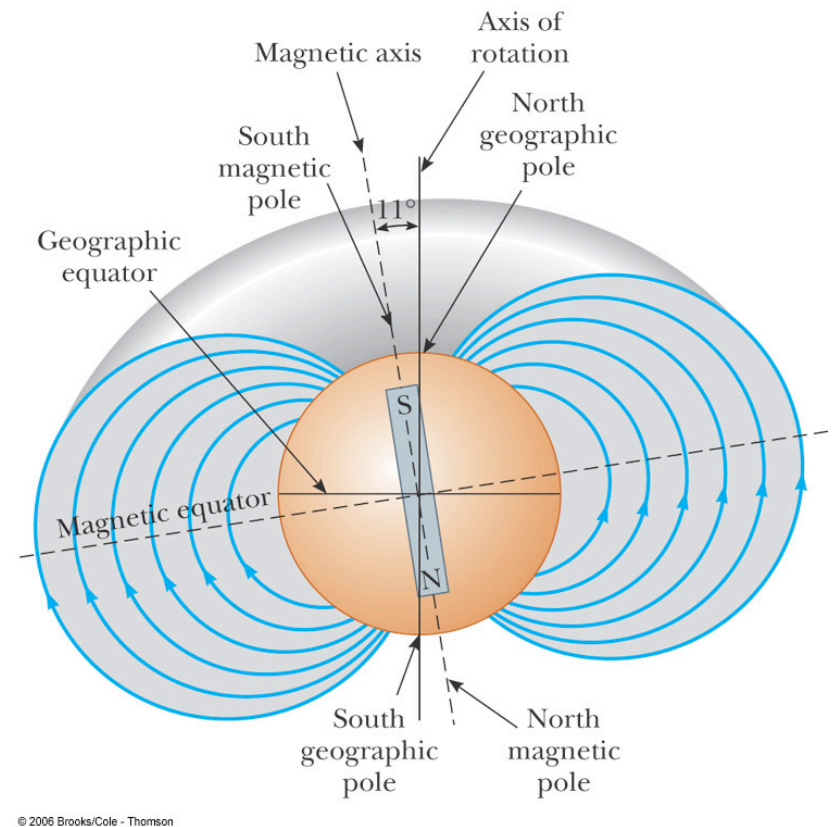


The N pole of a magnet got its name because it points roughly towards Earth's *north* pole. It points towards NE Canada, since that's the Earth's South magnetic pole.

Earth's B-field is actually upside down, and the dipole axis is offset slightly ( $\sim 11^\circ$ ) from Earth's rotation axis.



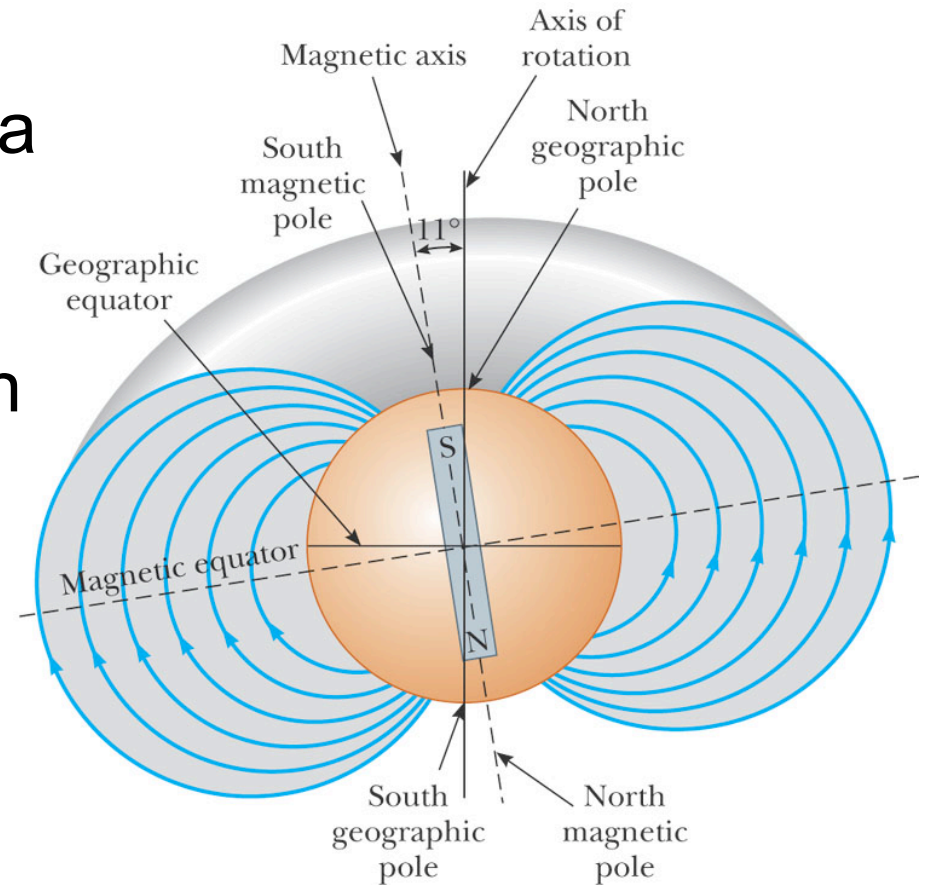
Earth's magnetic field has a vertical component at the surface:

Points upwards in southern hemisphere.

Points downwards in northern hemisphere.

Parallel to surface at magnetic equator.

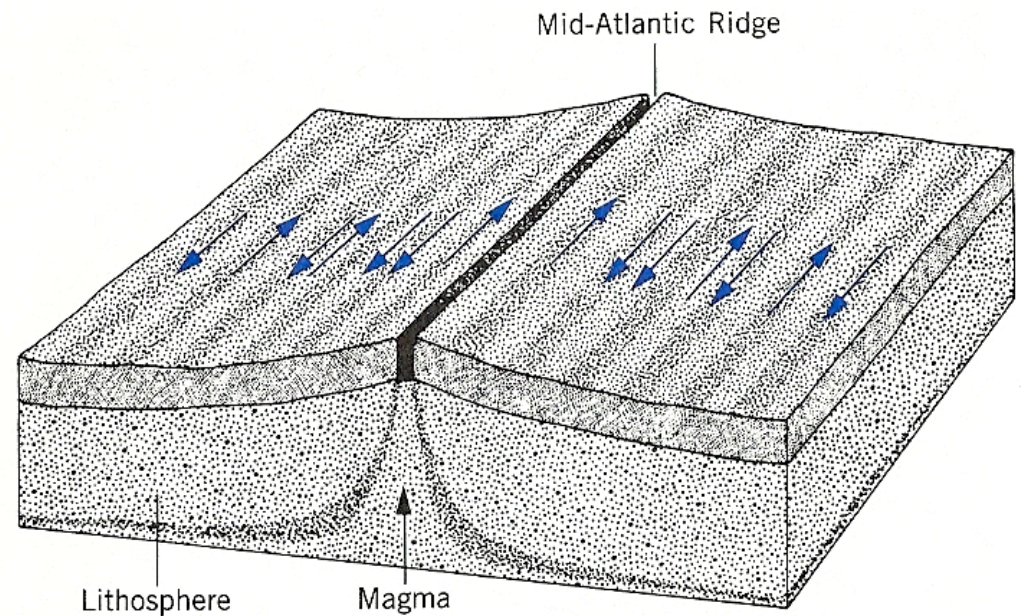
Source: Current/ convection in hot Fe liquid core



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Basalt is expelled from volcanoes and cools, and the B-field becomes imprinted.

Mid-Atlantic Ridge shows that B-field direction changes every  $\sim 0.1\text{--}1$  Myr



**Figure 13** As molten material emerges through a ridge in the ocean floor and cools, it preserves a record of the direction of the Earth's magnetic field at that time (arrows). Each segment might represent a time of 100,000 to 1,000,000 years.



Mars Global Surveyor (JPL/NASA)

Mars: No 'global' B-field like Earth's (no internal dynamo?)

Mars Global Surveyor's magnetometers: very strong crustal fields:

Older crust is magnetized much more strongly than newer crust

Indicates that very strong global magnetic fields existed, but only before ~4 billion yr ago



(from comicvine.com)





Mars Global Surveyor (JPL/NASA)

Meteor impacts: crust melts and reforms; any imprinted magnetism is lost

Mars' surface area/ volume ratio (prop.to  $4\pi r^2 / (4\pi/3)r^3$  prop.to  $1/r$ ) is larger than Earth's: lost interior heat & dynamo mechanism faster

Moon: also evidence for a B-field that was stronger in the past (from Lunar Prospector mission)



(from comicvine.com)

# Units of B-fields

Tesla [SI] and Gauss [cgs]

1 Tesla =  $10^4$  Gauss.

$$T = \frac{\text{Wb}}{\text{m}^2} = \frac{\text{N}}{\text{C} \cdot (\text{m/s})} = \frac{\text{N}}{\text{A} \cdot \text{m}}$$

Typical B-field strengths:

Earth's B-field at surface:  $0.5 \times 10^{-4} \text{ T} = 0.5 \text{ G}$

Refrigerator magnet:  $\sim 0.005 \text{ T}$

Bar magnets:  $0.01 \text{ T}$

MRI machine:  $1\text{-}5 \text{ T}$

Laboratory magnet:  $5 \text{ T}$

Superconducting magnet:  $20\text{-}30 \text{ T}$

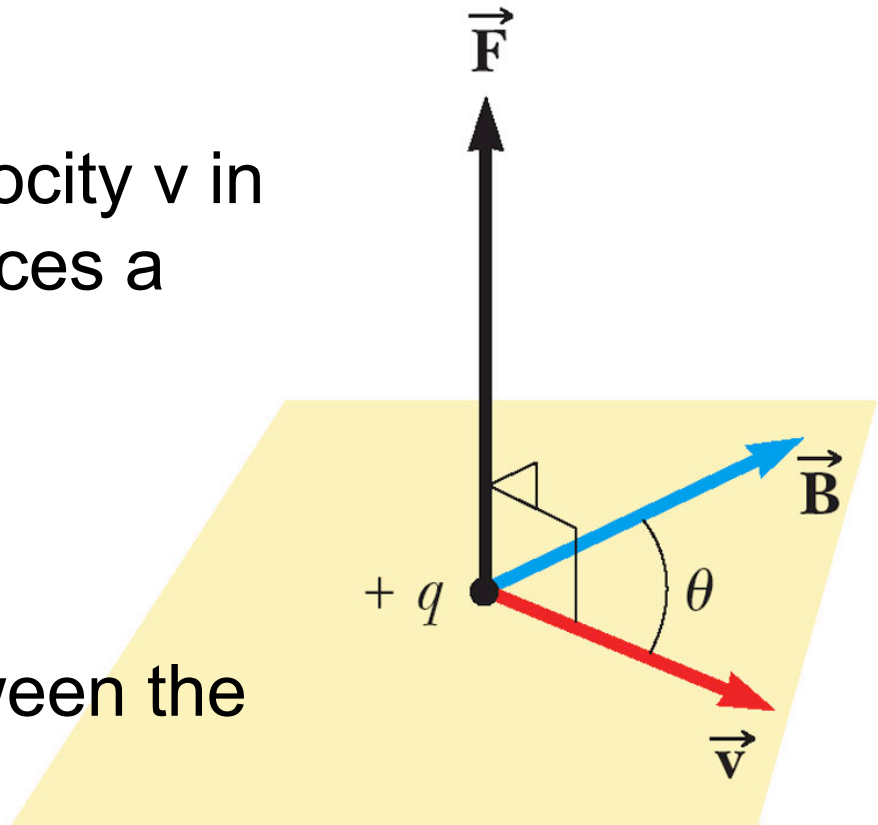
# Magnetic field produces a force on a moving charge

A charge  $q$  moving with velocity  $\vec{v}$  in a magnetic field  $\vec{B}$  experiences a force  $\vec{F}$  with magnitude:

$$F = q v B \sin(\theta)$$

$\theta$  is the smallest angle between the vectors  $\vec{v}$  &  $\vec{B}$

$\vec{F}$  is perpendicular to BOTH  $\vec{v}$  and  $\vec{B}$



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$$\mathbf{F} = q\mathbf{v}\mathbf{B}\sin(\theta)$$

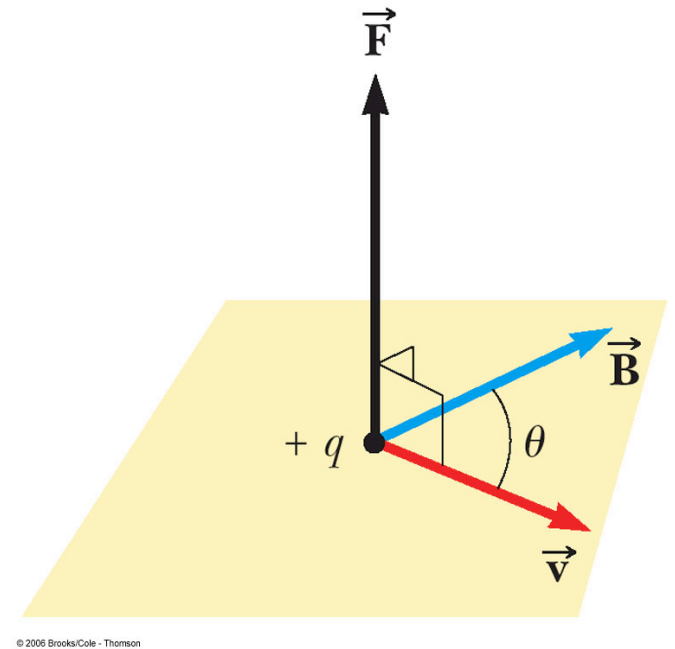
Magnitude of Force depends on both  $v$  and  $B$ :

If  $v=0$  or  $B=0$ , then  $F=0$

Force depends on angle  $\theta$ :

If  $\vec{B} \parallel \vec{v}$ ,  $F = 0$

Force is max. when  $\vec{v}$  and  $\vec{B}$  are perpendicular



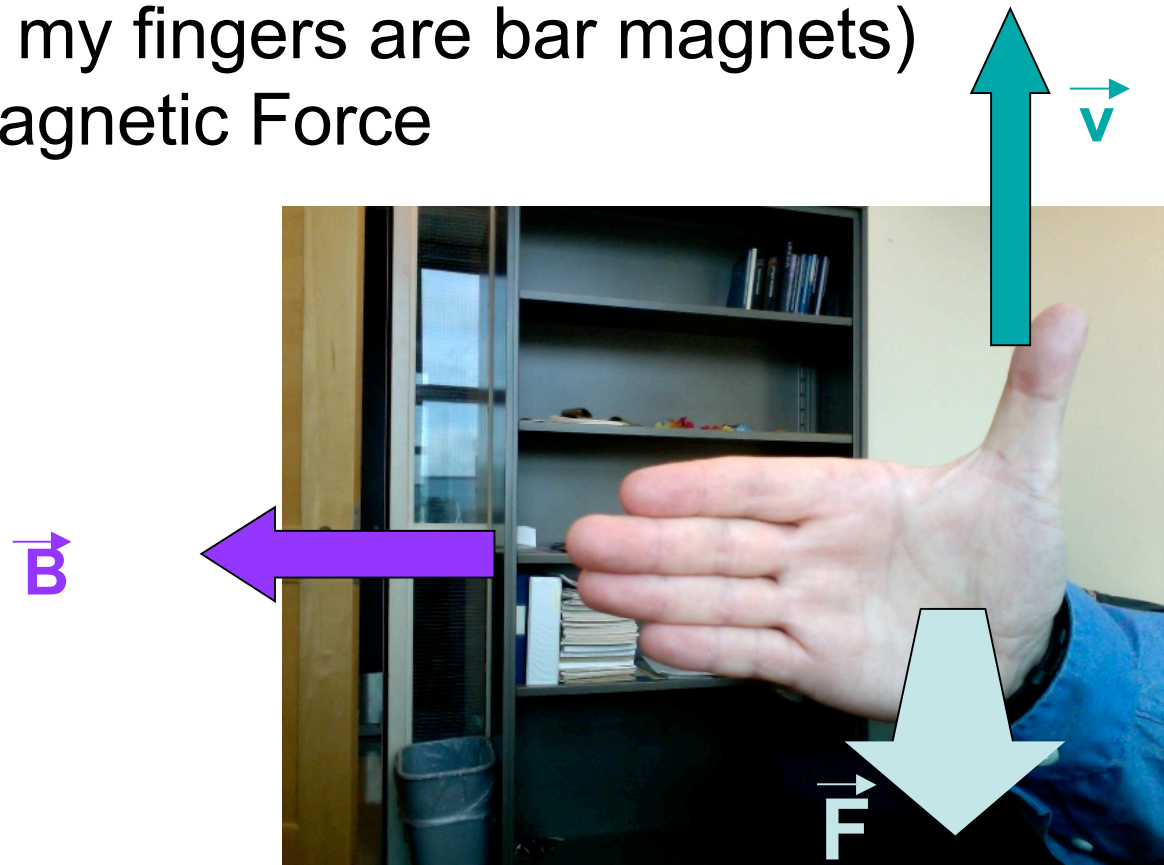
*Because  $\vec{F}$  is perp. to  $\vec{v}$ , magnetic forces cannot change a particle's speed, just its direction of motion.*

# Right-hand rule (my version)

Thumb =  $v$  (e.g., hitchhiking)

Fingers =  $B$  (like my fingers are bar magnets)

Out of Palm = Magnetic Force





Example: A proton is moving at  $1 \times 10^4$  m/s from left to right in a magnetic field of 0.4 T that's in the upward direction (in the plane of the page). Find the magnitude of the force vector. Find the direction. What would the force be if the particle was an  $e^-$ ?

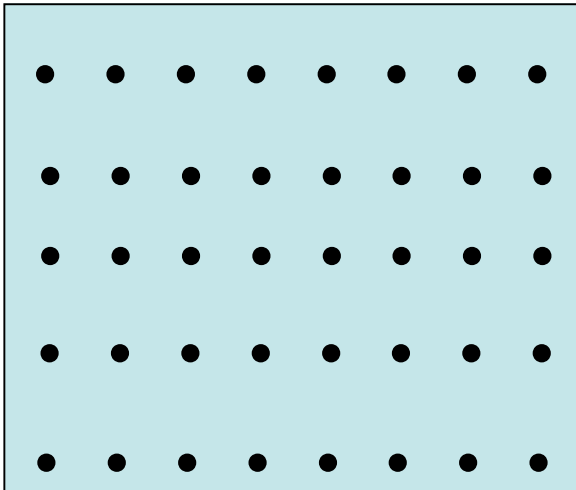
$$F = B * q * v * \sin\theta = 0.4\text{T} * 1.6 \times 10^{-19} \text{ C} * 10^4 \text{ m/s} * \sin 90^\circ = 6.4 \times 10^{-16} \text{ N}$$

Direction of force: out of the page towards you.

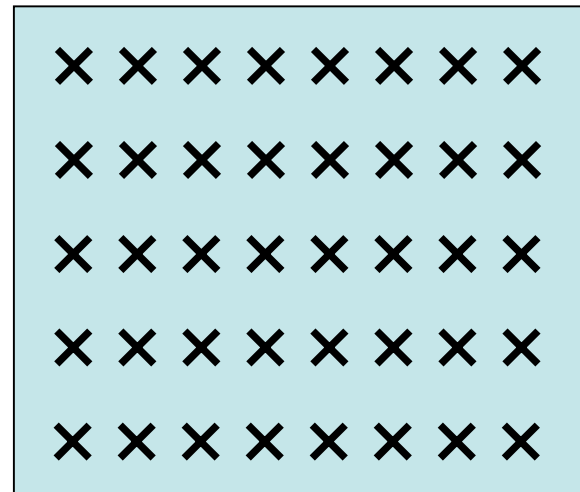
For  $e^-$ : Magnitude of force is same, but in the opposite direction.

# B-field notation

out of page:



into page:



think of the points/tails of arrows

# Velocity Selector

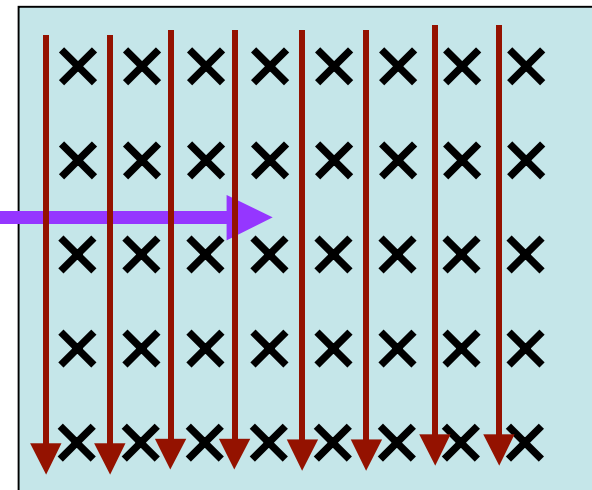
Perpendicular E and B fields can be used to select charged particles having a specific velocity

$F_B = evB$  (downward for  $e^-$ )

$F_E = eE$  (upward for  $e^-$ )

When  $F_B = F_E$ , forces cancel:  $evB = eE$

$v = E/B$ : electron with this velocity will be undeflected



$E$ : downward

$B$ : into page

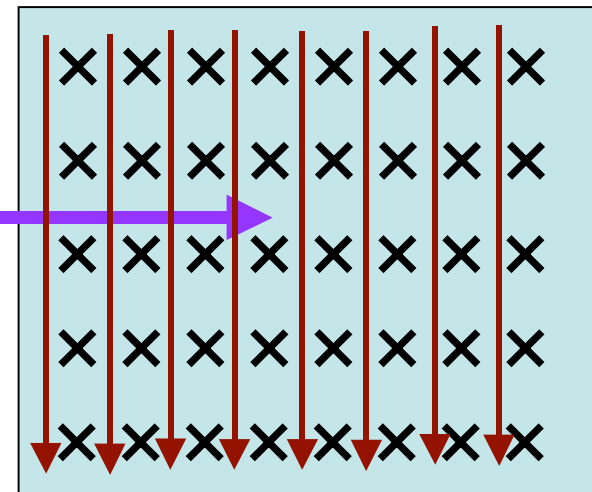
# Velocity Selector

Example: A velocity selector has perpendicular electric and magnetic field of  $E = 1000 \text{ V/m}$  and  $B = 0.3 \text{ T}$ . Find the velocity of the electrons that pass through undeflected. What would happen to faster electrons? Slower?

$$v = E/B =$$

$$1000 \text{ V/m} / 0.3 \text{ T} =$$

$$3.3 \times 10^4 \text{ m/s}$$



$E$ : downward

$B$ : into page

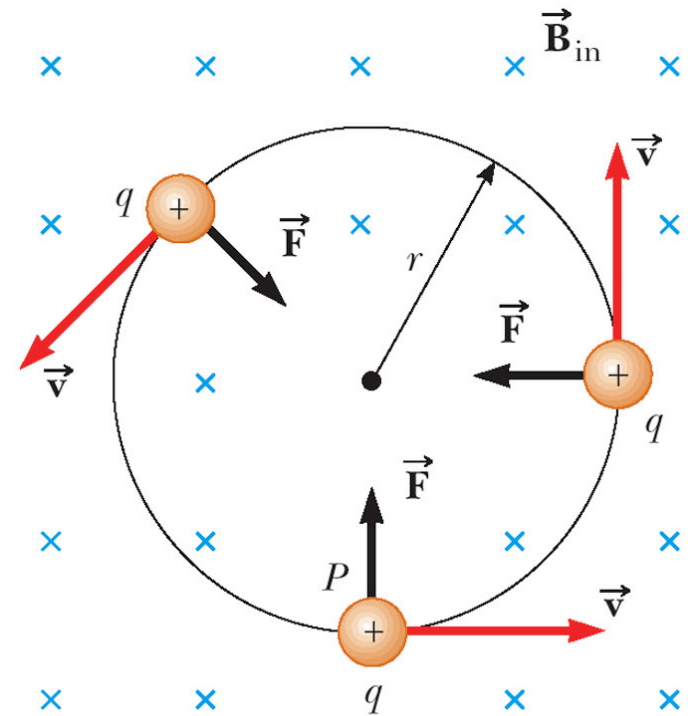
# Motion of a charged particle in a magnetic field

$\vec{F}$  is in a plane perpendicular to  $\vec{B}$ .

Particle's path remains in plane perpendicular to  $\vec{B}$ .

$$F = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$



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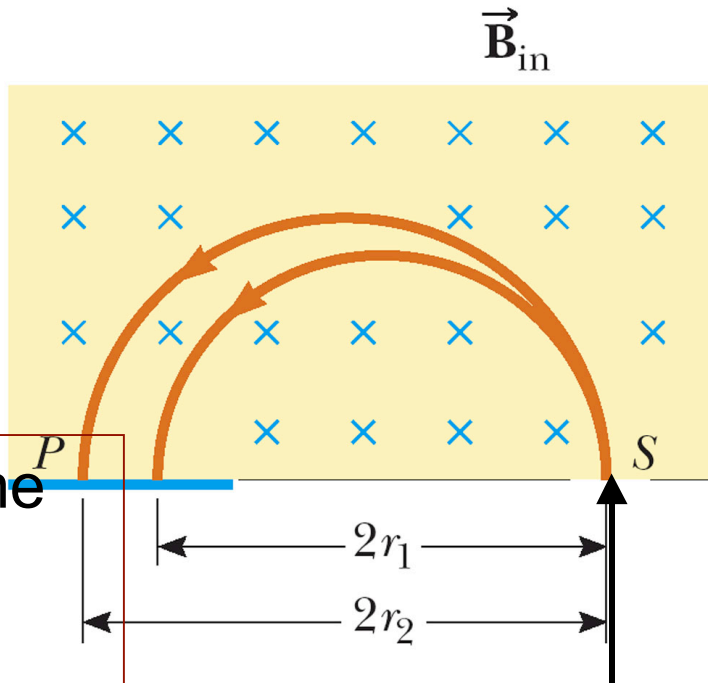


A proton with velocity  $v=1\times 10^6$  m/s is in a uniform B-field of 0.2 T. Find r:

$$r = mV / qB = \\ (1.67\times 10^{-27} \text{ kg} * 1\times 10^6 \text{ m/s}) / (1.6\times 10^{-19} \text{ C} * 0.2 \text{ T}) = \\ 5.2 \text{ cm}$$

# Mass Spectrometers

$$r = \frac{mv}{qB}$$



4. Ions with the same charge become separated by mass onto photographic plate: compute charge/mass ratio

3. Molecular ions injected into B-field at velocity  $v$

1. A sample is ionized (impacting with a  $e^-$  beam).  
2. Positive ions are accelerated by an E-field through veloc selector

Example: A mass spectrometer has a velocity selector at its inlet such that only  $q=+1$  ions with  $v = 1 \times 10^5$  m/s are permitted inside the mass spectrometer, where the B-field is 0.2 T. A mixture of gas containing  $\text{CO}_2^+$  is injected. But some of the  $\text{CO}_2$  contains Carbon-14. What radii are  $^{12}\text{CO}_2^+$  and  $^{14}\text{CO}_2^+$  rotated through, and what is their separation on the photographic plate?

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

Assume  $m_p = m_n$  for simplicity; ignore masses of electrons since they're 1800x less massive than protons/neutrons

$$\text{Mass } (^{12}\text{CO}_2^+) = (12+16+16) \times 1.67 \times 10^{-27} \text{kg} = 44 \times 1.67 \times 10^{-27} \text{kg} = 73.5 \times 10^{-27} \text{ kg}$$

$$\text{Mass } (^{14}\text{CO}_2^+) = (14+16+16) \times 1.67 \times 10^{-27} \text{kg} = 46 \times 1.67 \times 10^{-27} \text{kg} = 76.8 \times 10^{-27} \text{ kg}$$

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

$$r(^{12}\text{CO}_2^+) = mv/qB = (73.5 \times 10^{-27} \text{ kg} * 10^5 \text{ m/s}) / (1.6 \times 10^{-19} \text{ C} * 0.2 \text{ T}) = 23.0 \text{ cm}$$

$$r(^{14}\text{CO}_2^+) = mv/qB = (76.8 \times 10^{-27} \text{ kg} * 10^5 \text{ m/s}) / (1.6 \times 10^{-19} \text{ C} * 0.2 \text{ T}) = 24.0 \text{ cm}$$

The diameters of the circles traced out will be 46.0 and 48.0 cm, respectively.

The separation on the photographic plate will be 2.0 cm.

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

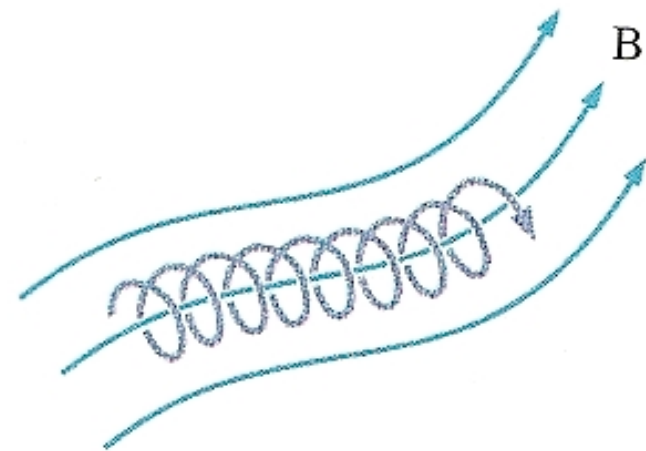
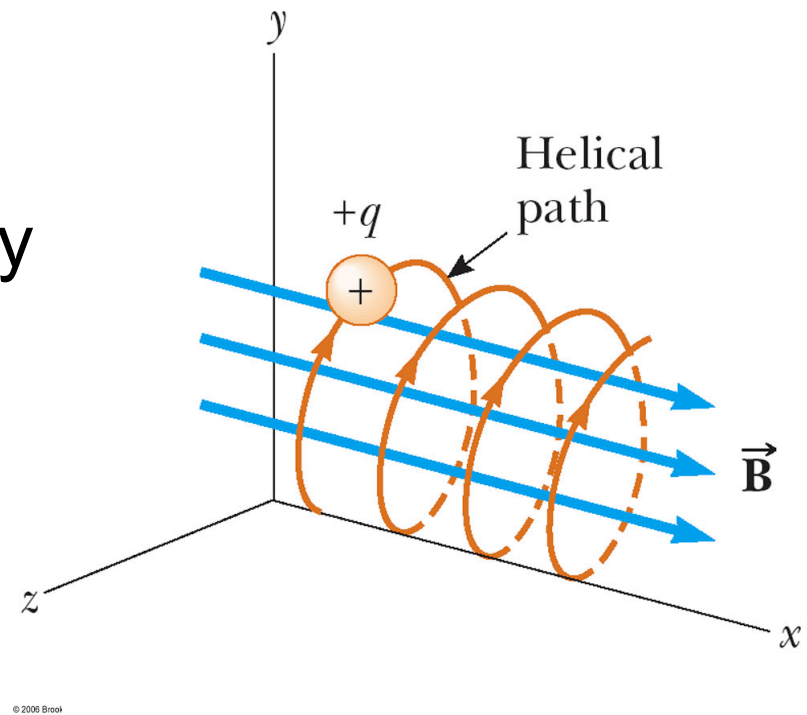


When  $\vec{B}$  and  $\vec{v}$  are not exactly perpendicular:

Motion  $\parallel$  to B-field is unaffected.

Motion perp. to B-field is circular.

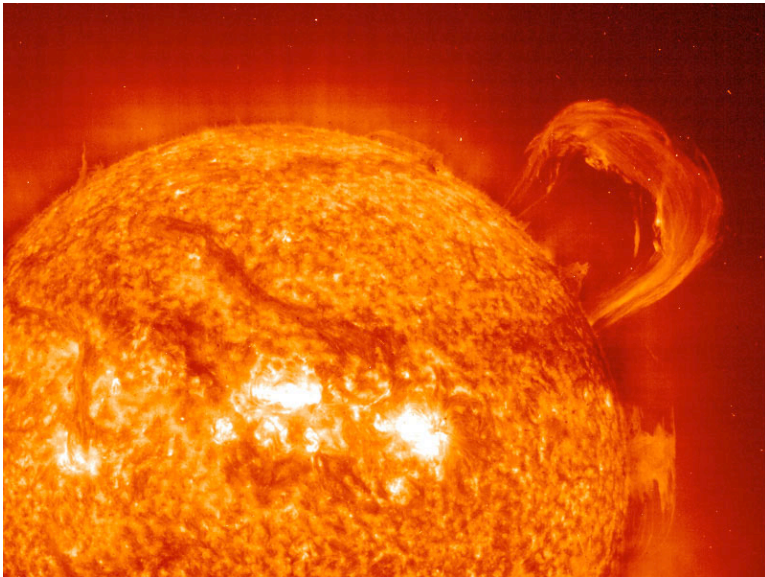
So a charge will follow a HELICAL path around field lines.



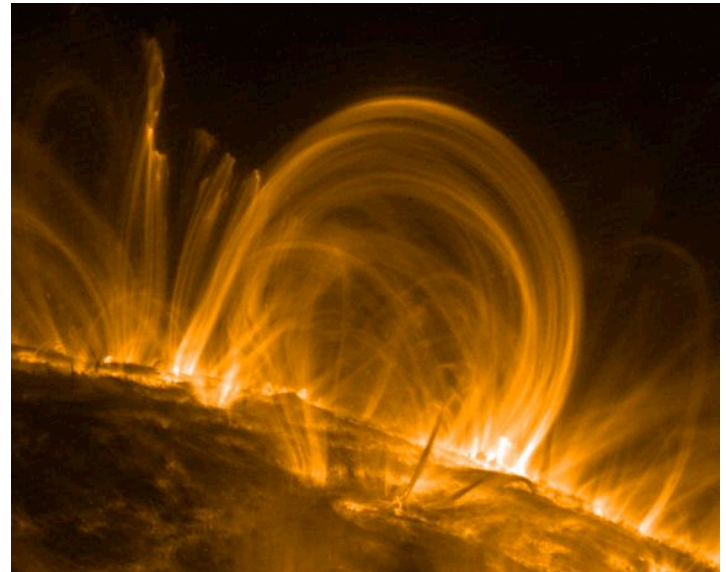
Example:

Solar Prominences: Charged particles in sun's corona move in helices along B-field lines, emit light & map out those B-field lines

Solar prominence viewed  
by SOHO:



Solar prominence viewed  
by TRACE:



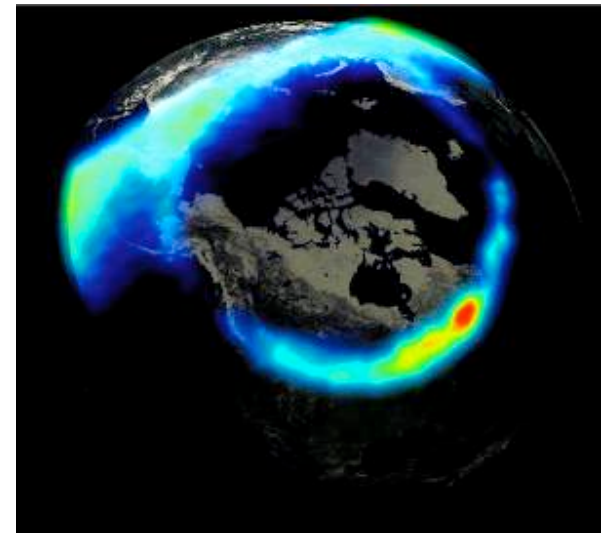
# Aurora

Charged particles from solar wind or solar flares get caught in B-field lines in Earth's B-field, funneled to the poles



[apod.nasa.gov](http://apod.nasa.gov)

Appears as circle surrounding magnetic pole  
(NASA's *Polar Sat.*)



[gsfc.nasa.gov](http://gsfc.nasa.gov)