Ch. 20: Induced Voltages & Inductance

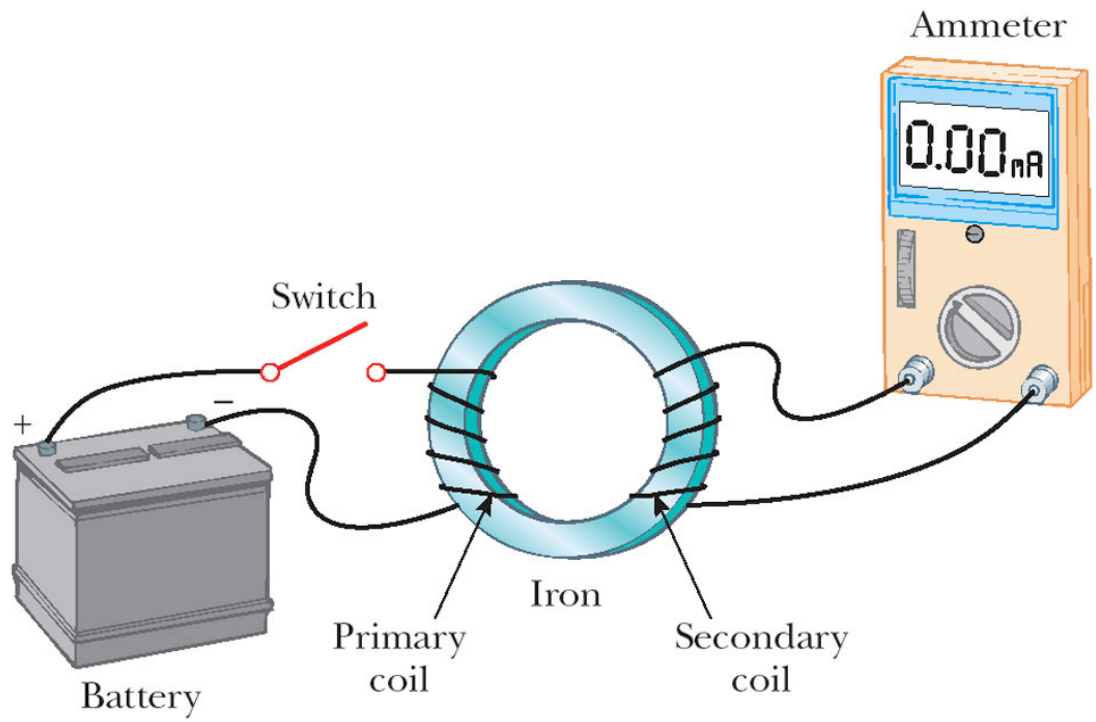
We've seen that electrical current can produce magnetic fields.

Can magnetic fields be used to produce electrical current?

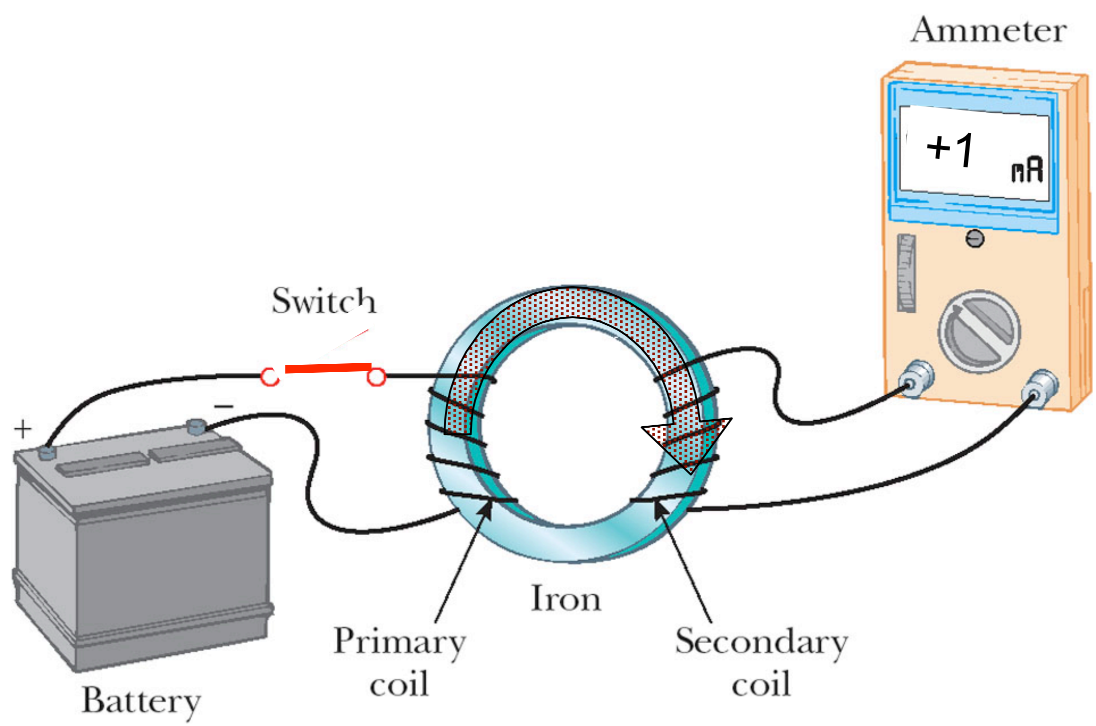
The answer, as discovered by Michael Faraday, is YES:

Applications include electrical generators, ground-fault interrupters, microphones

Faraday’s Experiments

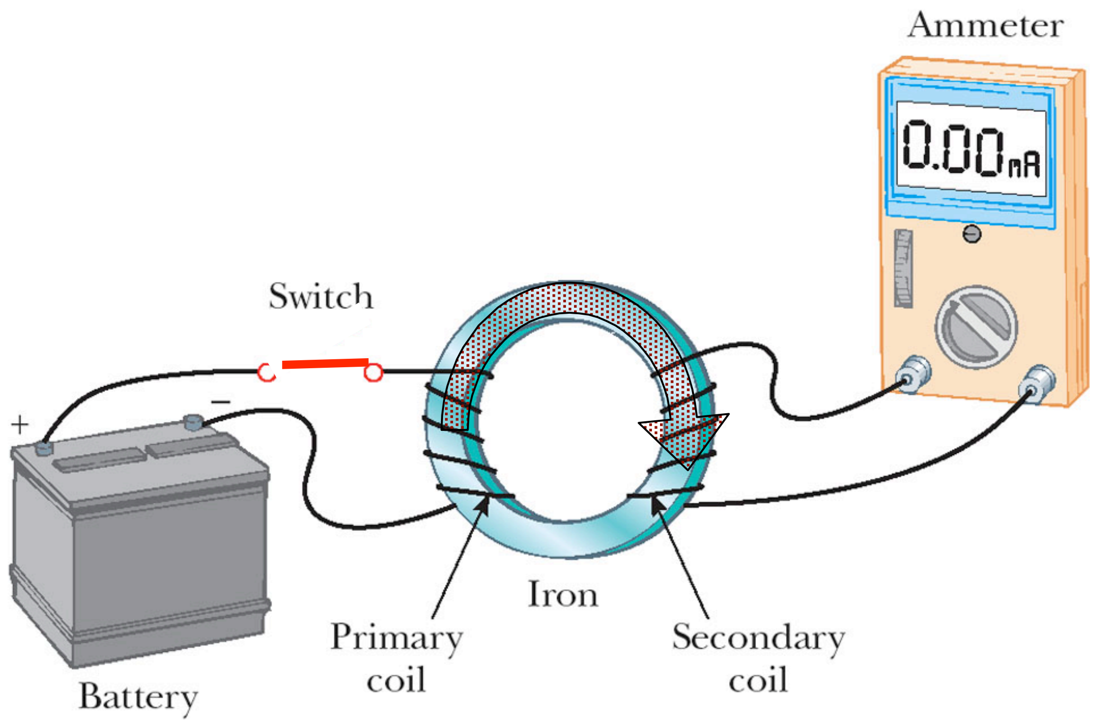


Faraday's Experiments



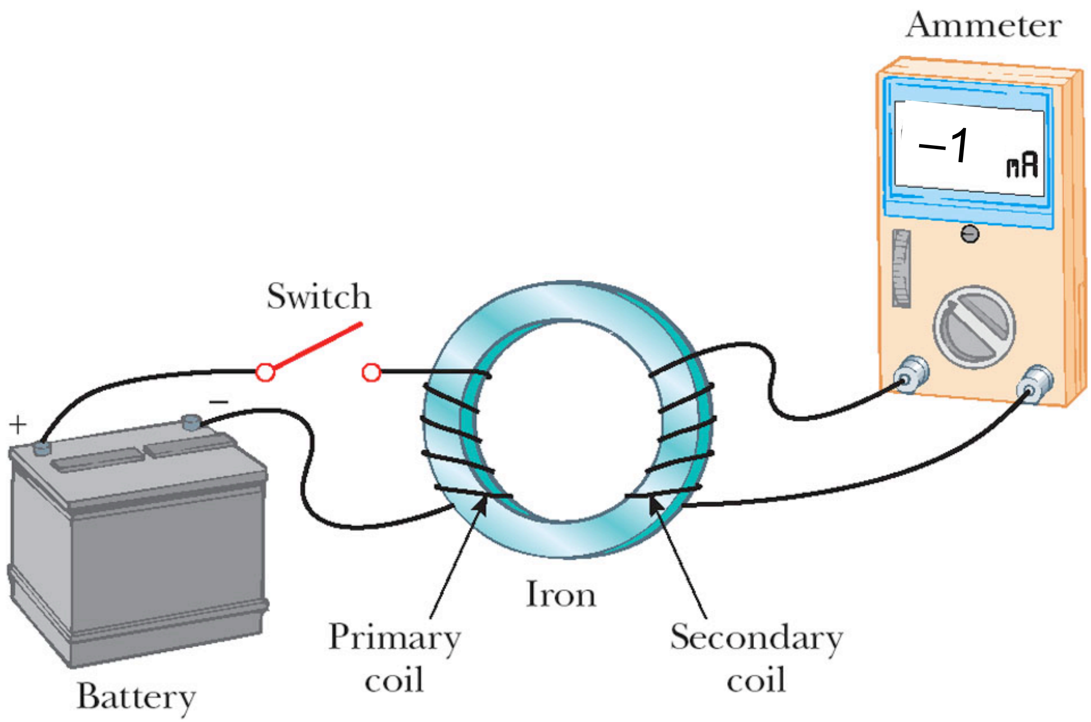
Close switch: immediately after switch closed, ammeter measures current in secondary circuit

Faraday's Experiments



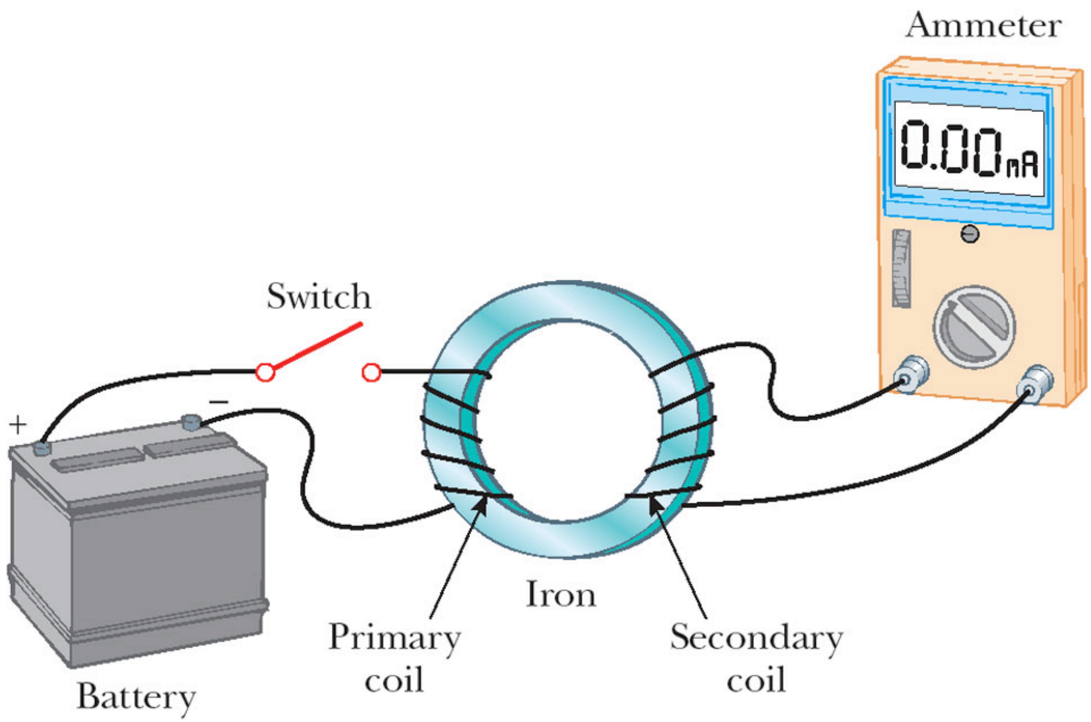
After the switch has been closed for a while, ammeter has returned to zero.

Faraday's Experiments



Open switch: immediately after opening switch, ammeter registers a current in the OPPOSITE direction

Faraday's Experiments



After the switch has been opened for a while, ammeter has returned to zero.

Conclusions from Faraday's experiment:

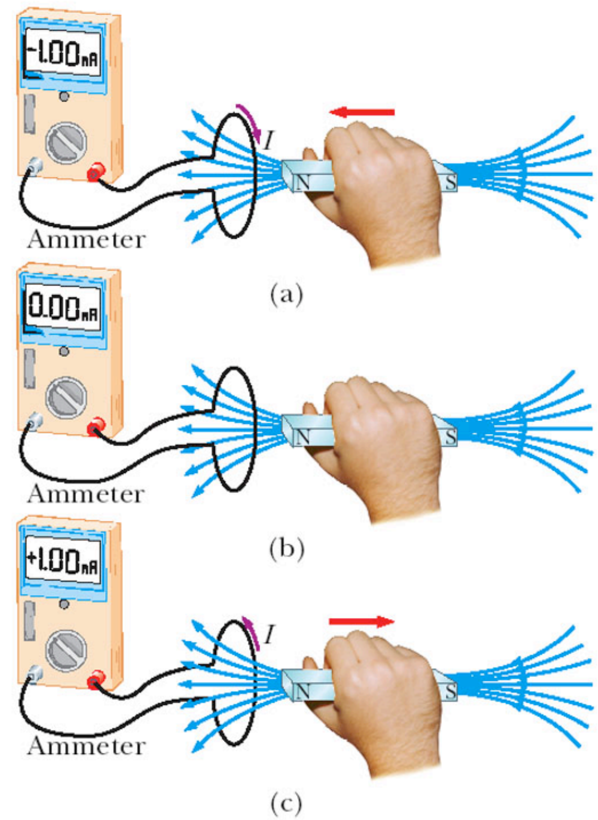
B-field going from OFF to ON-- induces a current in secondary circuit

B-field going from ON to OFF -- induces a current (opposite sign) in secondary circuit.

B-field level constant (zero or non-zero) -- no current induced!

Conclusion: The B-field itself does not induce any current -- **only a CHANGE in B-field.**

Electromagnetic Induction

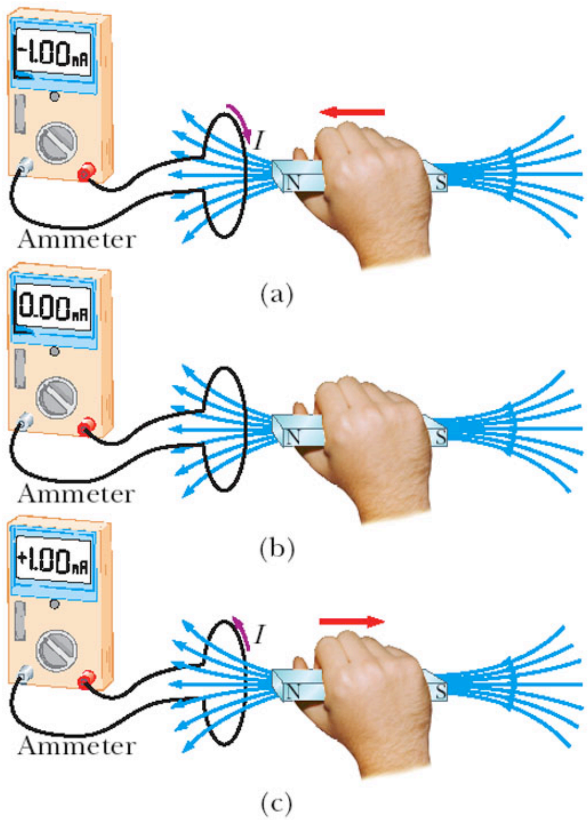


While a magnet is moving toward a loop of wire, the ammeter shows the presence of a current

While the magnet is held stationary, there is no current

While the magnet is moving away from the loop, the ammeter shows a current in the opposite direction

Electromagnetic Induction



If the magnet is held stationary and the LOOP is moving, you get the same effect.

A current is induced whenever there exists RELATIVE motion between the magnet & loop.

Direction of current depends on direction of motion.

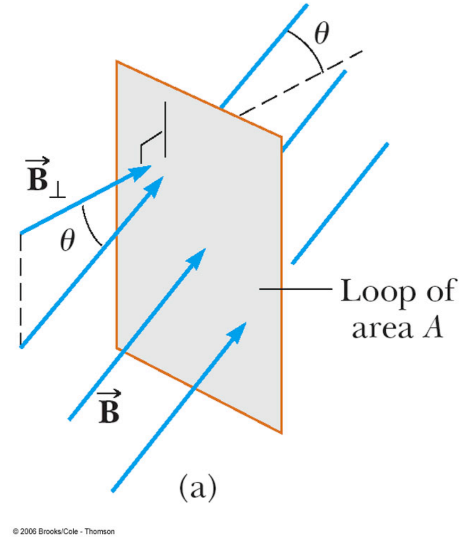
Magnetic Flux

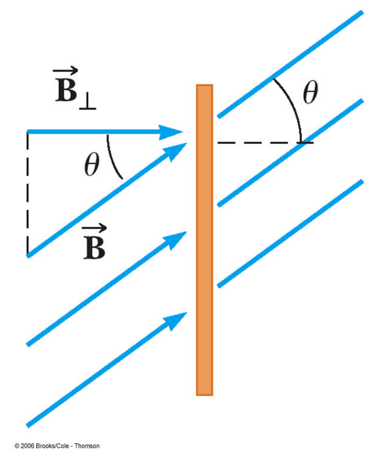
Assume B-field is uniform.
Area of loop = A.

Magn. Flux Φ_B through an area A:

$\Phi_B = B_\perp A = BA\cos\theta$

SI unit: Weber (Wb) = $T \times m^2$





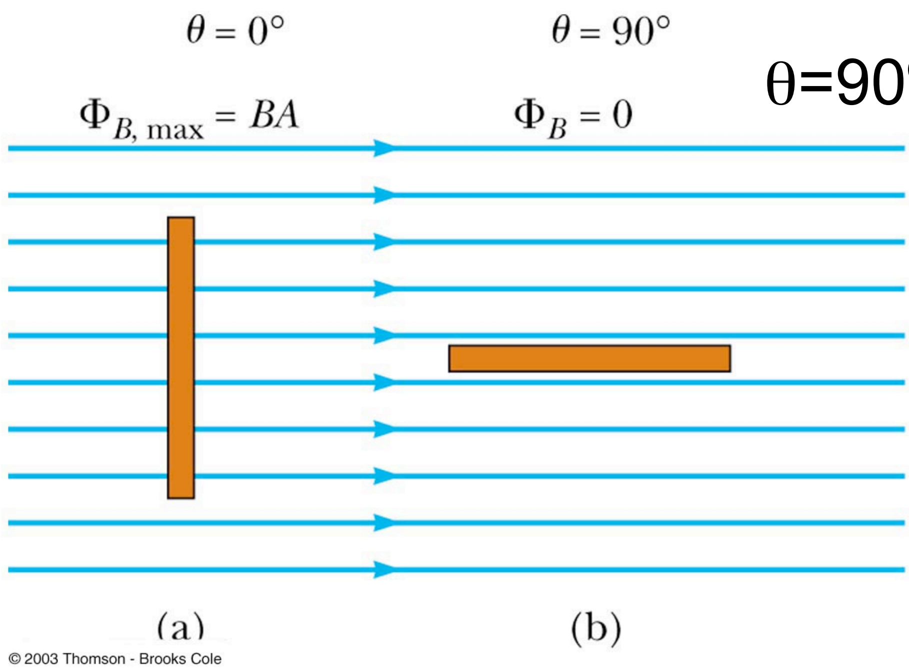
Φ_B is proportional to the total number of lines passing through the loop

Magnetic Flux

Edge view of loop in uniform B-field:

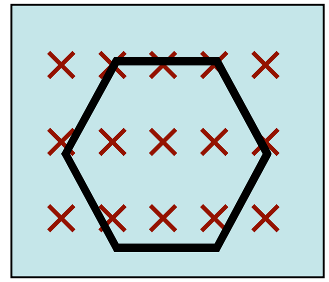
θ=0º: Φ_B is maximized: Φ_B = BA

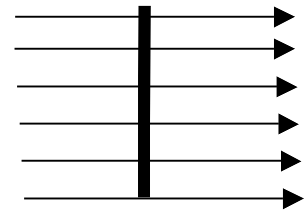
θ=90º: Φ_B is zero



Change B-field direction so θ=180º: Φ_B = –BA

Φ_B & $\Delta\Phi_B$





Example: A hexagon-shaped loop with area 0.5 m^2 is placed in a uniform B-field of 2 T such that the loop's normal is parallel to B. Calculate Φ_B.

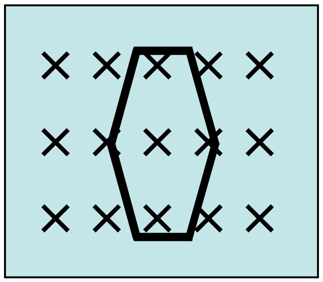
$\Phi_{B1} = BA\cos\theta = BA\cos 0^\circ = (2T)(0.5\ m^2) = 1.0$ Wb

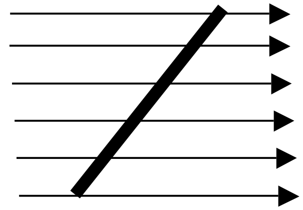
Suppose the B-field strength is halved. Calculate Φ_B now. Calculate $\Delta\Phi_B$.

$\Phi_{B2} = \Phi_{B1}/\ 2 = 0.5$ Wb

$\Delta\Phi_B = \Phi_{B2} - \Phi_{B1} = (1.0–0.5)\text{Wb} = 0.5$ Wb

Φ_B & $\Delta\Phi_B$





Example: Go back to B=2.0 T. Suppose the loop is rotated 45° as shown. Calculate Φ_B & $\Delta\Phi_B$.

$\Phi_{B2} = BA\cos\theta = BA\cos 45^\circ = (2T)(0.5\ m^2)(0.707) = 0.707\ Wb$

$\Delta\Phi_B = \Phi_{B2} - \Phi_{B1} = (1.0-0.707)Wb = 0.293\ Wb$

No magnetic monopoles

If we were to apply an analog of Gauss' law to MAGNETIC field lines for a Gaussian surface in any region of space, we'd find:

No magnetic monopoles means there's no "magnetic charge" to enclose

All B-field lines which enter the surface, must also leave it. So net Φ_B is zero.

FARADAY'S LAW OF MAGNETIC INDUCTION

The instantaneous EMF induced in a circuit equals the time rate of change of MAGNETIC FLUX through the circuit

If a circuit contains N tightly wound loops and the magn. flux changes by $\Delta\Phi_B$ during a time interval Δt, the average EMF induced is given by Faraday's Law:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

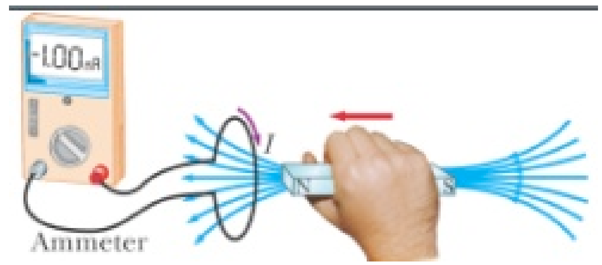
FARADAY'S LAW OF MAGNETIC INDUCTION

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

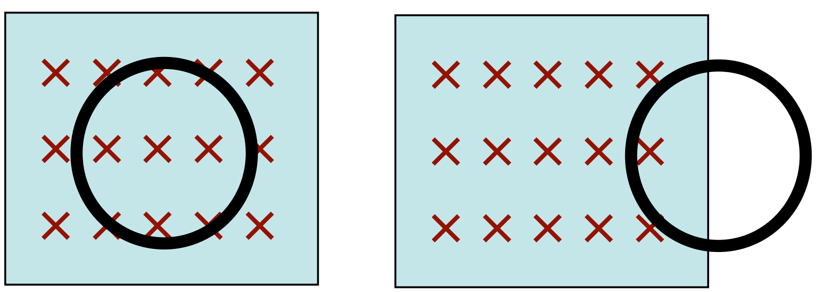
$\Phi_B = BA\cos\theta$

So EMF can be induced by changing any of B, A or θ.

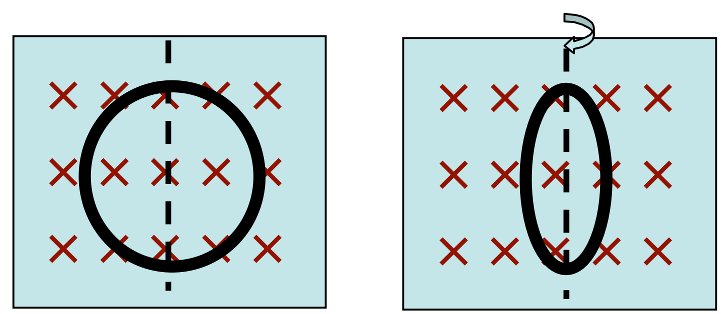
B: increase/decrease B-field strength



A: change area of loop inside B-field



θ: rotating coil in the field.



A singular circular coil with a radius of 20 cm is in a B-field of 0.2 T with the plane of the coil perpendicular to the field lines. If the coil is pulled out of the field in 0.30 s find the magnitude of the average emf induced during this interval.

$\varepsilon = –N\ \Delta\Phi_B/\Delta t = –N\ (\Phi_{B2} – \Phi_{B1})\ /\ \Delta t$

$\Phi_{B1} = BA = (0.2T)(\pi(0.2m)^2) = 0.025\ Wb$

$\Phi_{B2} = 0$

$N = 1$

$\varepsilon =\ 1(0.025Wb)/(0.3s) =$ **0.084V.**

Another example:

A 25-turn circular coil of wire with a diameter of 1.0 m is placed with its axis aligned with the Earth's B-field, which has a magnitude of 0.5×10^{-4} T = 0.5G. During a time interval of 0.2 s, it's flipped 180º. What's the magnitude of the average EMF generated during this time?

$\Phi_{B1} = BA$

$\Phi_{B2} = -BA$

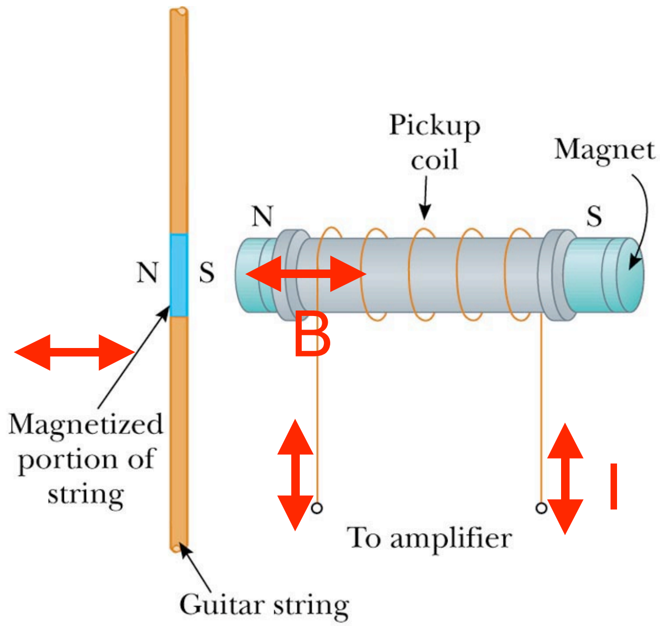
$\Delta\Phi_B = \Phi_{B2} - \Phi_{B1} = -2BA$

$\varepsilon = -N\,\Delta\Phi_B/\Delta t = -25(2BA)/\Delta t$

$\varepsilon = -25(2)(0.5\times10^{-4}\ \text{T})(\pi(0.5\text{m})^2)\ /\ 0.2\ \text{s}$

$\boldsymbol{\varepsilon = 9.8\times10^{-3}}$ **V.**

Application: Electric Guitars

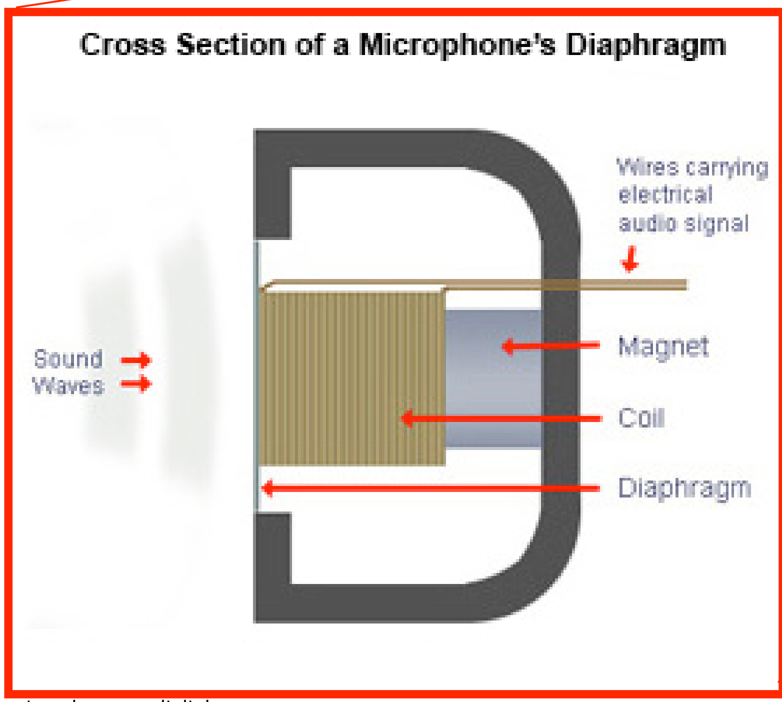


(a)

changing magnetic flux induces voltage in pickup coil

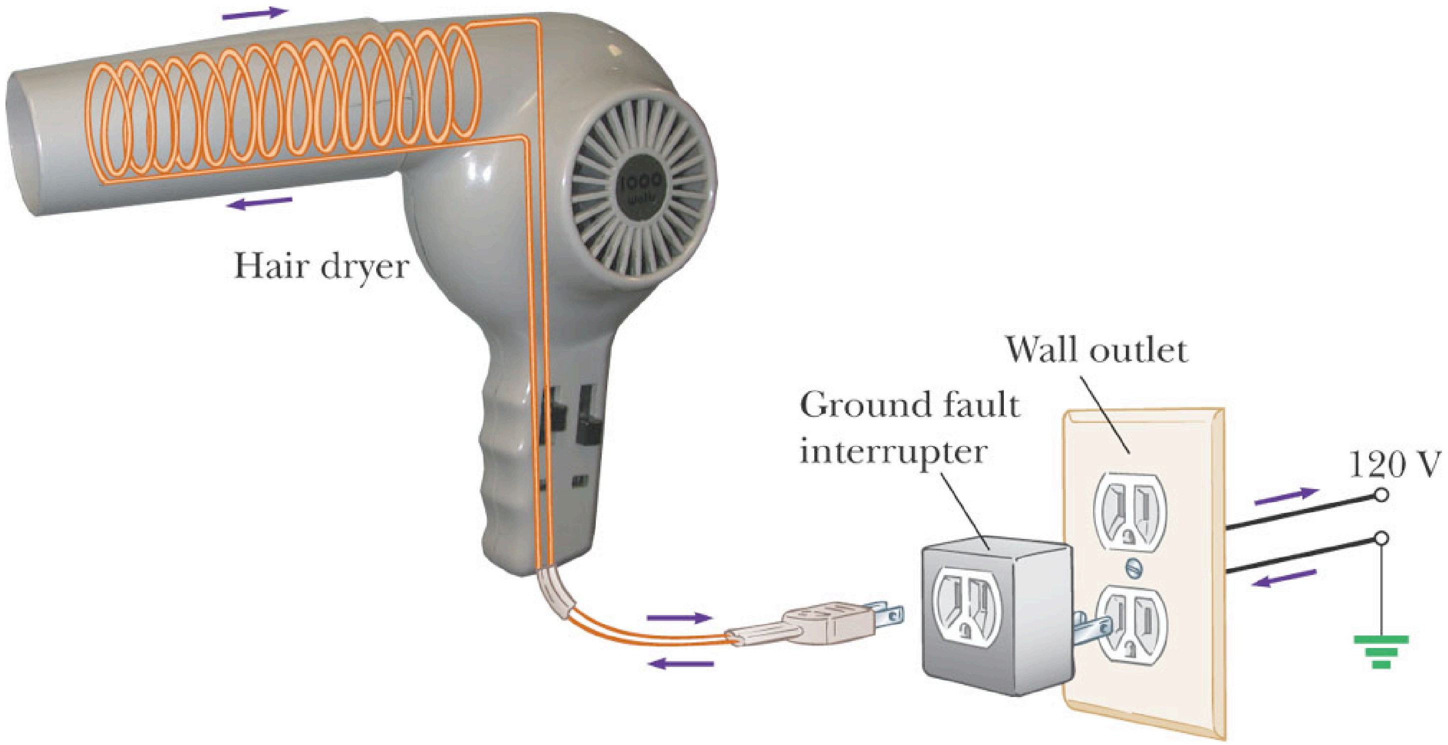
Microphones / acoustic pick-ups

(me in 1998, BTW)



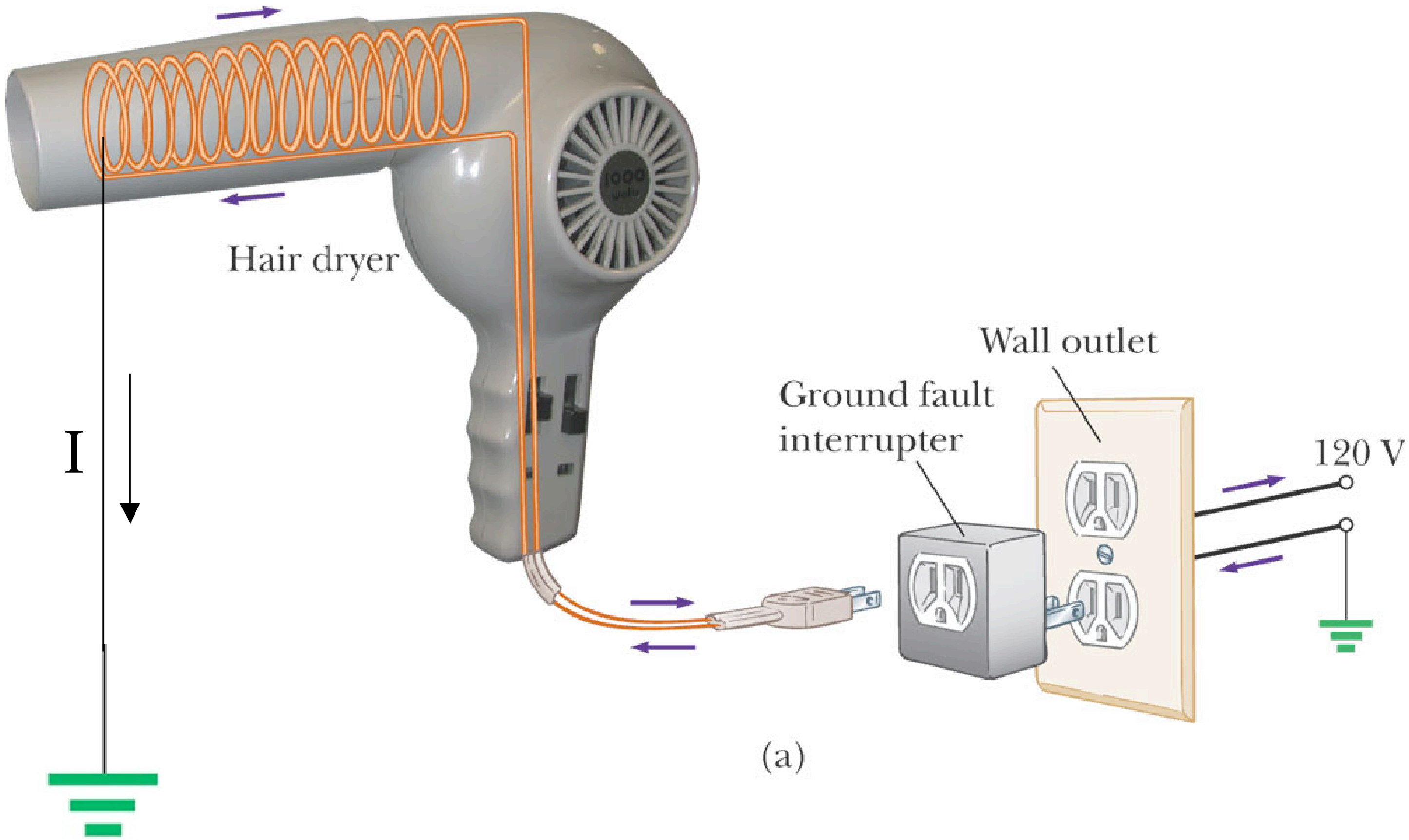
microphones.audiolinks.com

Ground Fault Interrupters



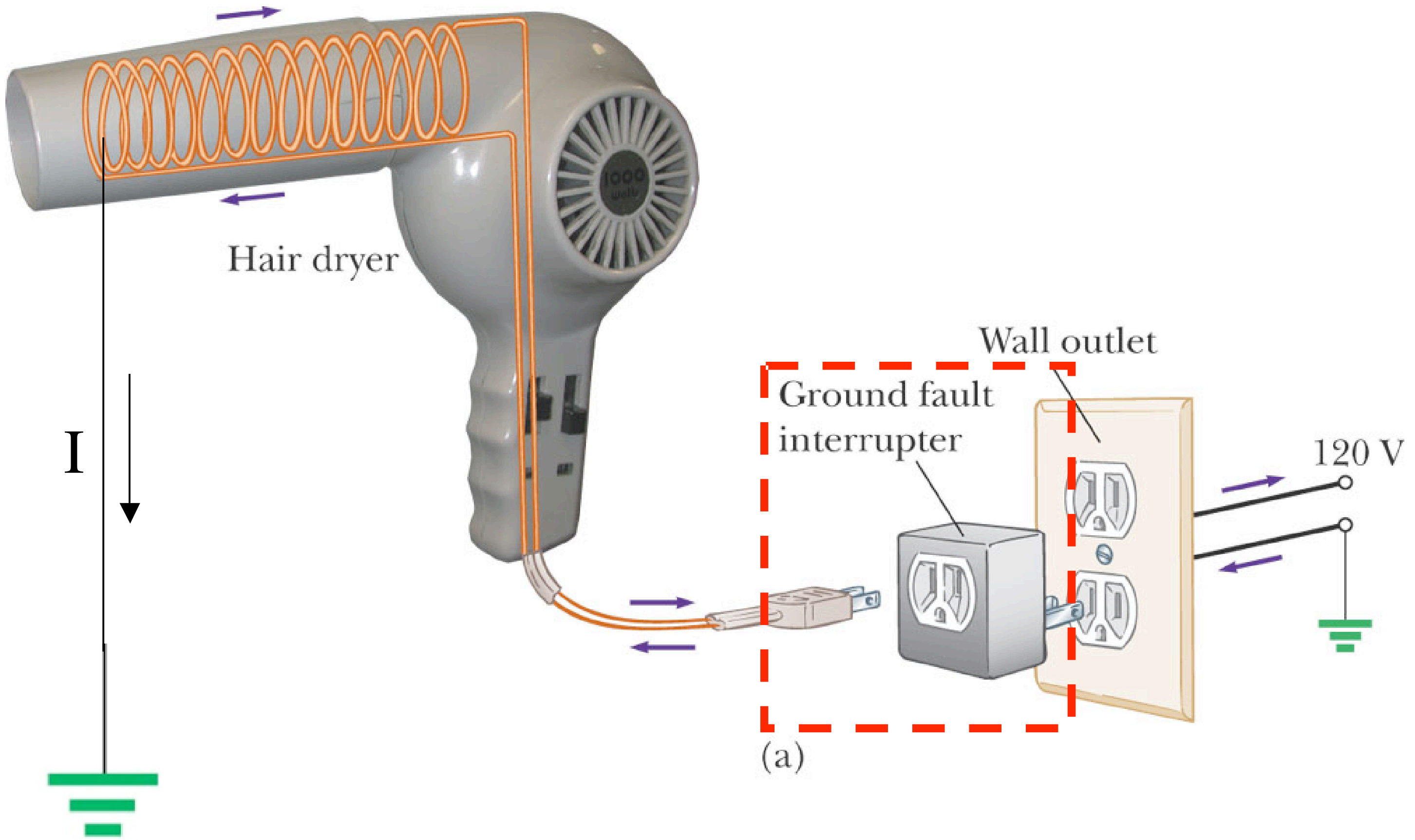
(a)

Ground Fault Interrupters



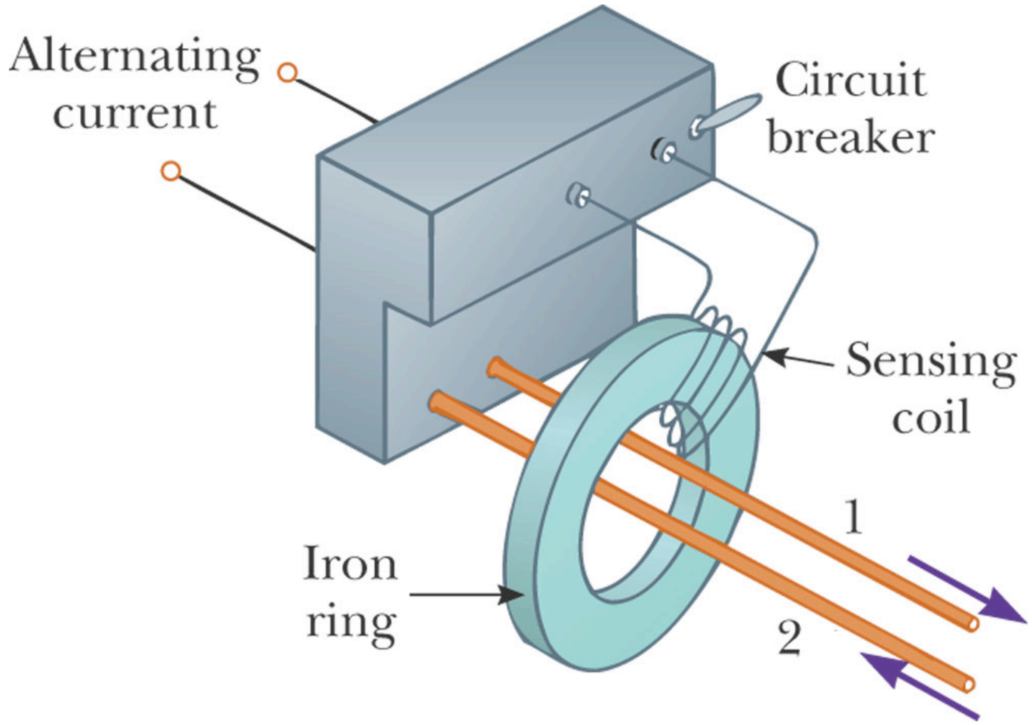
If a connection to ground is accidently made…

Ground Fault Interrupters



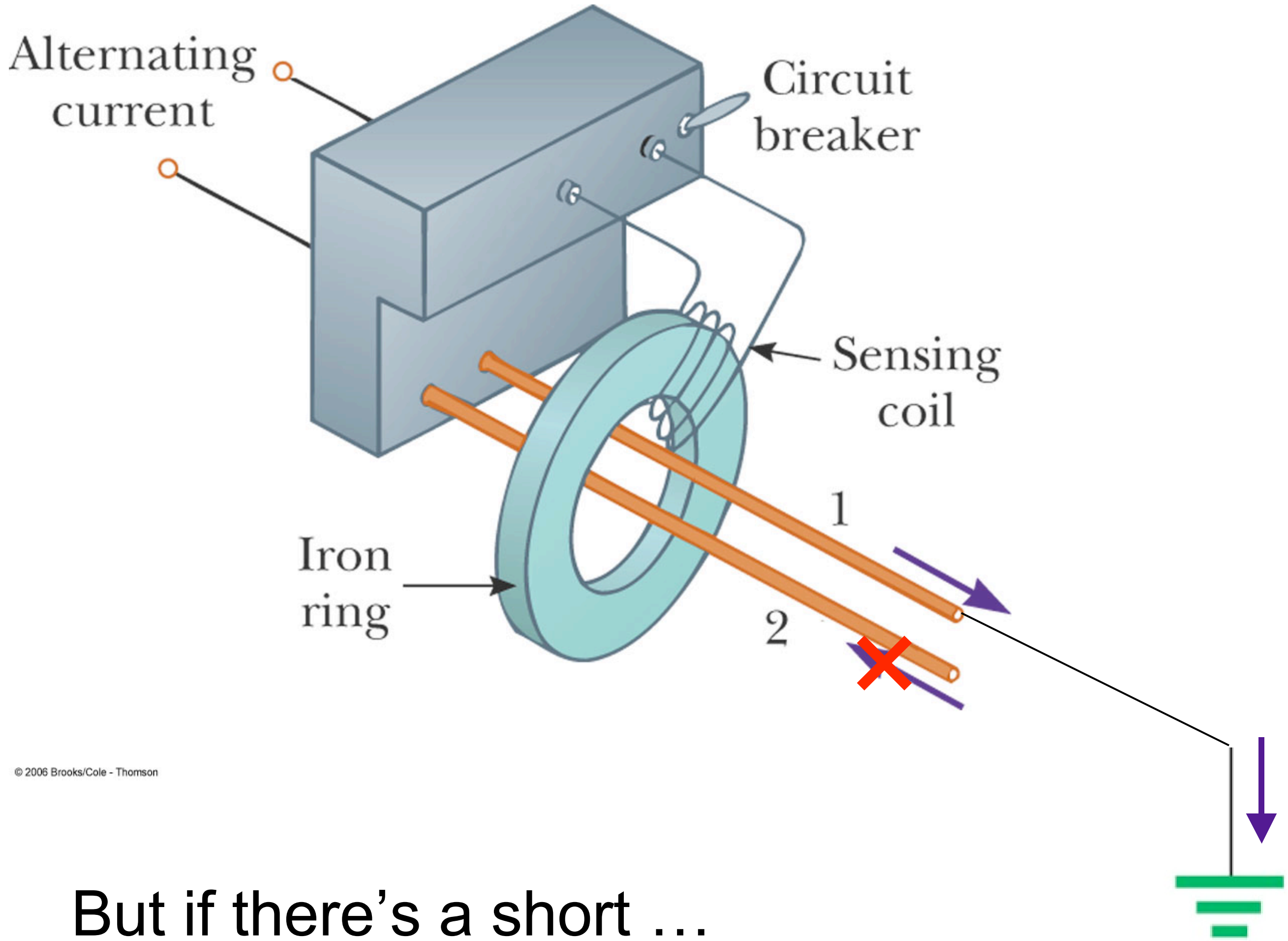
… the ground fault interrupter shuts off the device

Ground Fault Interrupters



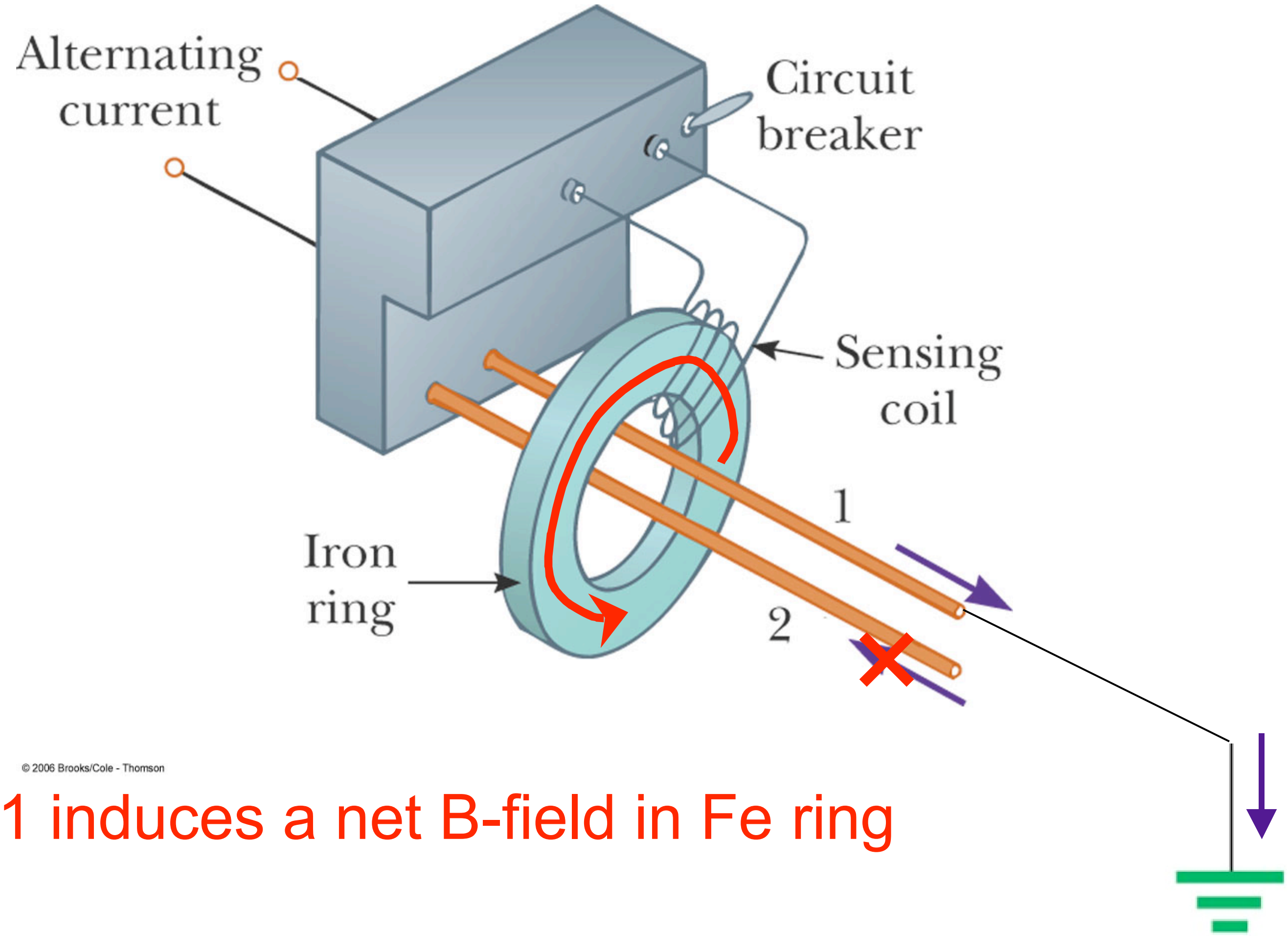
Under normal operation, I_1 & I_2 are equal and opposite. Net B-field through Fe ring is zero.

Ground Fault Interrupters



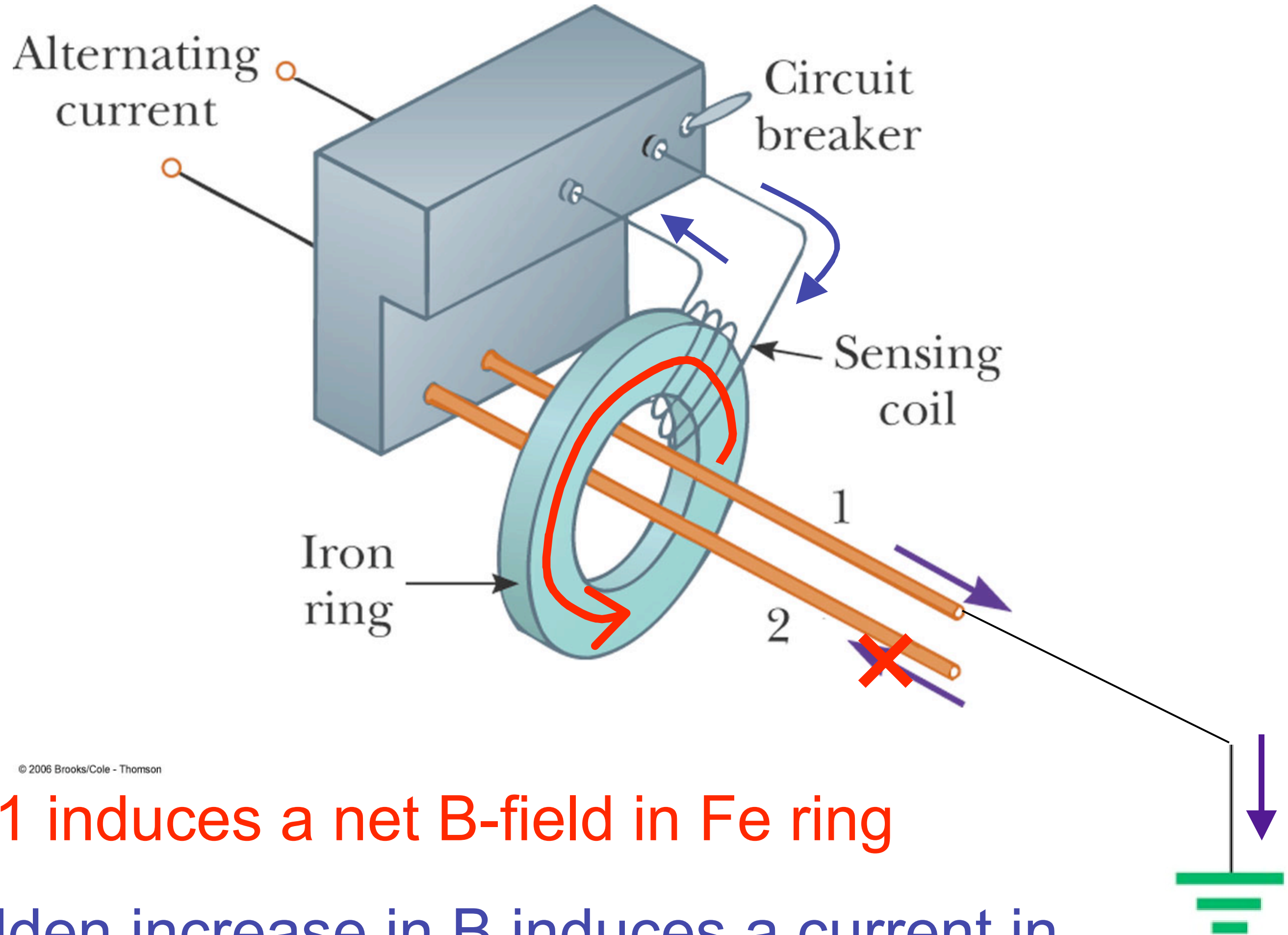
But if there's a short …

Ground Fault Interrupters



… wire 1 induces a net B-field in Fe ring

Ground Fault Interrupters



… wire 1 induces a net B-field in Fe ring

The sudden increase in B induces a current in sensing coil, which triggers the circuit breaker

Lenz's Law:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

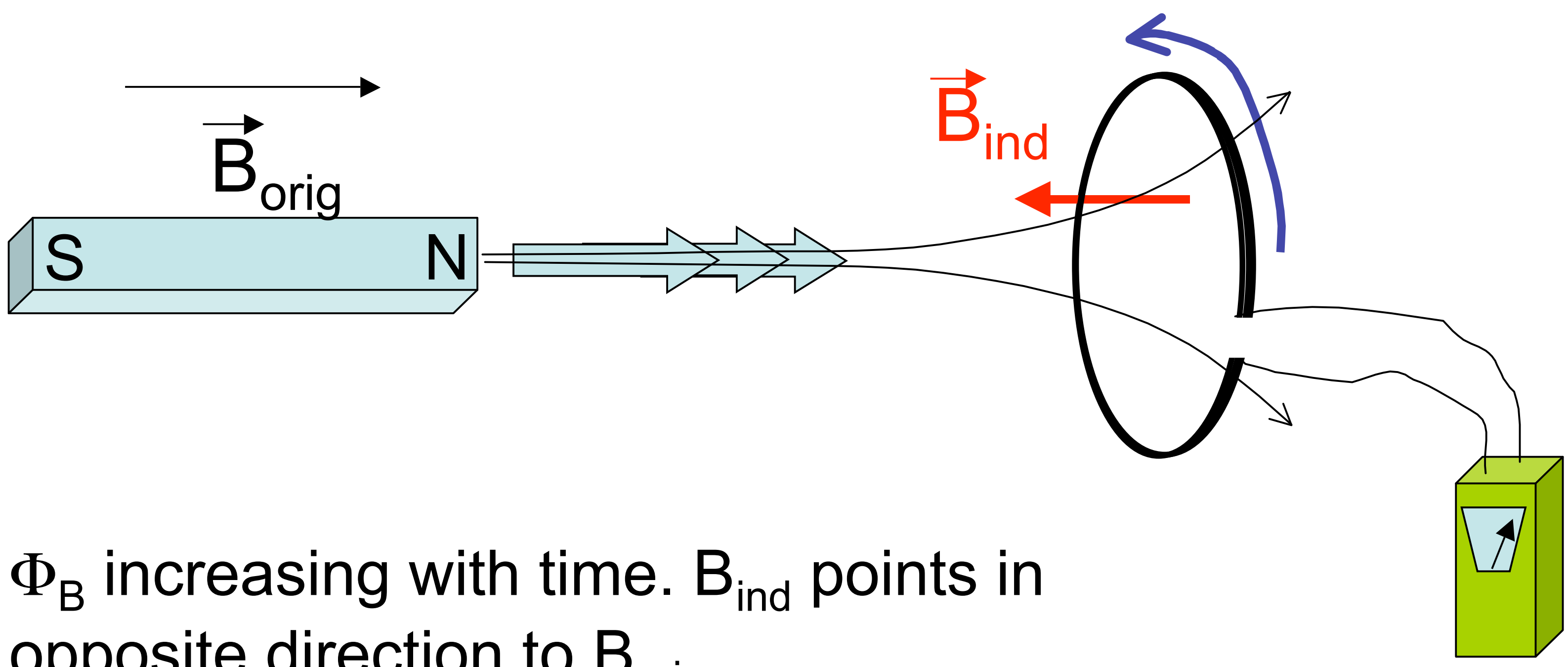
Determines the direction the induced current flows.

<u>The current caused by the induce EMF travels in the direction that creates a magnetic field whose flux OPPOSES the CHANGE in the original magnetic flux.</u>

i.e., the induced current will flow to try to maintain the original magnetic flux thru the loop.

Lenz's Law:

Induced B-field B_{ind} "shores up" original Φ_B if original Φ_B is shrinking with time, or opposes the original Φ_B if original Φ_B is growing with time.



Φ_B increasing with time. B_{ind} points in opposite direction to B_{orig}.

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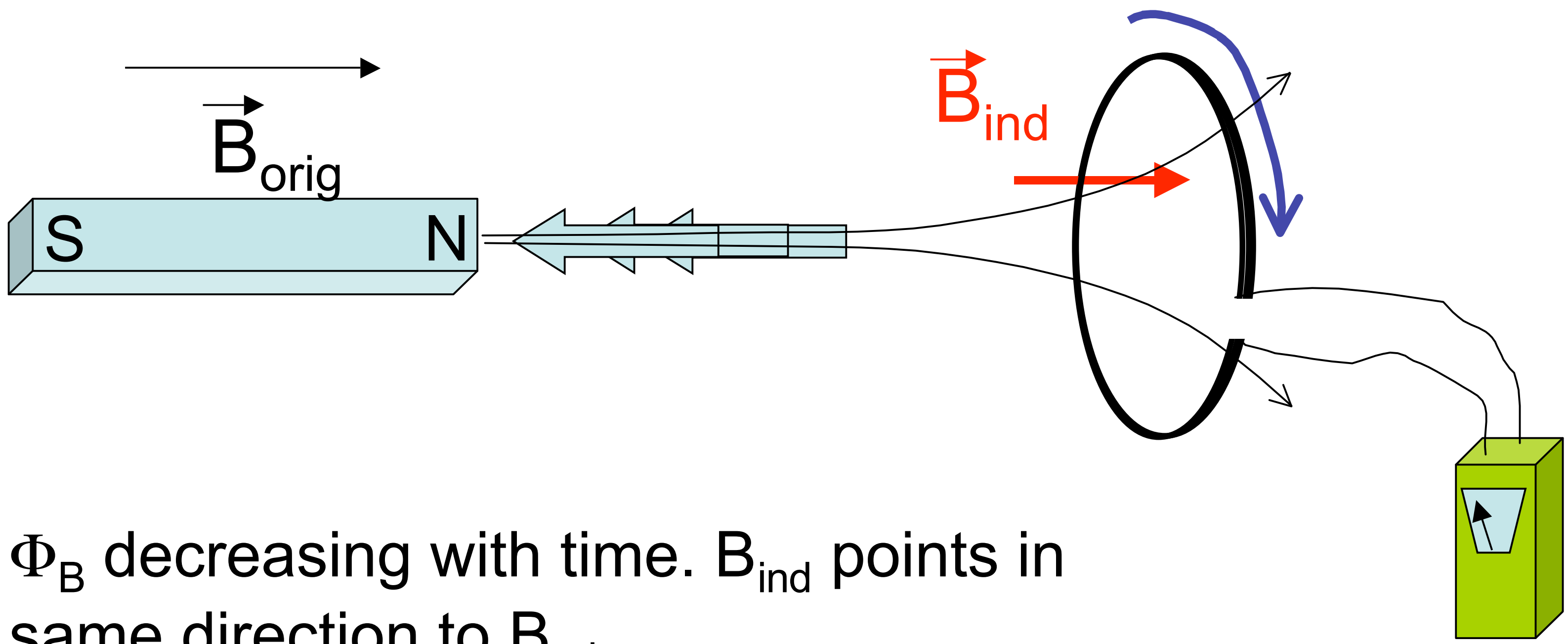

Φ_B decreasing with time. B_{ind} points in same direction to B_{orig}.

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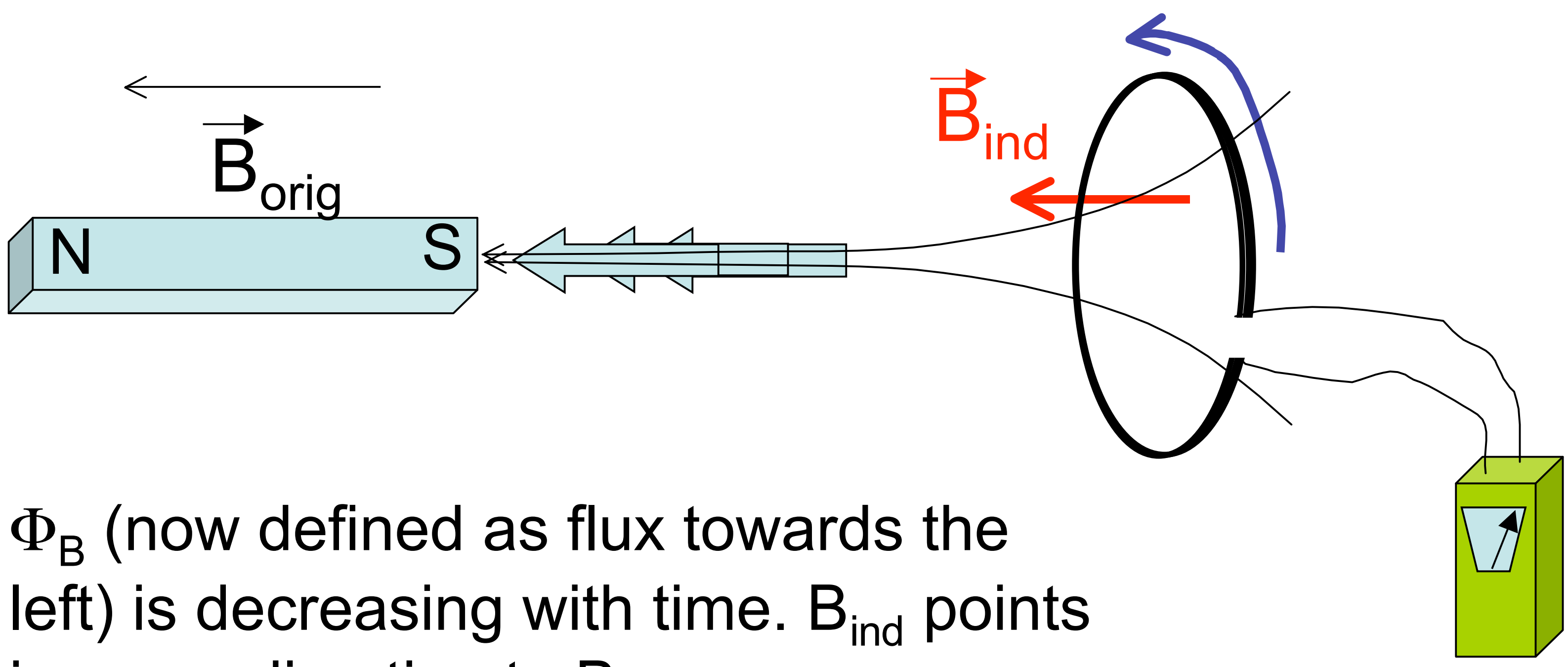

Φ_B (now defined as flux towards the left) is decreasing with time. B_{ind} points in same direction to B_{orig}.

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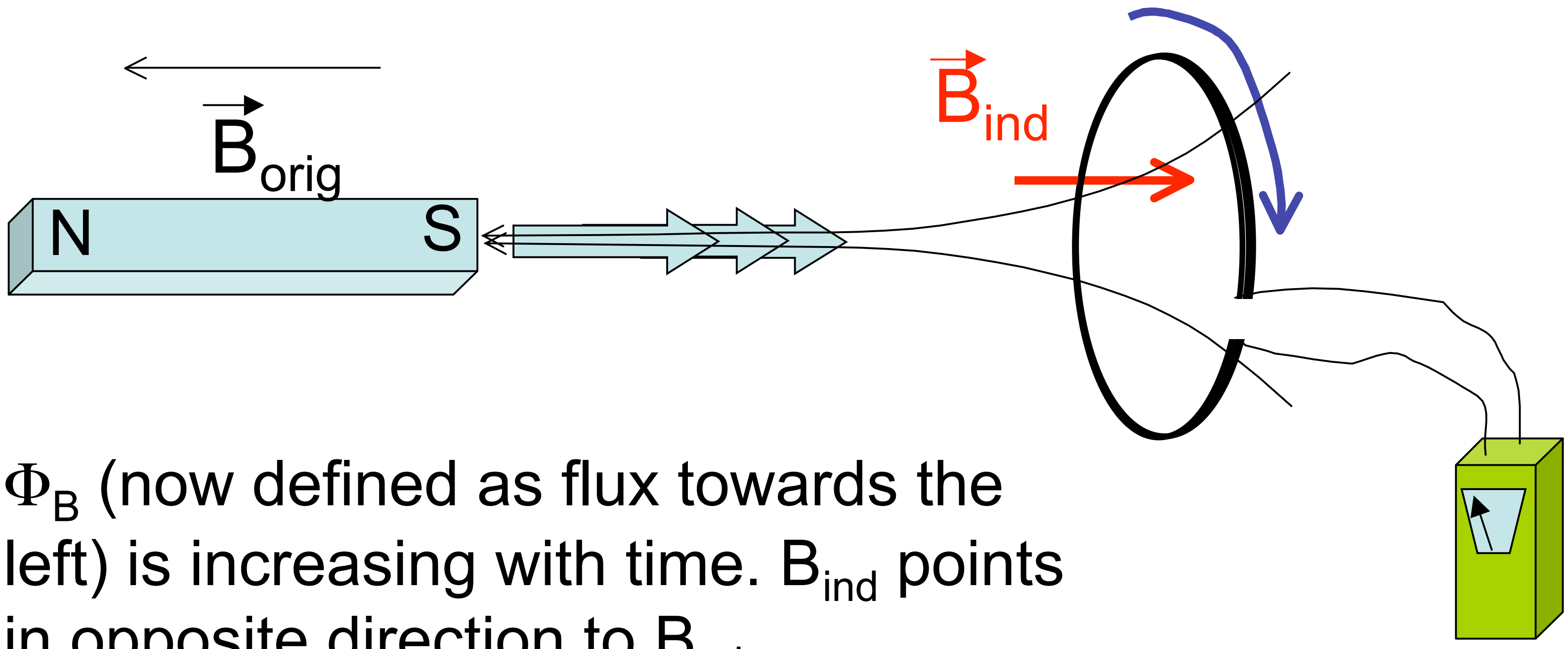

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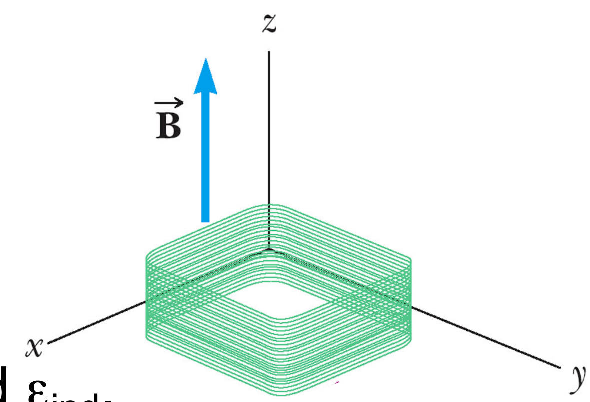
Example 20.2

Given: a square coil with 25 turns. Each side is 1.8cm. The total resistance of the coil 0.35Ω. A uniform B-field is applied in the +z direction.



a) If B changes from 0 T to 0.5 T in 0.8 s, find ε_{ind}.

First, find $\Phi_{Binitial}$ & Φ_{Bfinal}:

$\Phi_{Binitial} = B_{init} A \cos 0^o = 0$

$\Phi_{Bfinal} = B_{final} A \cos 0^o = 0.5T(0.018m)^2 = 1.62\times10^{-4}$ Wb

$\Delta\Phi_B = \Phi_{bfinal} - \Phi_{Binitial} = 1.62\times10^{-4}$ Wb

Then use Faraday's Law:

$\varepsilon_{ind} = -N\,\Delta\Phi_B/\Delta t = -25(1.62\times10^{-4}\text{ Wb}) / 0.8\text{s} = -5.06\times10^{-3}$ V

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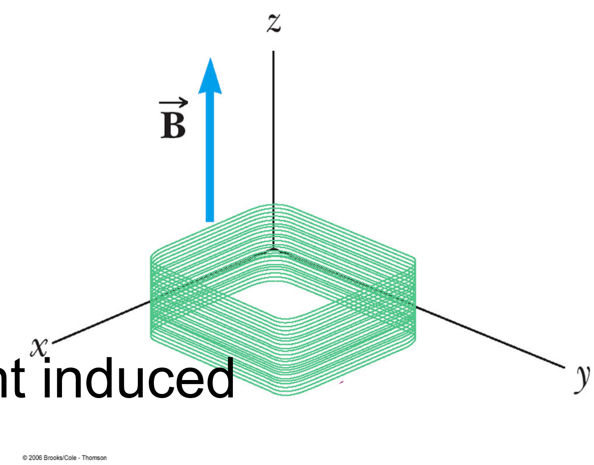

b) Find the magnitude & direction of the current induced while the field is changing.

Use Ohm's Law to find the magnitude of I

$I = \Delta V/R = 5.06\times10^{-3}\ V\ /\ 0.35\Omega = 1.45\times10^{-2}\ A$

The direction is found from Lenz's Law: $\vec{B}$ is increasing up through the loop. Φ_B is also positive and increasing. The induced B-field will thus point downward. The induced current will circulate clockwise (right-hand rule with thumb pointing downward).

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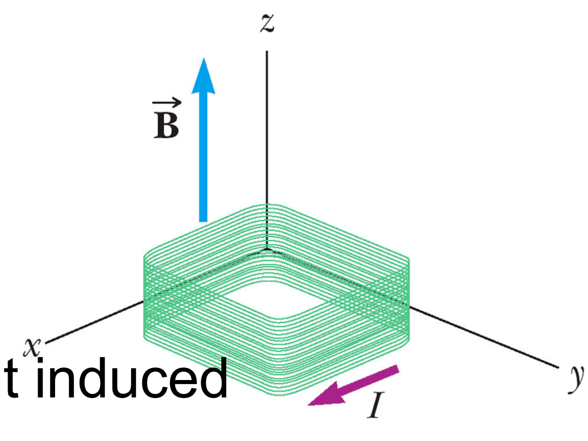

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