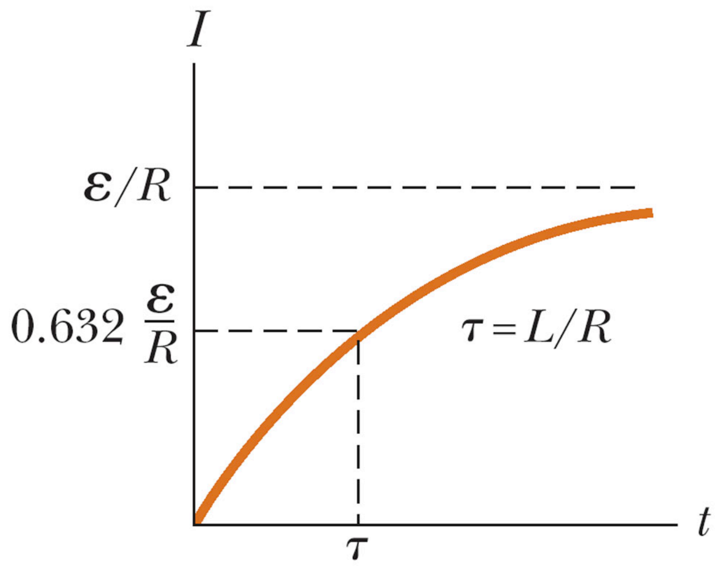
$+\varepsilon - IR - \varepsilon_L = 0$

$+\varepsilon - IR - L(\Delta I/\Delta t) = 0$



Current would like to increase to its maximum value of ε/R, but can't do it instantaneously.

Right after t=0, I is at a minimum and ΔI/Δt is at a maximum: $+\varepsilon - \overset{0}{IR} - L(\Delta I/\Delta t) = 0$

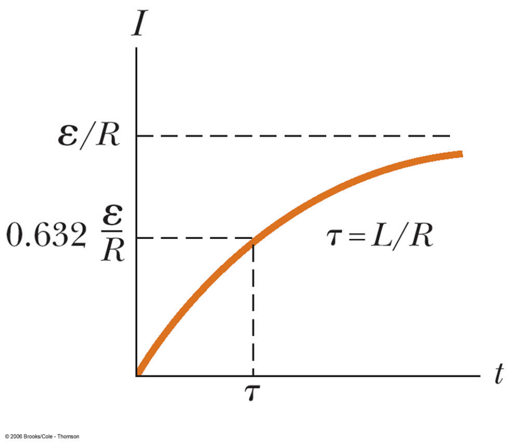


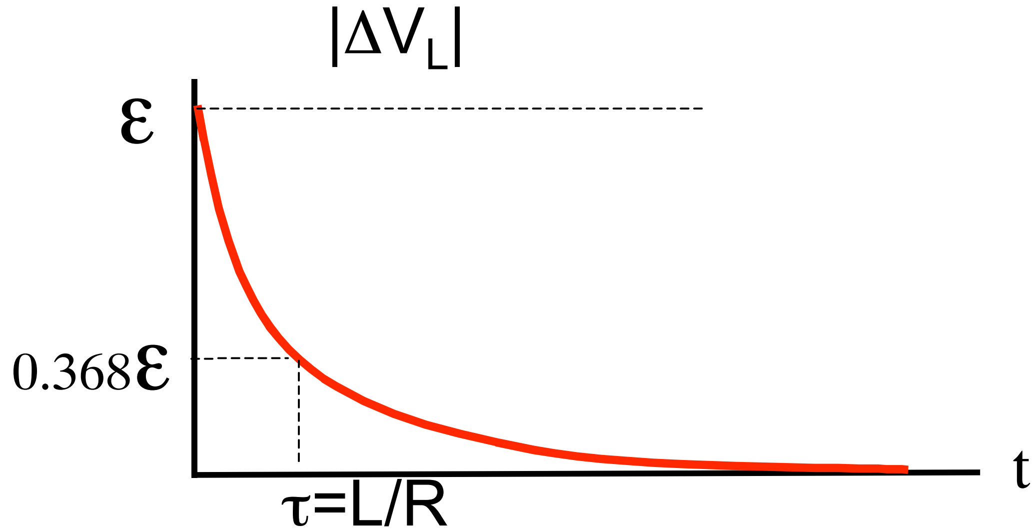
time constant τ

As time proceeds, $\Delta I/\Delta t$ lessens, $|\Delta V_L| = L(\Delta I/\Delta t)$ decreases, I increases and approaches ε/R.

$$+\varepsilon - IR - L\cancelto{0}{(\Delta I/\Delta t)} = 0$$

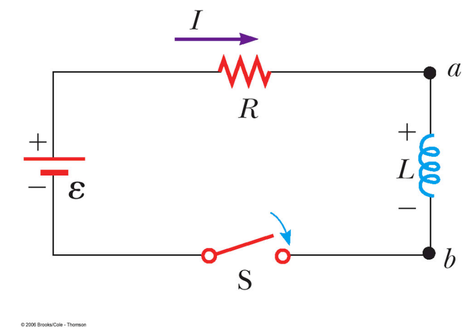
I(t):





	$t=0$		$t \to \infty$
I	0	$(\varepsilon/R)(1-e^{-(t/\tau)})$	ε/R
$\lvert\Delta V_R\rvert$	0	$\varepsilon(1-e^{-(t/\tau)})$	ε
$\lvert\Delta V_L\rvert$	ε	$\varepsilon(e^{-(t/\tau)})$	0

Example for an RL circuit



Given a 9-volt battery connected in series with a 3 mH inductor and a 0.5 Ω resistor. The switch is closed at t=0.

a: Find the time constant τ:

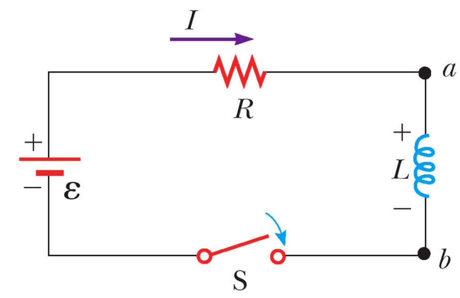
$\tau = L/R = 3 \times 10^{-3} H/0.5\Omega = 6 \times 10^{-3}$ sec = 6 msec

b: What is the current when $t=\tau$?

At $t=\tau$, the current will be 63.2% of the maximum value, $I_{max} = \varepsilon/R = 9V/0.5\Omega = 18A$

$I(t=\tau) = 0.632 I_{max} = 0.632(18A) = 11.4A$

Example for an RL circuit



Given a 9-volt battery connected in series with a 3 mH inductor and a 0.5 Ω resistor. The switch is closed at t=0.

c: Find the magnitude of the voltage drop across the resistor at

$t=0$, $t=\tau$, and $t=\infty$.

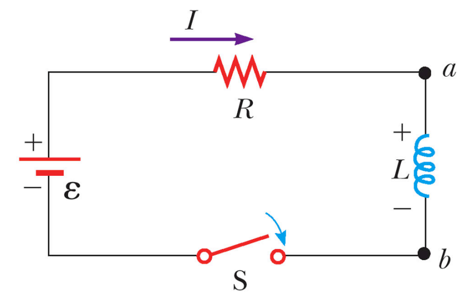
Hint: determine the current at these 3 times:

At $t=0$, $I = 0$. so $\Delta V_R = IR = 0$

At $t=\tau$, $I=0.632\times I_{max} = 0.632(\varepsilon/R)$ so $\Delta V_R = IR = 11.4\text{A} \times 0.5\ \Omega = 5.69\ \text{V}$

At $t=\infty$, $I = I_{max} = \varepsilon/R$ so $\Delta V_R = IR = 18\text{A} \times 0.5\ \Omega = 9\ \text{V} = \varepsilon$

Example for an RL circuit



d: Find the magnitude of the voltage drop across the inductor at

$t=0$, $t=\tau$, and $t=\infty$.

Hint: Recall from Kirchhoff's Rule:

$\varepsilon - \Delta V_R - \Delta V_L = 0.$

$\varepsilon = \Delta V_R + \Delta V_L$

At $t=0$, $\Delta V_R = 0$, so $\Delta V_L = \varepsilon$.

At $t=\tau$, $\Delta V_R = 0.632\varepsilon$ so $\Delta V_L = \varepsilon - \Delta V_R = IR = 9V - 5.69\ V = 3.31V$

(At $t=\tau$, $\Delta V_L = \varepsilon/e = 0.328\varepsilon$)

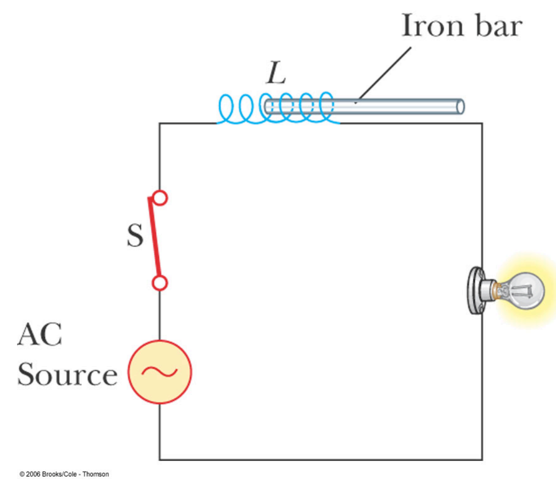
At $t=\infty$, $\Delta V_R = \varepsilon$, so $\Delta V_L = 0$ (expected since $\Delta I/\Delta t \rightarrow 0$ as $t \rightarrow \infty$)

Quick Quiz 20.5

In this RL circuit, driven by an AC source of EMF, the inductor is an air-core solenoid.

The switch is closed and after some time, the light bulb glows steadily.

An iron rod is inserted into the solenoid, thereby increasing the strength of the B-field inside the solenoid. As the rod is being inserted, what happens to the brightness of the light bulb?



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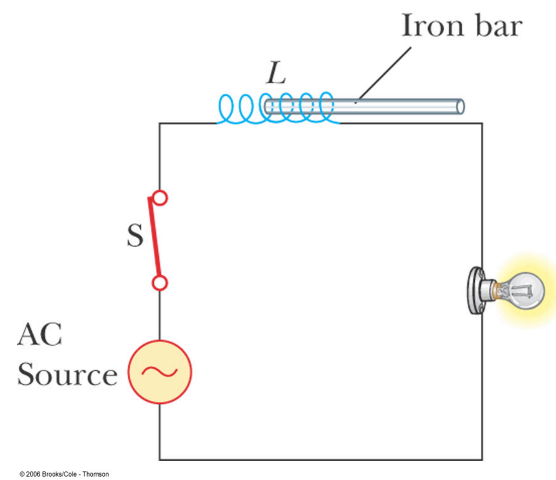

Answer: L is increasing. So $|\Delta V_L| = L(\Delta I/\Delta t)$ is increasing.

$\varepsilon = \Delta V_R + \Delta V_L$

ΔV_R is decreasing, the power dissipated in the lightbulb decreases, and its brightness **decreases.**