19.4: Force on a Current-carrying Conductor

Total force = Force on each charge $\times$ # of charges in the wire

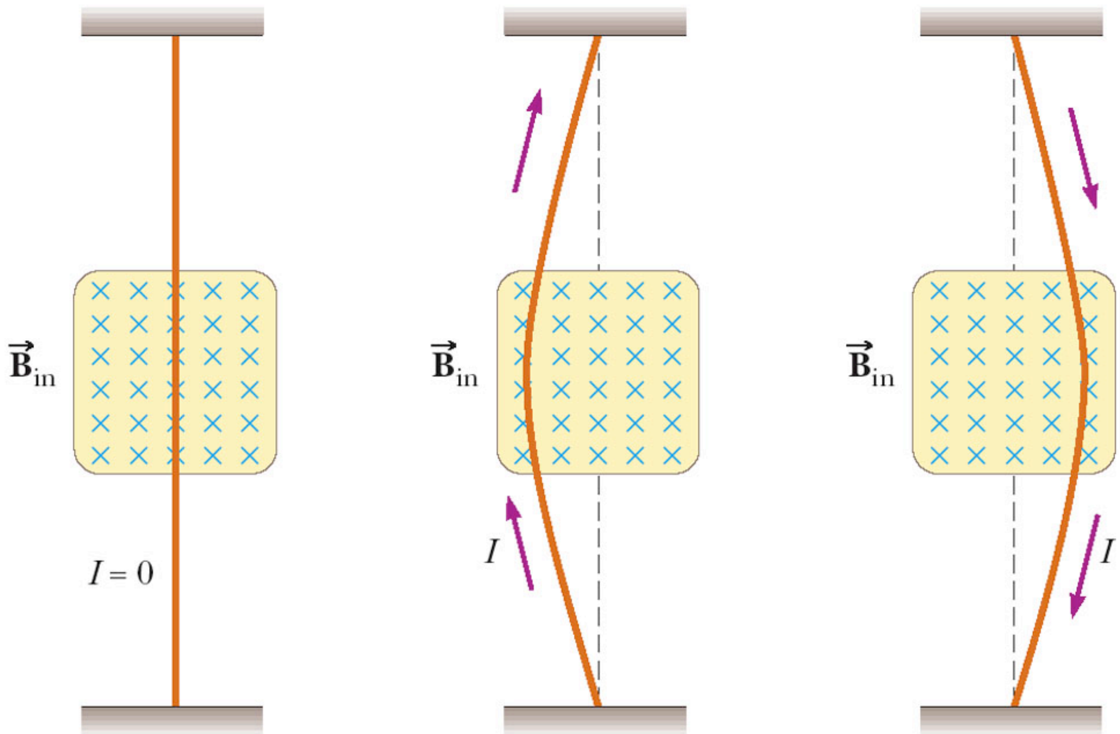
$F_B = (q\, v_{dr}\, B \sin\theta)\,(n\, A\, \ell\,)$ $\quad$ ($A \times \ell$ = vol.; n = charges/vol)

But recall that $I = \Delta Q/\Delta T$
and $\Delta Q/\Delta T$ = (q of each charge) $\times$ (# of charges per second)

# of charges per second = $A\, n\, v_{dr}$
$(m^2)\,(m^{-3})\,(m/s)$

So $I = n\, A\, v_{dr}\, q$

$F_B = B\, I\, \ell \sin(\theta)$ $\quad$ or $\quad$ $F_B / \ell = B\, I \sin(\theta)$



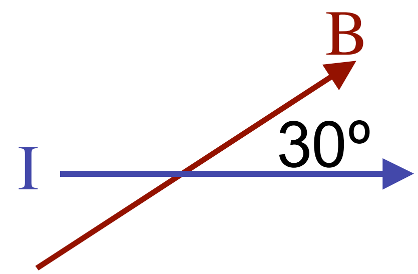
Reminder: Direction of deflection is given by right hand rule:

I = thumb

B = fingers

F_B = out of your palm

Example: Your magnet produces a B-field with strength 0.01 T in the plane of the page as shown. A wire carries 1 Amp from left to right, such that $\theta = 30^o$. What's the force per unit length (and its direction)?



$F_B / \ell = B\, I \sin 30^o = 0.01T \times 1A \times 0.5 = 0.005$ N/m (out of page)

So it's not much.

Suppose the current is 6 Amps and the B-field is 3 T.
$F_B / \ell = 9$ N/m (which is slightly more substantial)

Consider a power transmission line carrying 100 Amps from W to E. What's the total force on 100m of wire due to the Earth's magnetic field (assume a North component only)?
In which direction is the wire deflected (assume charge carriers are e^{-}'s)?
Reminder: the Earth's B-field is 0.5 Gauss = 0.5×10^{-4} T

$F_B = B\, I\, \ell \sin\theta$
$= (0.5 \times 10^{-4}\ \mathrm{T})(100\mathrm{A})(100\mathrm{m}) = 0.5\ \mathrm{N}.$

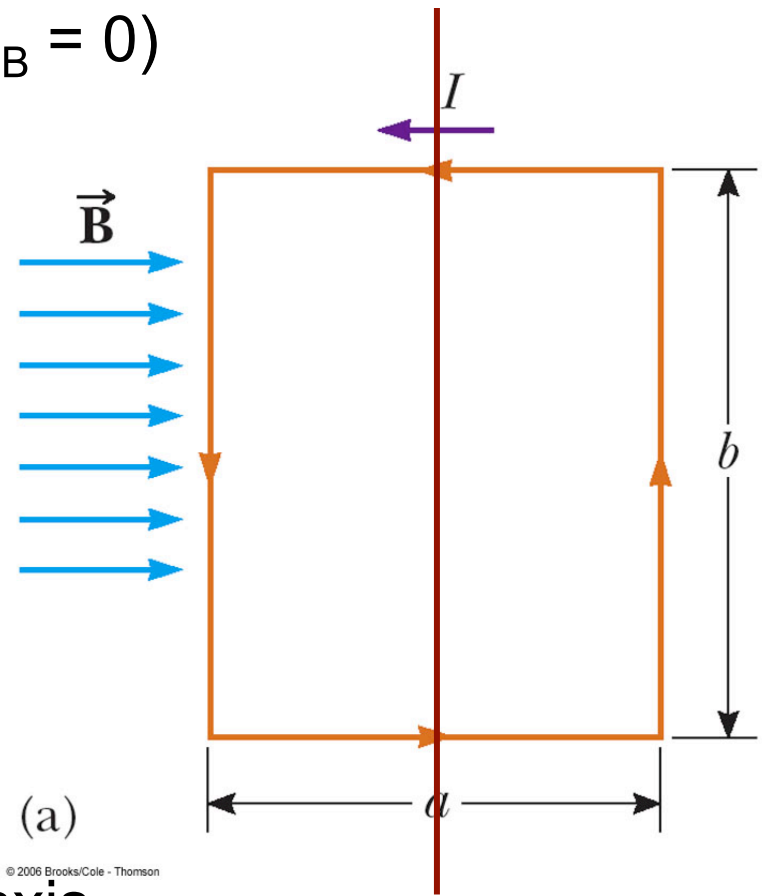
So, again, it's not much.

Direction of deflection = downward

Force on a square loop of current in a uniform B-field.

$F_{top} = 0$ ($\theta = 180°$; $\sin\theta = 0$; so $F_B = 0$)
$F_{bottom} = 0$

$F_{left} = I\,b\,B$ (out of page)
$F_{right} = I\,b\,B$ (into page)

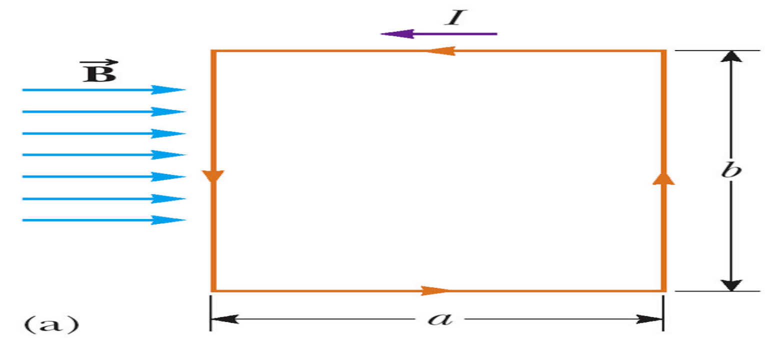


(a)

Assume loop is on a frictionless axis

What's the TORQUE
on the current loop?

Fig. 19.15b in text is the view along the axis, from the bottom towards the top.

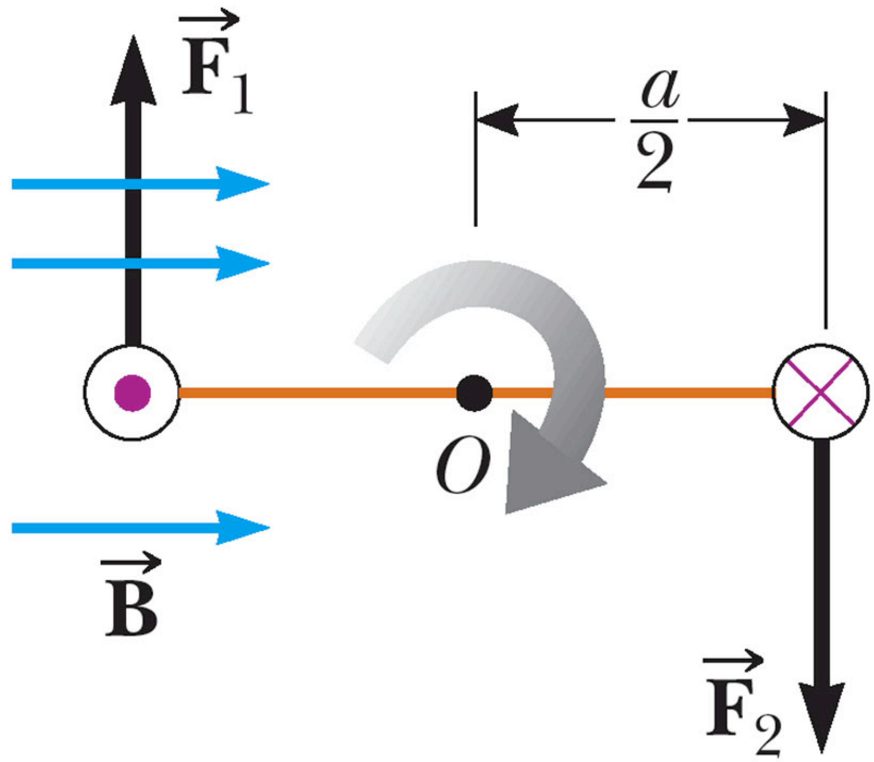


Reminder: torque = $\vec{F} \times \vec{r}$ = F r sinθ

$\tau = F_{left}\ a/2 + F_{right}\ a/2 =$
(B b I + B b I) a/2 = B I A

A = area; θ=90º here

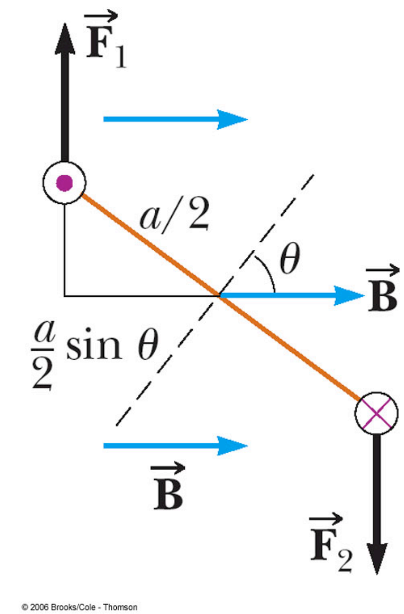
Note direction of torque: clockwise



For 1 loop:

$\tau = BIA \sin\theta$

$\tau_{max} = BIA$



For N turns: Total current = NI

$\tau = BIAN \sin\theta$

Magnetic Moment $\vec{\mu}$ = IAN

$\vec{\mu}$ always points perp. to the plane of the loops (points along the normal)

$\tau = \mu B \sin\theta$

A coil consisting of 100 turns, each carrying 3A of current and having an area 0.2 m^2, is oriented such that its normal makes a angle of 45° with a B-field of 0.5T. Find the total torque on the coil. What's the direction of rotation?

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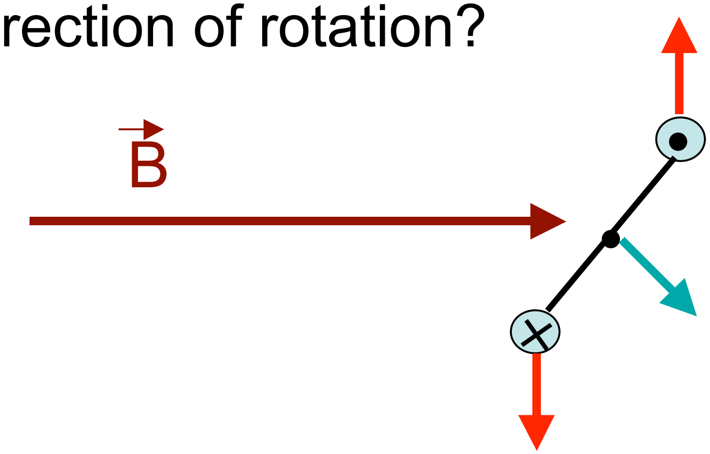

τ = BIANsinθ = (0.5T)(3A)(100)(0.2m^2)sin45º = 21.2 Nm

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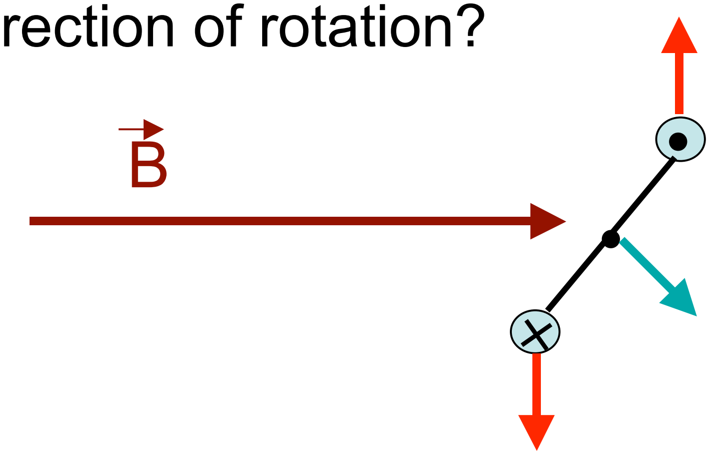
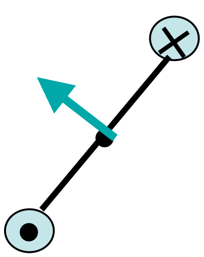


τ = BIANsinθ = (0.5T)(3A)(100)(0.2m^2)sin45º = 21.2 Nm

What would happen if the current were flowing in the opposite direction?



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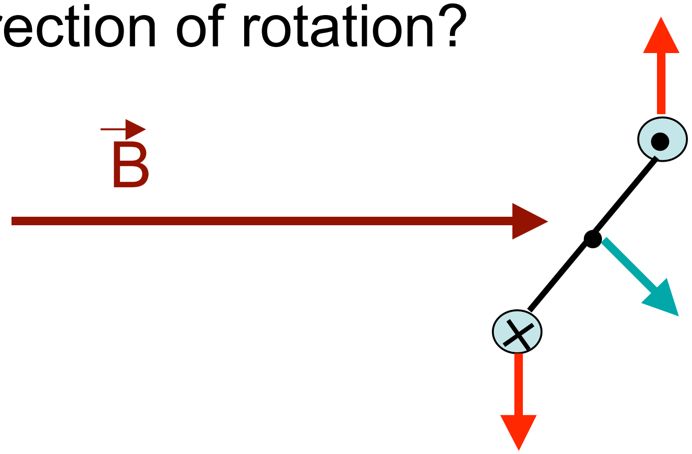
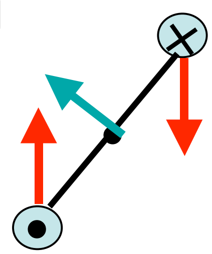
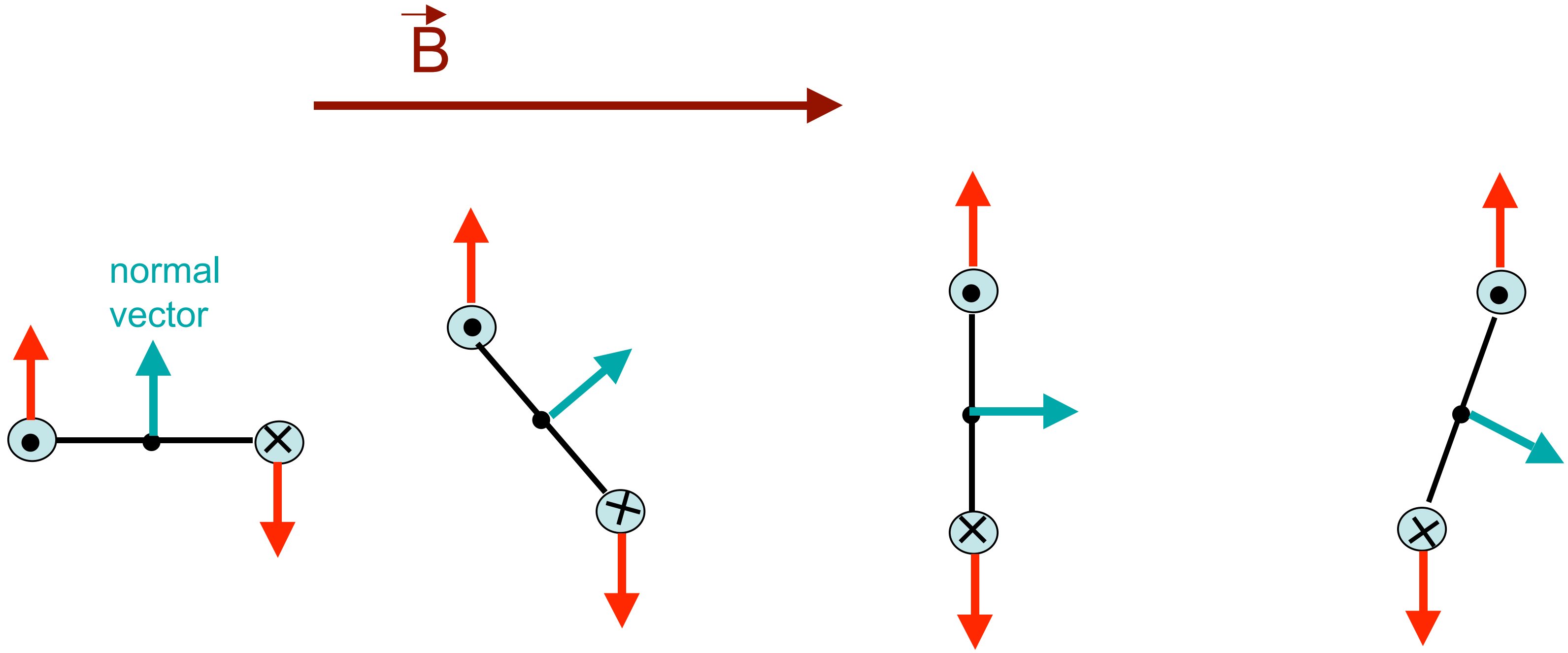


$$\tau = BIAN\sin\theta = (0.5T)(3A)(100)(0.2m^2)\sin 45^\circ = 21.2 \text{ Nm}$$

What would happen if the current were flowing in the opposite direction?



Same magnitude of τ, but rotation is now CW



#1: θ=90º

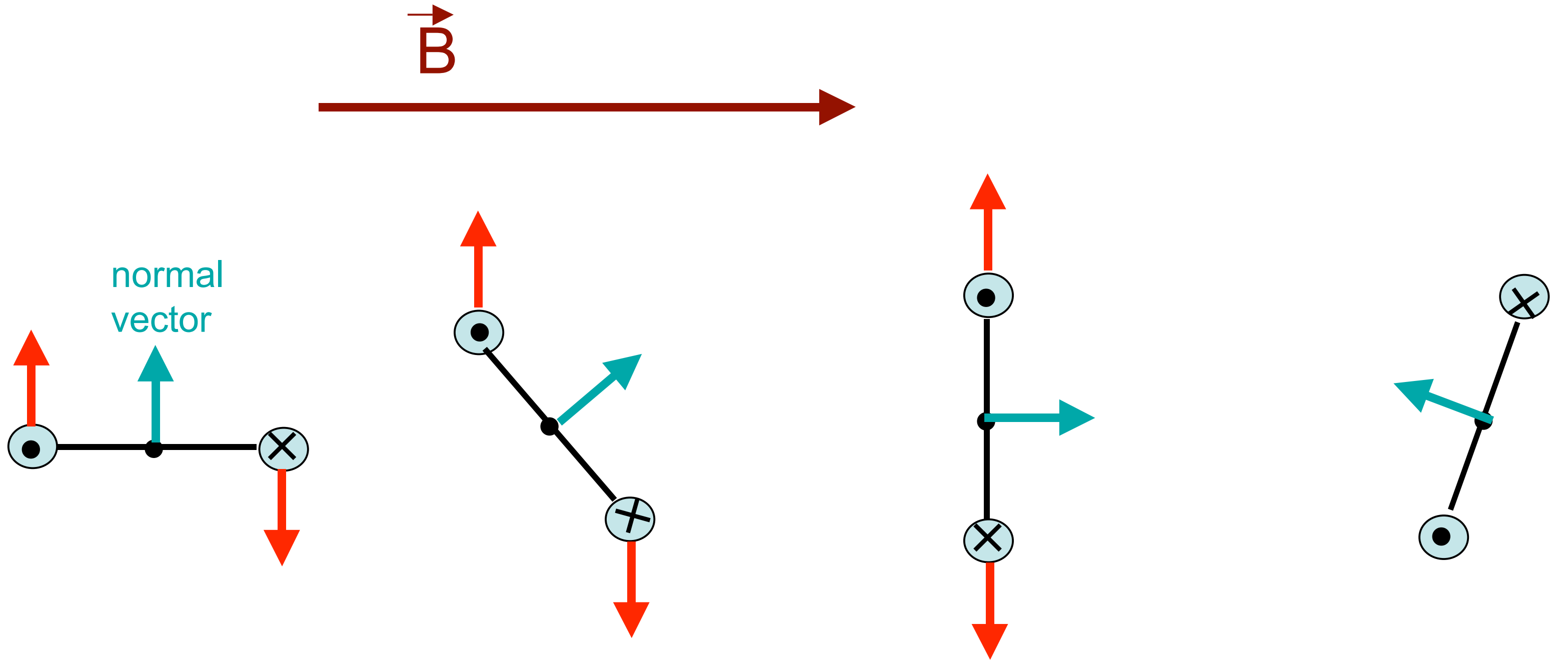
#2: 0º < θ <90º

#3: θ=0º

#4: 90º < θ <180º

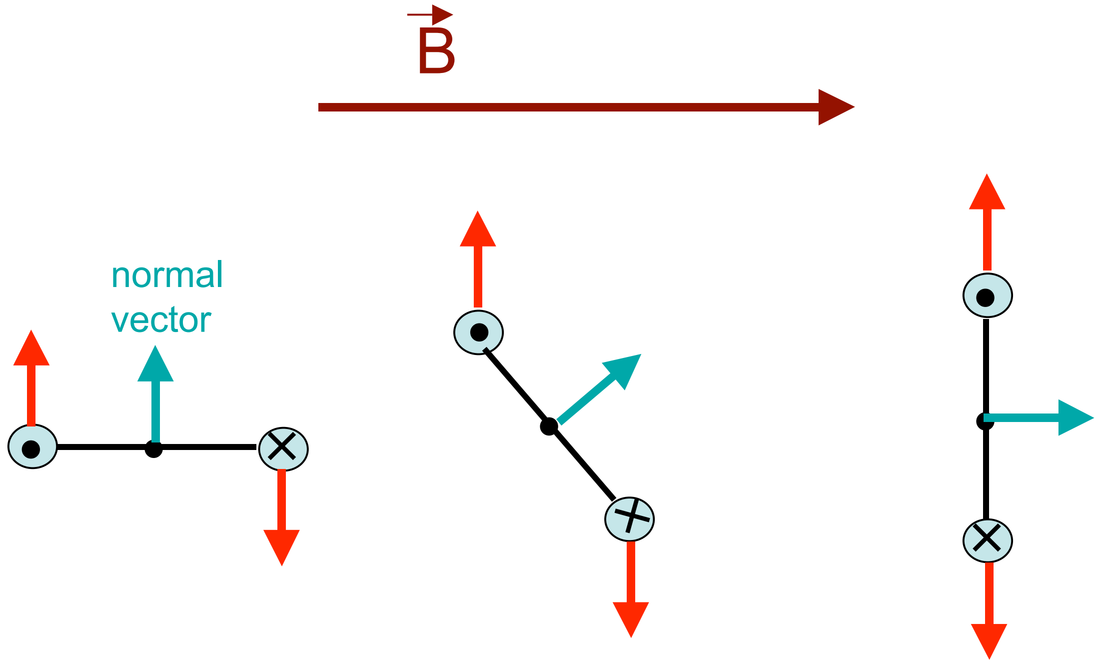
torque acts to align plane of loop perpendicular to B-field (align normal vector with B-field), as in #3

(if released from rest in this position, it won’t rotate)



#1: θ=90º	#2: 0º < θ <90º	#3: θ=0º	#4: 90º < θ <180º

As loop is rotating, what would happen if we switched the direction of current immediately after #3?



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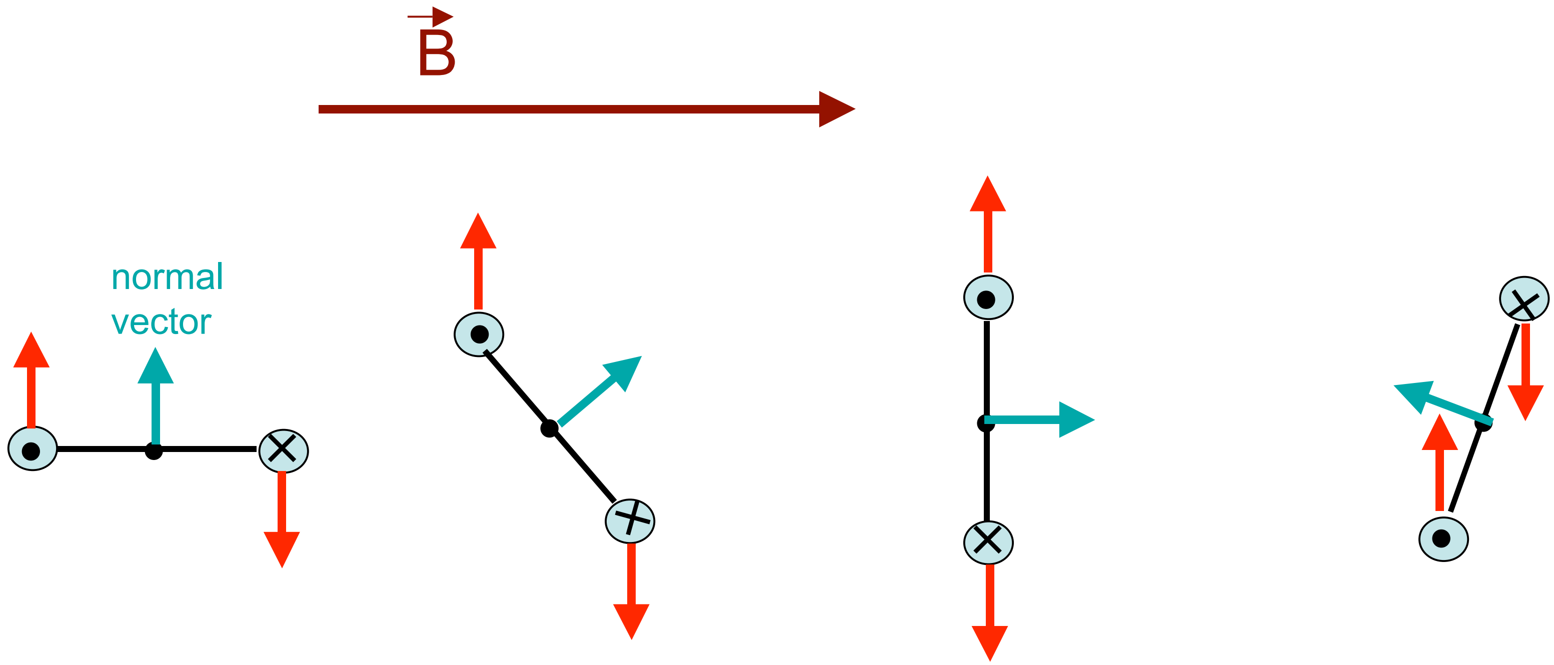


#1: θ=90º

#2: 0º < θ <90º

#3: θ=0º

#4: 90º < θ <180º

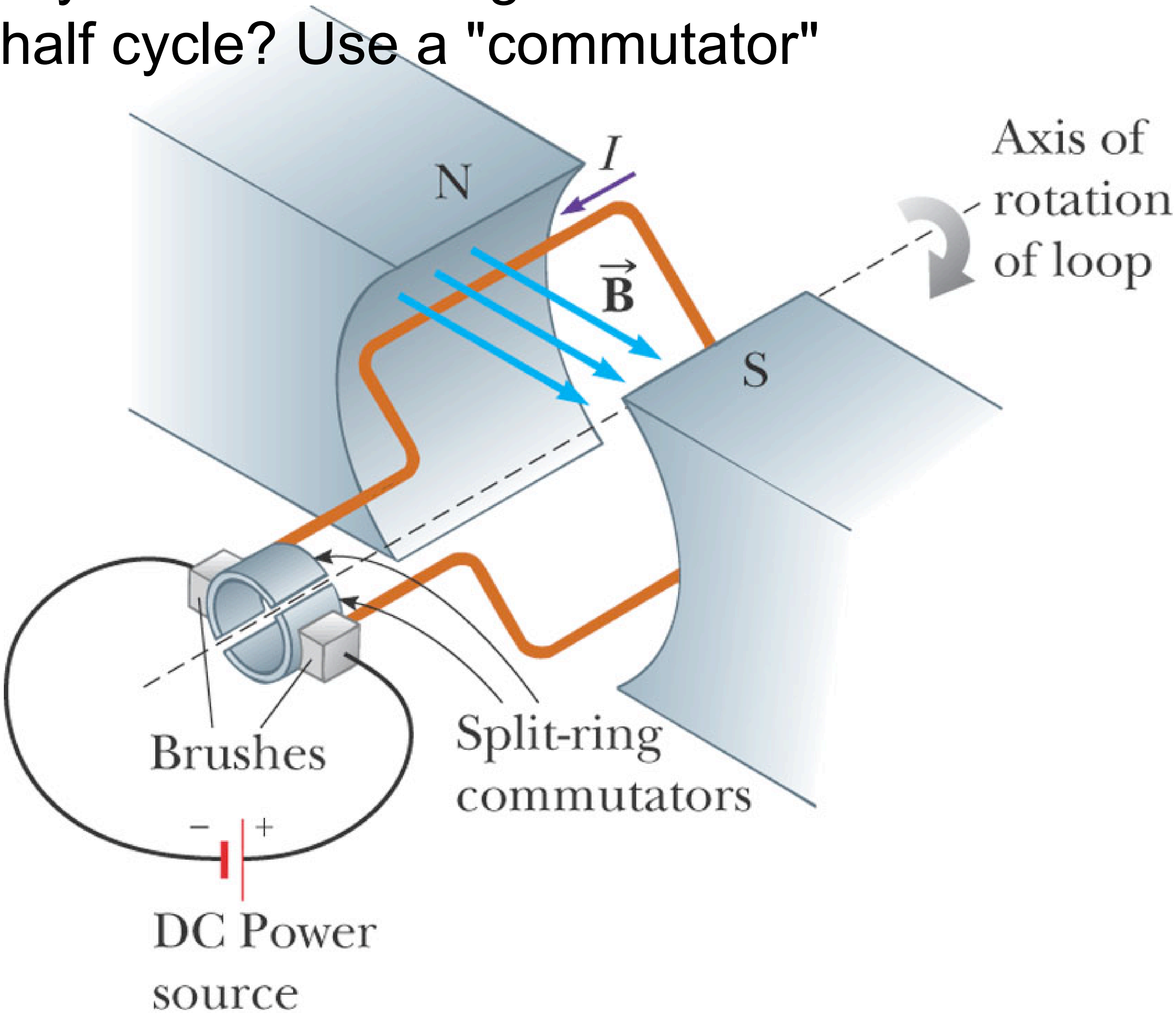
As loop is rotating, what would happen if we switched the direction of current immediately after #3?

The loop would continue to rotate clockwise!

Electric motors

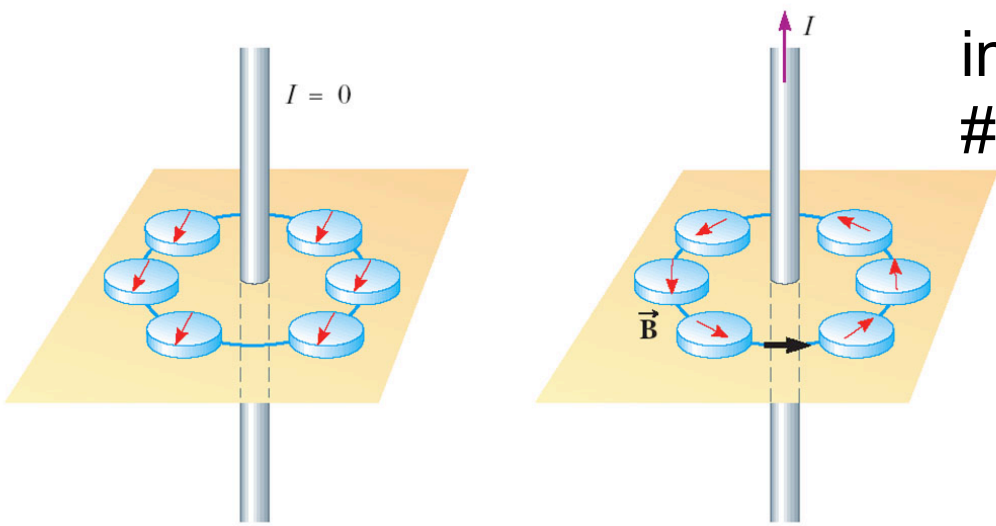
- If direction of current is switched every time τ is about to change sign, then τ will never change sign!
- Loop will rotate nonstop: we have an electric motor (electrical energy converted to mechanical (rotational) energy)!
- Fans, blenders, power drills, etc.
- Use AC current (sign changes naturally), or if you only have DC current available….

How do you switch the sign of current every half cycle? Use a "commutator"

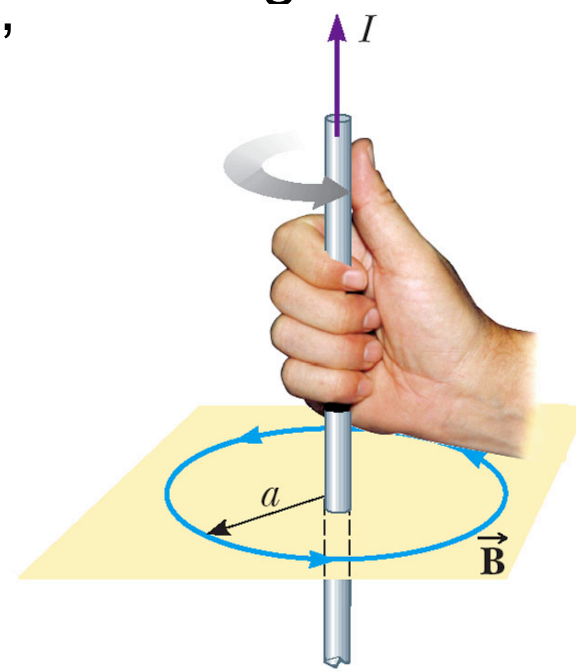


Magnetic Field of a long straight wire; Ampere's Law

In 1819, Oersted (Denmark) noticed that a magnet (compass needle) was deflected when current was drawn through a nearby wire. 1820: compasses in a horizontal plane:

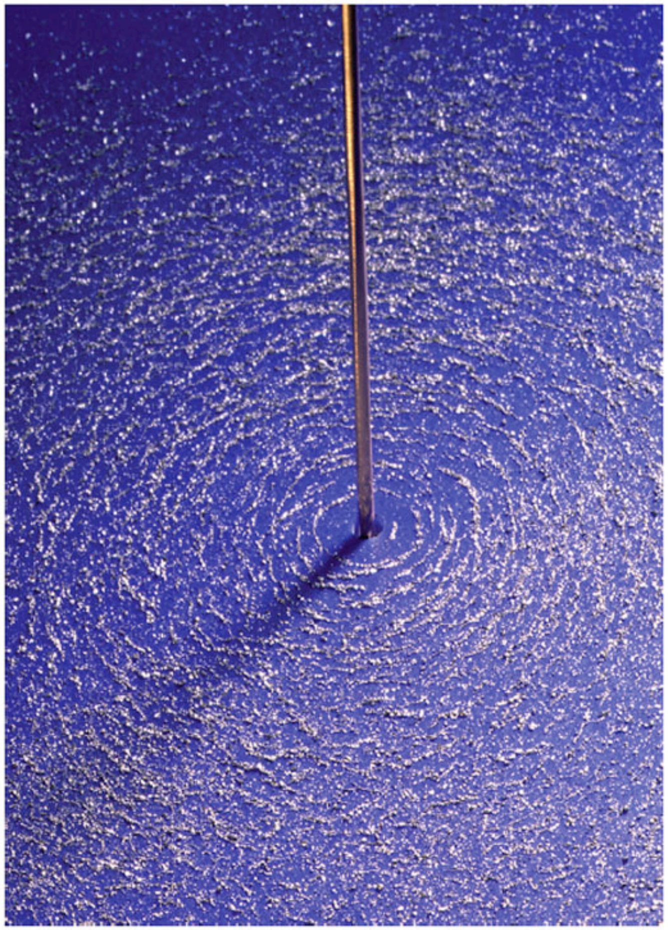


Direction of deflection indicated "right-hand rule #2'



Magnetic Field of a long straight wire

B-field lines form concentric circles:



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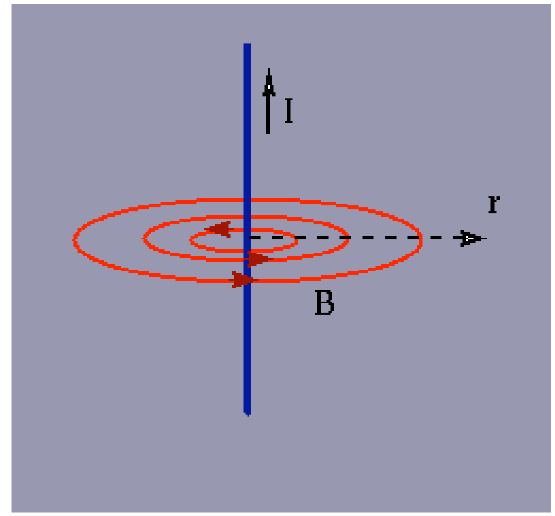
Notice that the iron filings are more strongly aligned closer to the wire

Magnitude of $\vec{B}$ is the same everywhere along a given radius: $|\vec{B}|$ depends only on r (& physical constants)

Magnetic Field of a long straight wire

The magnitude of the field at a distance r from a wire carrying a current of I is



$$\boxed{B = \frac{\mu_o I}{2\pi r}}$$

$\mu_o = 4\,\pi \times 10^{-7}\ \mathrm{T{\cdot}m / A}$

μ_o is called the *permeability of free space*

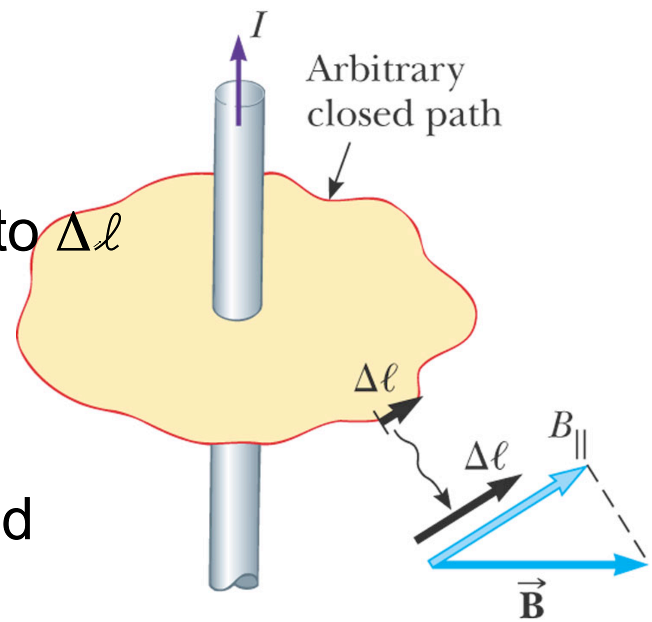
Ampere's Law: A more general method for deriving magnitude of B-field due to sources of current

1. Construct a closed path consisting of short segments, each of length $\Delta\ell$

2. B = magnetic field
$B_{||}$ = Component of B-field PARALLEL to $\Delta\ell$

Consider the product $B_{||} * \Delta\ell$

3. Sum of these products over the closed path $= \mu_0$ I



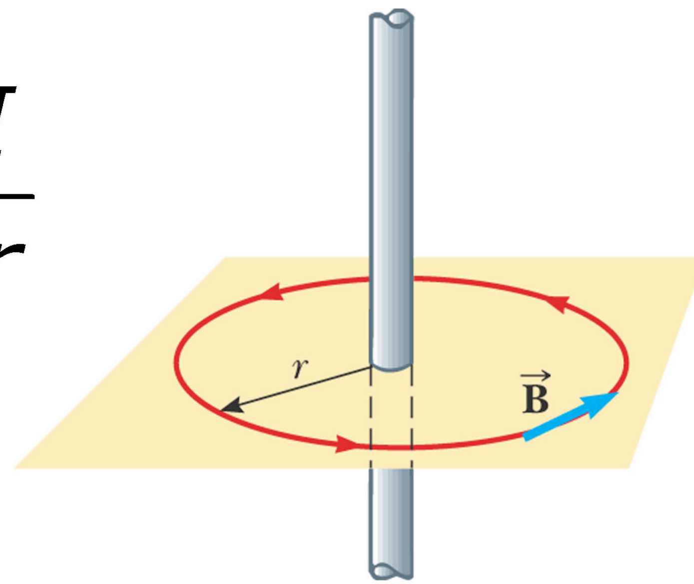
$$\Sigma B_{||}\,\Delta\ell = \mu_o\, I$$

Take advantage of symmetry. When B-field lines are circles, we choose a circular Amperian loop. B is already || to the circle at all points on the circle

$\sum B_{||}\Delta\ell$ = $B_{||}$ * circumference

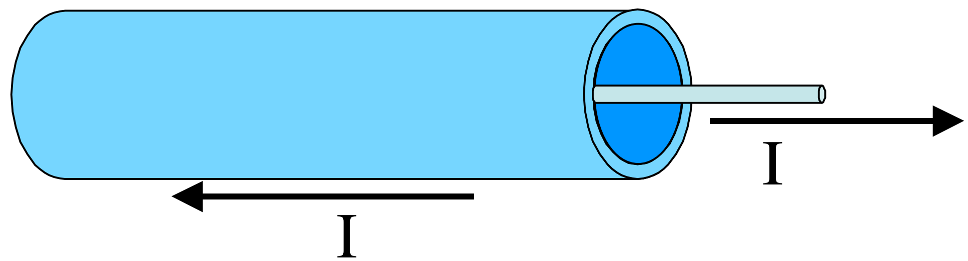
= $B_{||}$ * $2 \pi r$ = $\mu_0 I$

Rearrange to get $$B = \frac{\mu_o I}{2\pi r}$$



Example: A Co-axial cable

Inner conductor and outer conductor of radius R_{out} carrying currents in opposite directions.

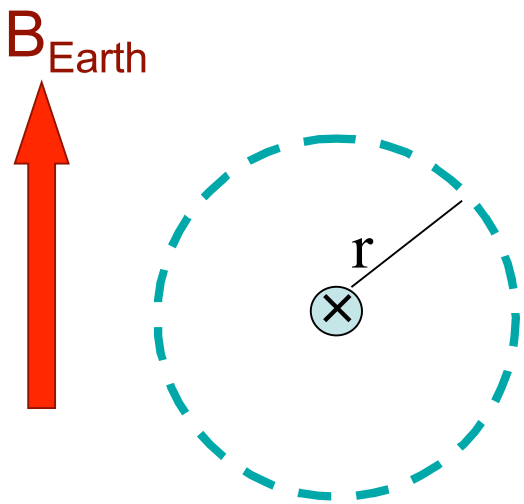


What's B at $r > R_{out}$?

KEY: Remember SUPERPOSITION!

$+I \; + \; -I$ = zero total current, so B-field = 0.

Example: A wire carrying 5A of current travels vertically into the page. At what distance r will the B-field equal the Earth's B-field (which points northward) at the surface, 0.5 Gauss? If 4 compasses are placed N,S,E,W of the wire at this radius, how will each compasses' needle be deflected?



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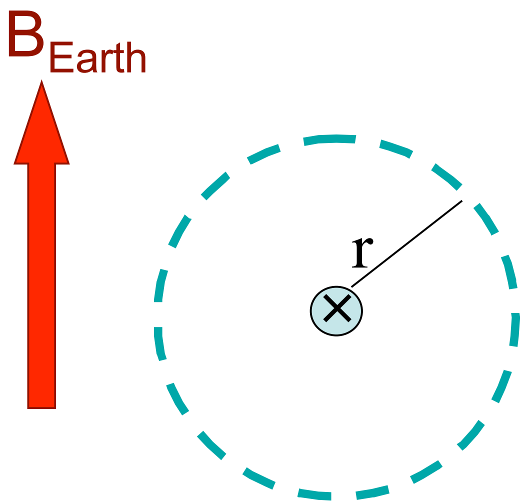

$$B = \frac{\mu_o I}{2\pi r}$$

$r = \mu_0 I / 2\pi B$

$r = (4\pi \times 10^{-7}\ \mathrm{T\ m / A})(5\mathrm{A}) / (2\ \pi\ 0.5\times10^{-4}\mathrm{T})$

$= (2 \times 10^{-7}\ \mathrm{T\ m / A})(5\mathrm{A}) /\ \ 0.5\times10^{-4}\mathrm{T}$

$= 2 \times 10^{-2}$ m = 2 cm.

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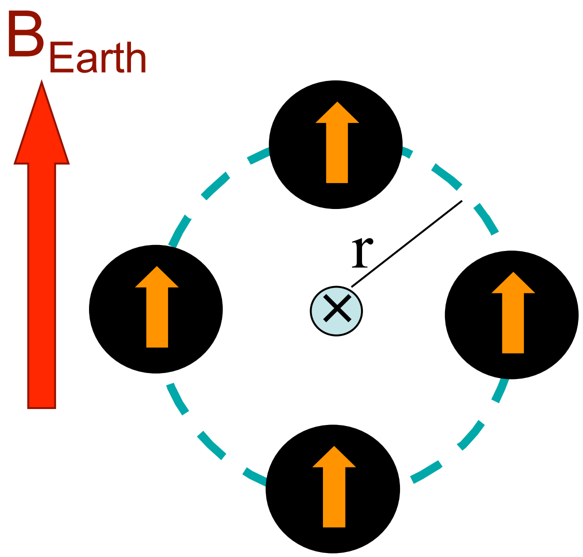

$$B = \frac{\mu_o I}{2\pi r}$$

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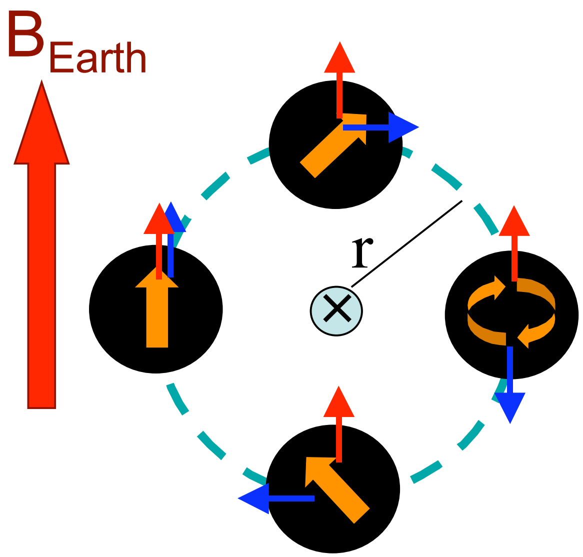

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$= (2 \times 10^{-7}\ T\ m / A)(5A) /\ \ 0.5\times10^{-4}T$
$= 2 \times 10^{-2}\ m = 2\ cm.$

The E compass will spin freely ($F_{B\_Earth} = F_{B\_wire}$).
The W compass will really point north, as it feels 2 * F_{B_Earth}
The N compass will point NE
The S compass will point NW

A power line 20m above the ground carries a current of 1000 A from E to W. Find the magnitude and direction of the B-field due to the wire at the ground below the line, and compare it to the Earth's B-field. Repeat for a wire 10m above ground.

At 20m:
$B = \mu_0 I / 2\pi r$
$= (4\pi \times 10^{-7}\ \mathrm{T\ m / A} * 1000\ \mathrm{A}) / (2\ \pi\ 20\mathrm{m})$
$= 1 \times 10^{-5}\ \mathrm{T}$ -- about a factor of 3-5 smaller than the Earth's B field (0.3-0.5 Gauss).

At 10m: $B(r=10) = 2 \times B(r=20) = 2 \times 10^{-5}\ \mathrm{T}$ -- still just smaller than the Earth's B-field

Direction of B-field when you're below the wire: south