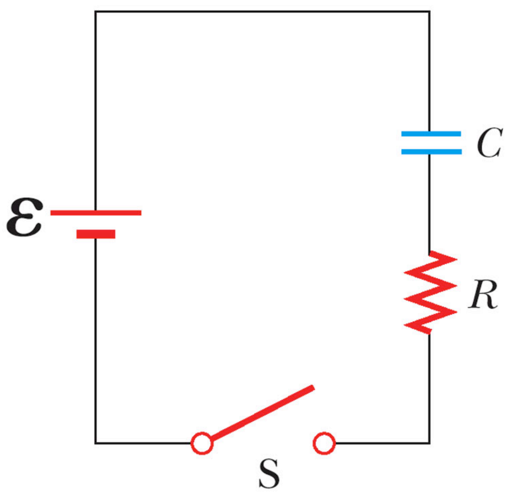
18.5 RC Circuits

Introduction to time-dependent currents and voltages.

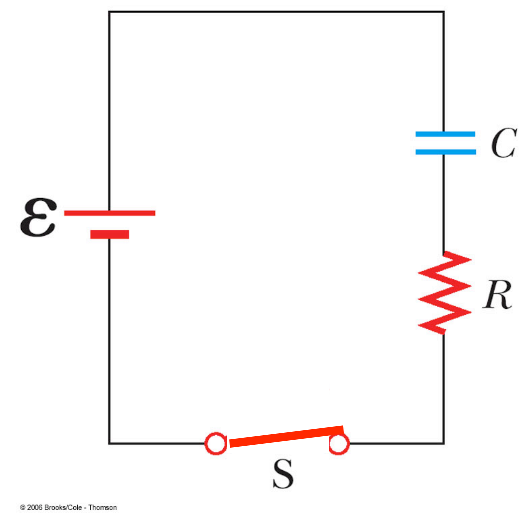
Applications: timing circuits, clocks, computers, charging + discharging capacitors

An RC circuit

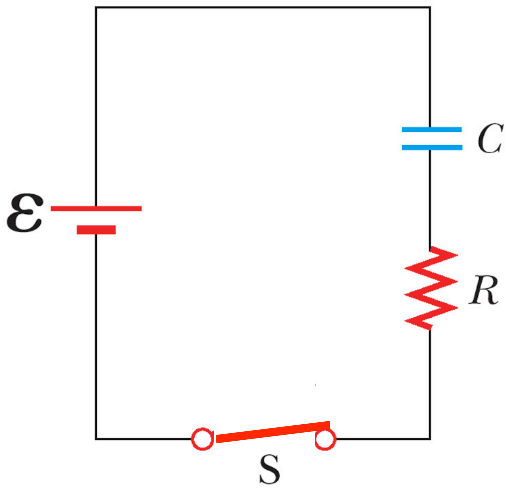


RC circuit: charging

At time t=0, close Switch



An RC circuit



At time t=0, close switch

Initially (at t=0), $Q = 0$.
ΔV_C (at t=0) = $Q(t=0) / C = 0$

Loop Rule: $\Sigma \Delta V = 0 = +\varepsilon - \Delta V_R - \Delta V_C$

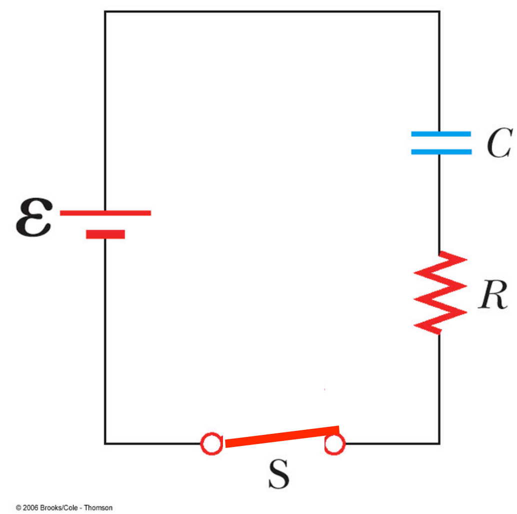
$+\varepsilon - IR - \Delta V_C = 0$

So when charge = 0 (which occurs at t=0),
$\varepsilon = IR$ and so $I = \varepsilon/R$

(at this instant, capacitor has no effect)

RC circuit: charging

Immediately after time t=0: Current starts to flow. Charge starts to accumulate on Capacitor (at a rate $I=dQ/dt$).



As Q increases over time, $\Delta V_C = Q/C$ also increases.

But remember $\Delta V_C + \Delta V_R = \varepsilon$.

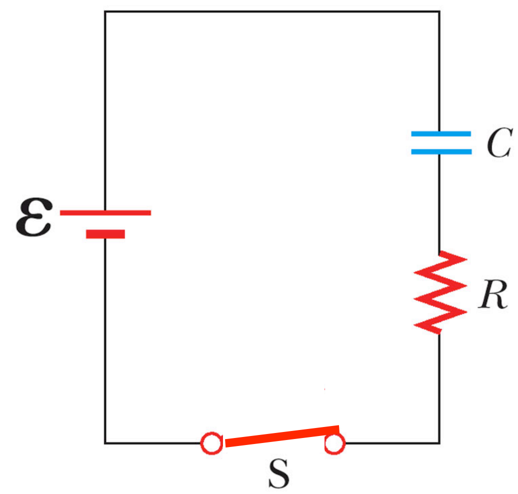
So ΔV_R is decreasing over time.

And I through the resistor = $\Delta V_R/R$ is also decreasing.

RC circuit: charging

After a very long time:

Charge accumulates until Q reaches its maximum:

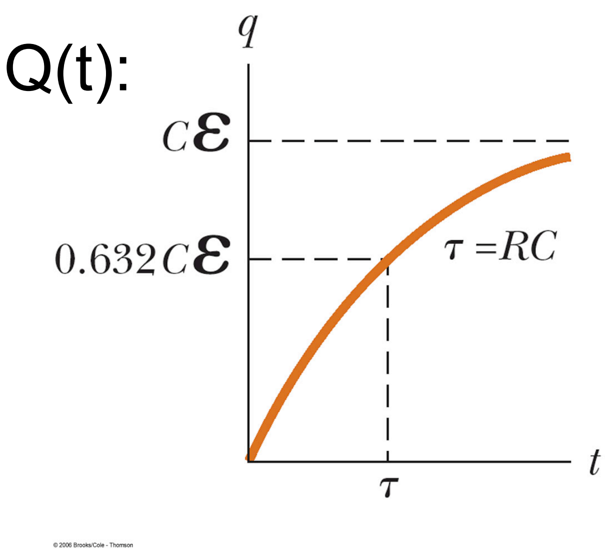


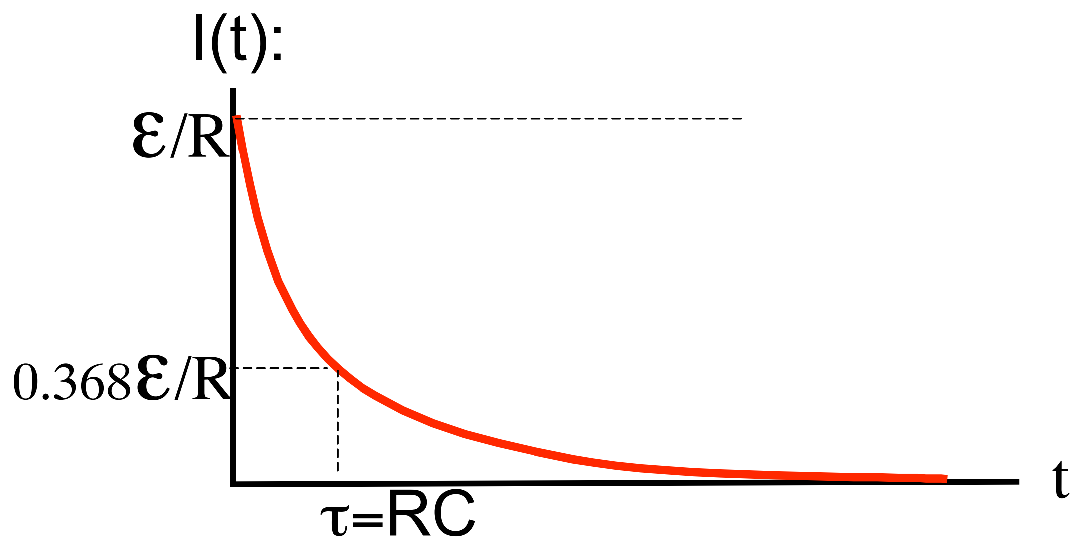
ΔV_C goes to ε. Total Q on the capacitor goes to Cε.

$+\varepsilon = \Delta V_C + \Delta V_R$

ΔV_R goes to zero. And I goes to zero.

RC circuit: charging





	t=0		$t \to \infty$
ΔV_C	0	$\varepsilon(1-e^{-(t/\tau)})$	ε
Q	0	$C\varepsilon(1-e^{-(t/\tau)})$	$C\varepsilon$
ΔV_R	ε	$\varepsilon(e^{-(t/\tau)})$	0
I	ε/R	$(\varepsilon/R)(e^{-(t/\tau)})$	0

Exponential decay

Rate of decay is proportional to amount of species.

Other applications: Nuclear decay, some chemical reactions

Atmospheric pressure decreases exponentially with height

If an object of one temperature is exposed to a medium of another temperature, then the temperature difference between the object and the medium undergoes exponential decay.

Absorption of electromagnetic radiation by a medium (intensity decreases exponentially with distance into medium)

Time constant $\tau = RC$

RC is called the time constant: it's a measure of how fast the capacitor is charged up.

It has units of time:
$RC = (V/I)(q/V) = q/I = q / (q/t) = t$

At $t = RC$, $Q(t)$ and $\Delta V_C(t)$ go to $1 - 1/e = 0.63$ of the final values

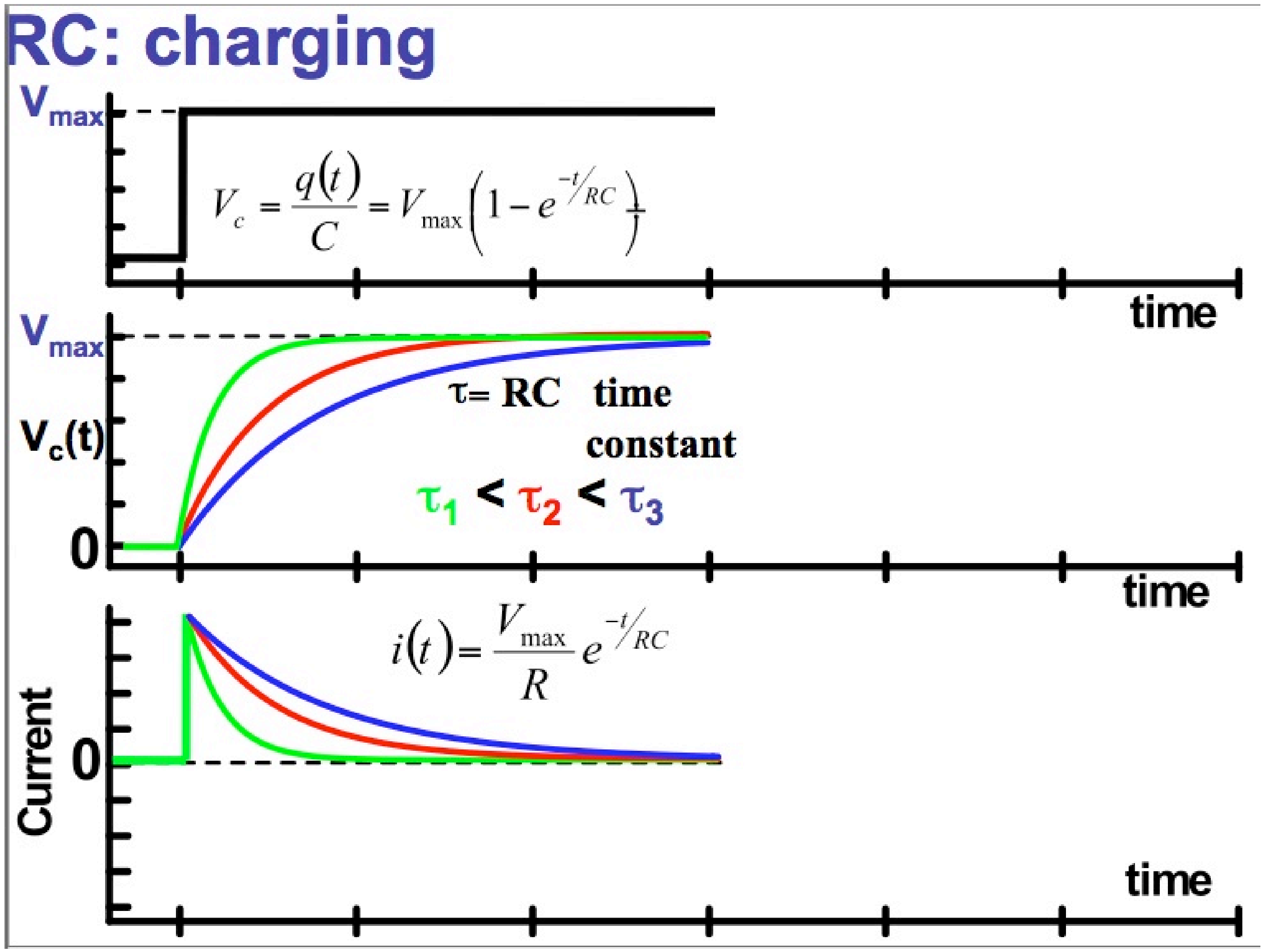
At $t = RC$, $I(t)$ and $\Delta V_R(t)$ go to $1/e$ of the initial values

Time constant τ = RC

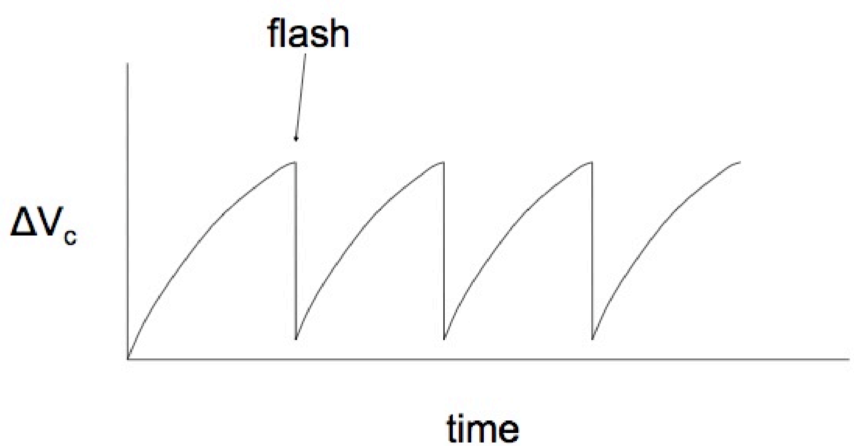
Think about why increasing R and/or C would increase the time to charge up the capacitor:

When charging up: τ will increase with C because the capacitor can store more charge.
Increases with R because the flow of current is lower.

Time constant τ = RC



You want to make a so-called flasher circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges once every 5.0 sec. If you have a 10 microfarad capacitor what resistor do you need?

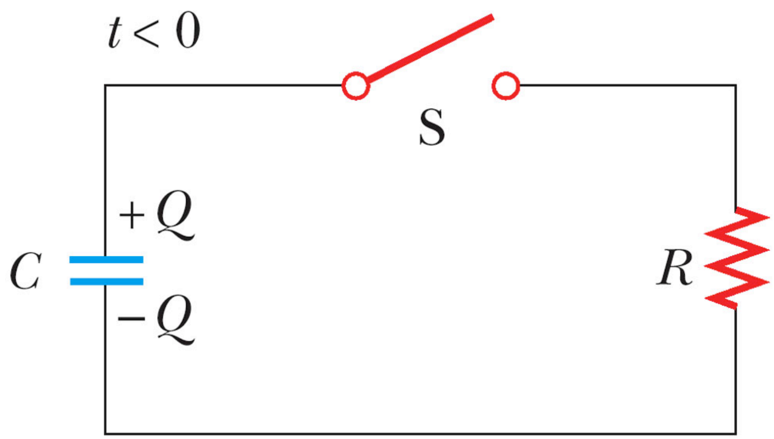


Solution: Have the flash point be equal to 0.63 $\Delta V_{C,max}$

$\tau = RC \;\rightarrow\; R = \tau/C = 5s/10^{-6}F = 5 \times 10^{5}$ Ohms

This is a very big resistance, but 5 seconds is pretty long in "circuit" time

Discharging an RC circuit:

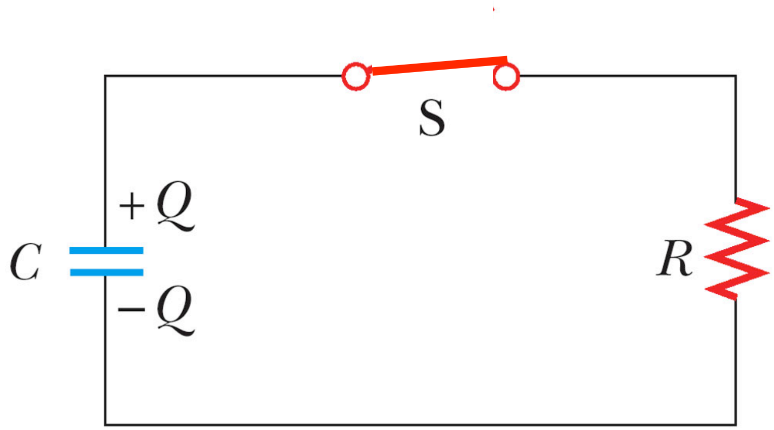


First, disconnect from EMF source.

Q is at maximum value, $Q_{max} = C\varepsilon$

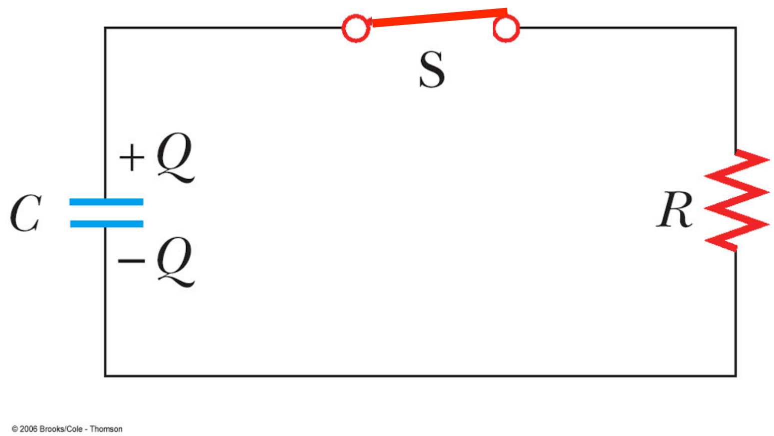
ΔV_C is at maximum value of $\Delta V_{C,max} = \varepsilon$

Discharging an RC circuit:



Then close switch at time t=0.

Discharging an RC circuit:



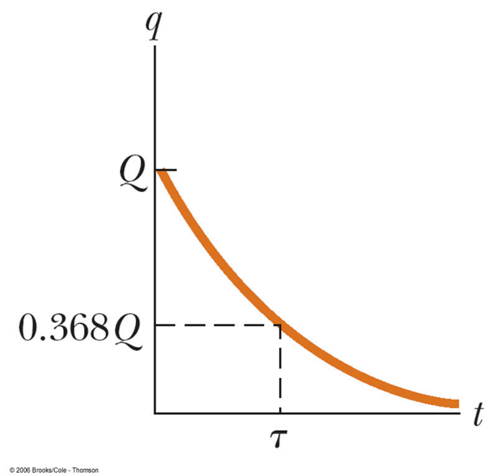
Circuit now has only R and C.

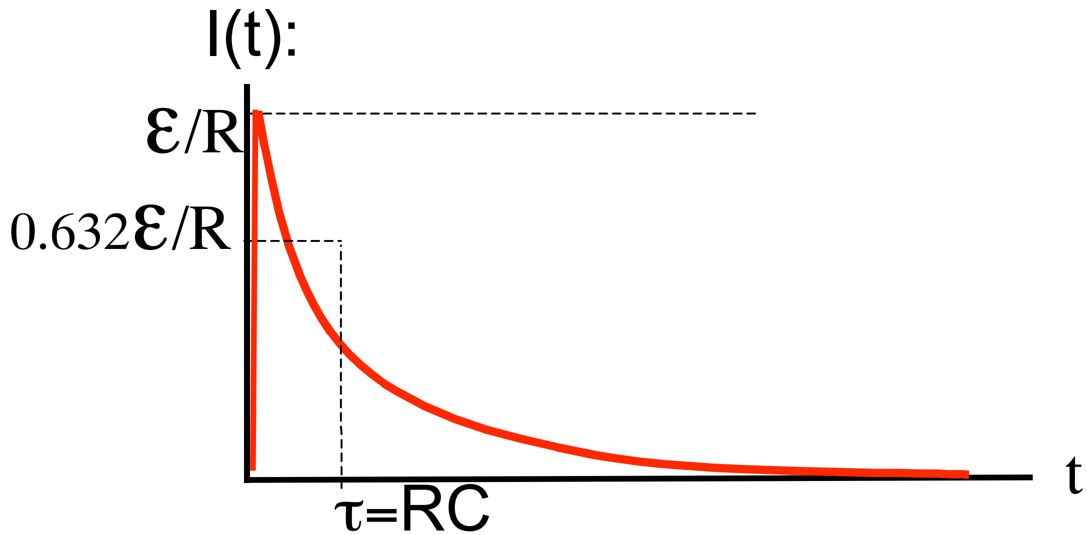
From loop rule: $-\Delta V_C - \Delta V_R = 0$

As capacitor discharges, Q and ΔV_C decrease with time.

ΔV_R will track ΔV_C

So $I = \Delta V_R/R$ will jump from 0 to ε/R at t=0, then exponentially decay





	t=0		$t \rightarrow \infty$
ΔV_C	ε	$\varepsilon(e^{-(t/\tau)})$	0
Q	$Q_{max}=C\varepsilon$	$C\varepsilon(e^{-(t/\tau)})$	0
ΔV_R	ε	$\varepsilon(e^{-(t/\tau)})$	0
I	ε/R	$\varepsilon/R(\ e^{-(t/\tau)})$	0

Think about why increasing R and/or C would increase the time to discharge the capacitor:

τ will increase with C because there is more stored charge in the capacitor to unload. τ increases with R because the flow of current is lower.

Given a 12 μF capacitor being discharged through a 2000 Ω resistor. How long does it take for the voltage drop across the resistor to reach 5% of the initial voltage?

Solution:

First, calculate τ: $\tau = 2000\Omega * 12\text{x}10^{-6}$ F = 24 ms

Then: $V = V_0 \exp(-t/\tau)$

$V / V_0 = \exp(-t/\tau)$

Take ln of both sides: $\ln(V/V_0) = -t/\tau$

Solve for t: $t = -\tau^*\ln(V/V_0) = -0.024\text{ s } (\ln(0.05)) = 0.072$ sec