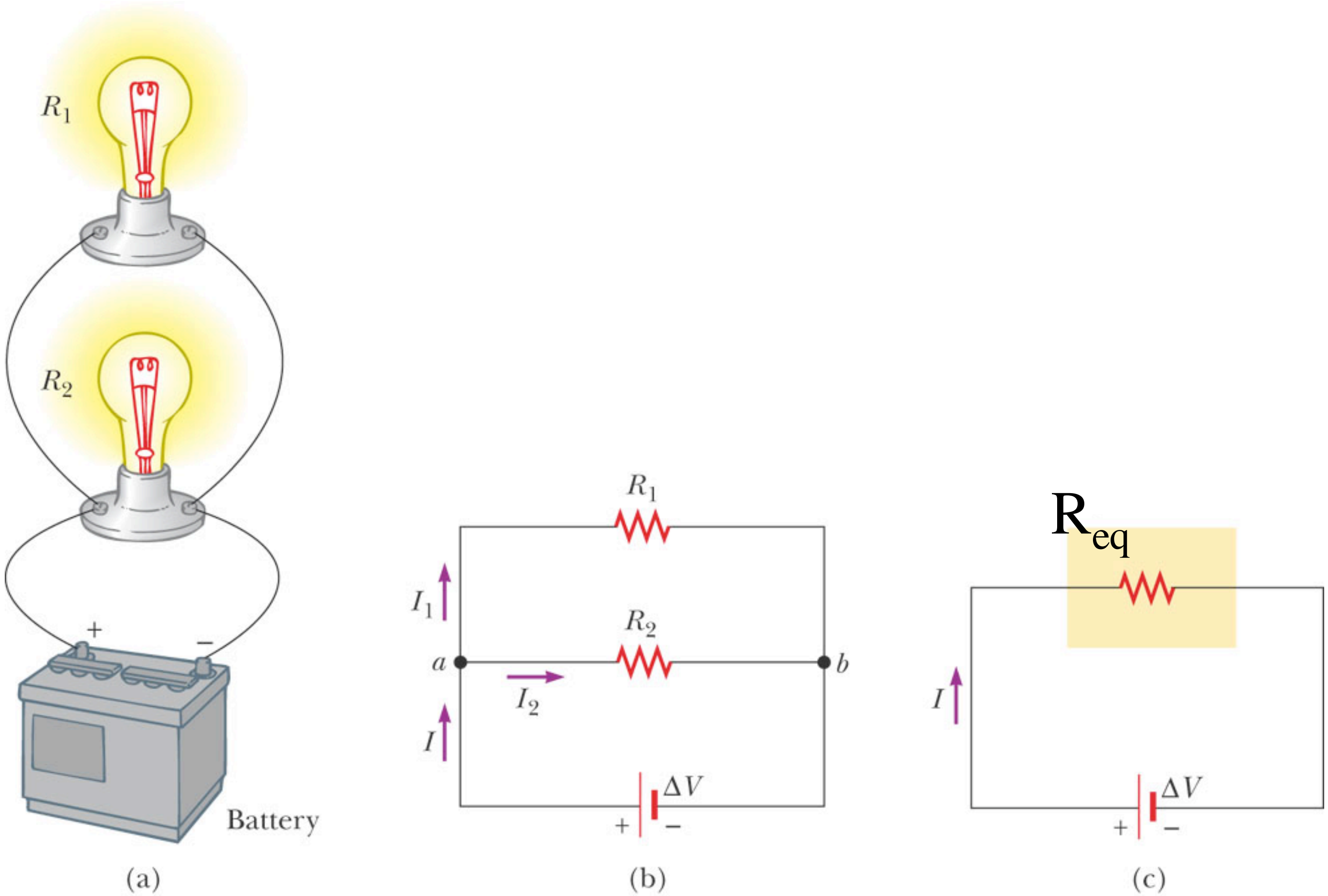
Resistors in Parallel



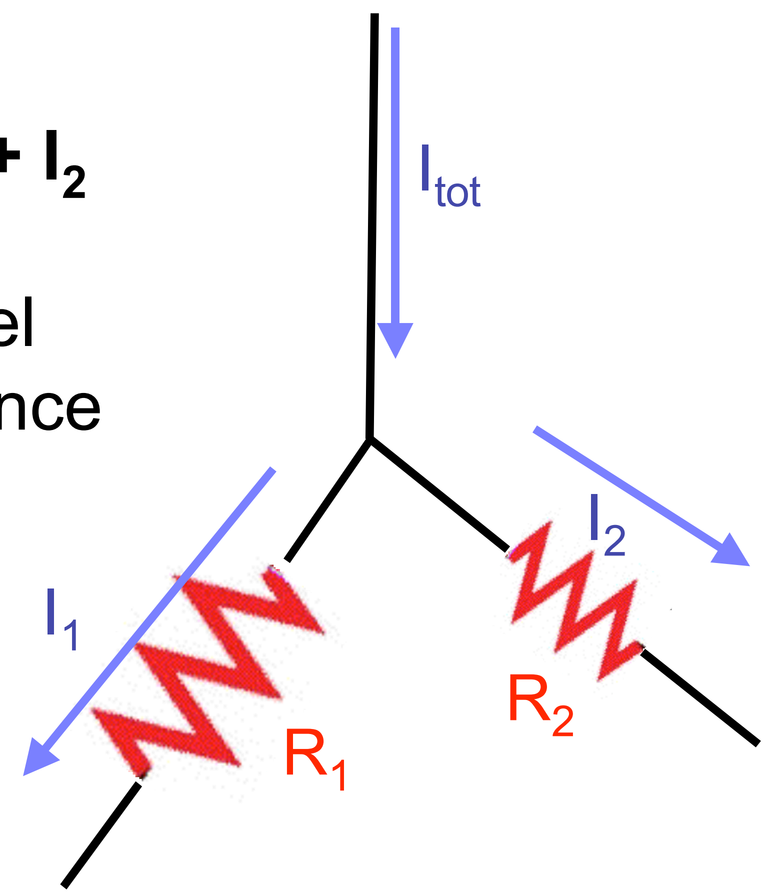
What happens at a junction?

Initial current I_{tot} splits up:
I_1 through R_1 and I_2 through R_2

Charge is conserved: $I_{tot} = I_1 + I_2$

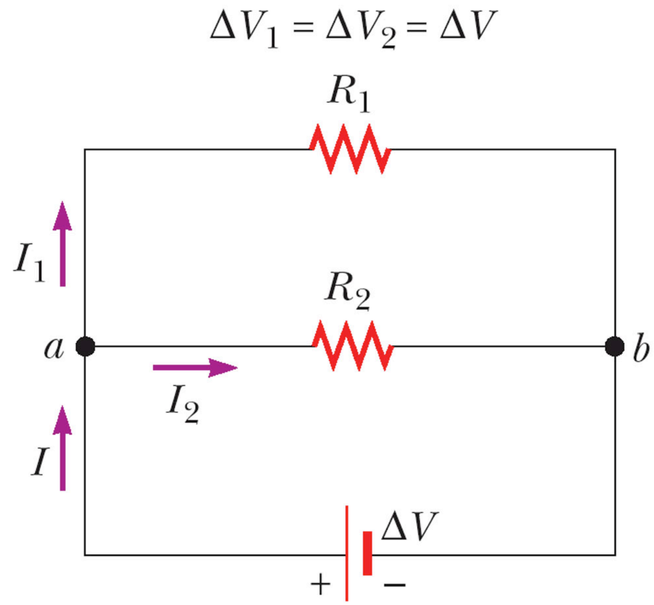
More charge will be able to travel through the path of least resistance

If $R_1 > R_2$, then $I_2 > I_1$



Resistors in Parallel

Note: ΔV across each resistor is the same



$I = I_1 + I_2 = \Delta V / R_1 + \Delta V / R_2 = \Delta V(1/R_1 + 1/R_2)$

$\Delta V = I / (1/R_1 + 1/R_2)$

$\Delta V = I\, R_{eq}$

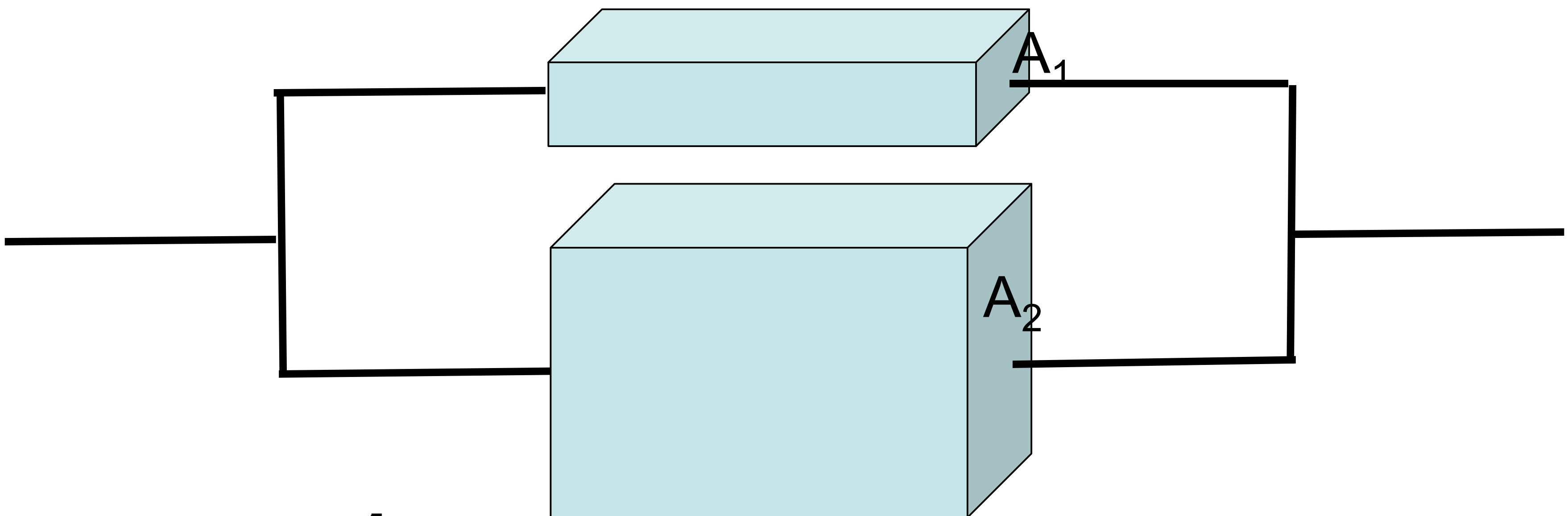
$R_{eq} = 1 / (1/R_1 + 1/R_2)$

$1/R_{eq} = (1/R_1 + 1/R_2)$

For N resistors in parallel:

$1/R_{eq} = 1/R_1 + 1/R_2 + \ldots + 1/R_N$

Understanding the parallel law



$$R = \rho \frac{L}{A}$$

R is prop.to 1/A

$A_{tot} = A_1 + A_2$

A_{tot} prop.to $1/R_1 + 1/R_2$

R_{tot} prop.to $1/A_{tot}$

$1/R_{tot}$ prop.to $1/R_1 + 1/R_2$

Example:

Find the current in each resistor.

$I_1 = \Delta V / R_1$ = 18V/3Ω = 6A

$I_2 = \Delta V / R_2$ = 18V/6Ω = 3A

$I_3 = \Delta V / R_3$ = 18V/9Ω = 2A

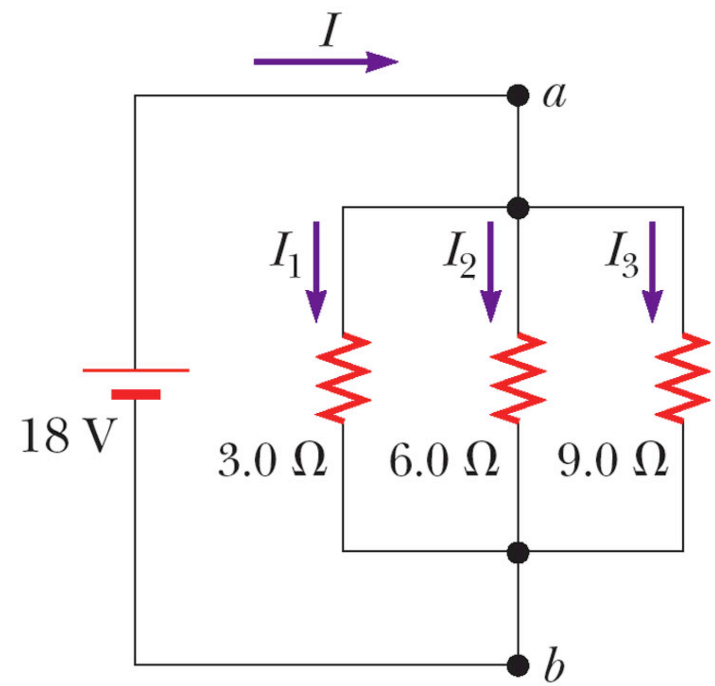
(Total I = 11A)

Find the power dissipated in each resistor:

$P_1 = I_1 \Delta V$ = 6A ×18V = 108 W

$P_2 = I_2 \Delta V$ = 3A ×18V = 54 W

Py = $I_3 \Delta V$ = 2A ×18V = 36 W



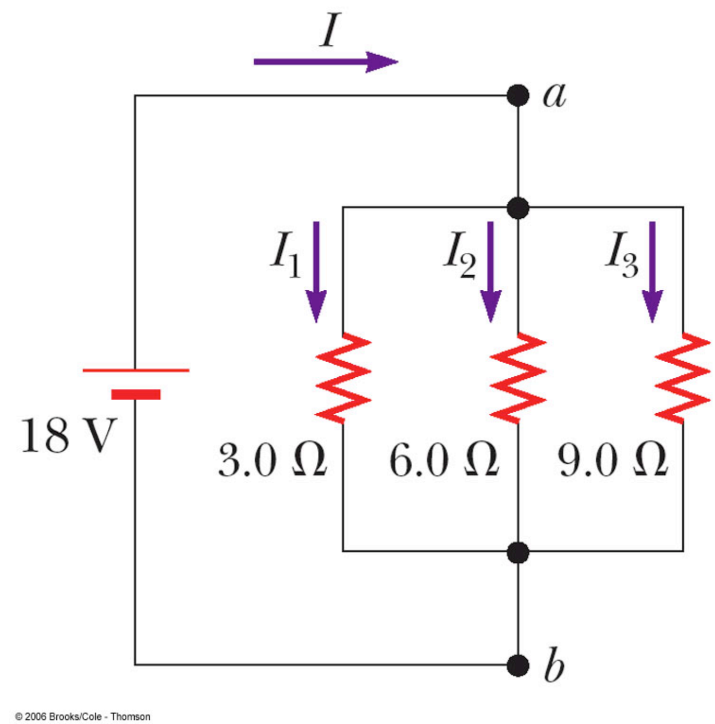
Total P = 198 W

Example:

Find R_{eq}:

$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 =$
$1/(3\Omega) + 1/(6\Omega) + 1/(9\Omega) =$
$11/(18\Omega)$

$R_{eq} = (18/11)\Omega = 1.64\ \Omega$



Find the power dissipated in the equivalent resistor:

$P = (\Delta V)^2/R_{eq} = (18V)^2/1.64\Omega =$
198 W

Also, $P = I\Delta V = 11A \times 18V =$
198W

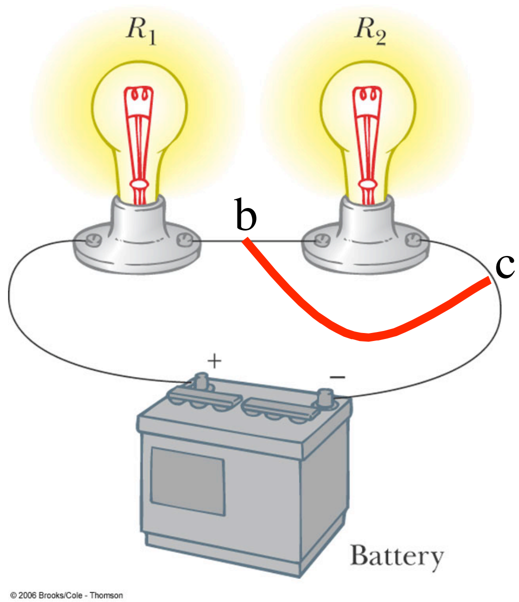
Comparing resistors and capacitors

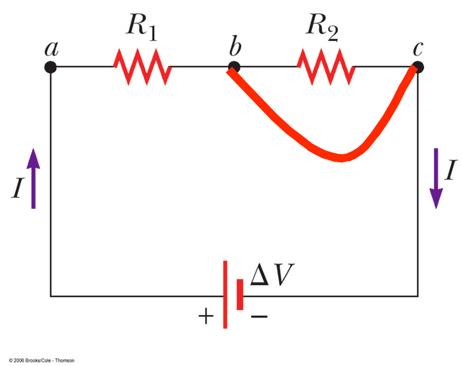
Resistors in series are like capacitors in parallel. Resistors in parallel are like capacitors in series.

$R \propto L$ and $C \propto 1/L$

$R \propto 1/A$ and $C \propto A$

Quiz 18.1





Use a piece of conducting wire to connect points b & c, bypassing R_2.

What happens to the brightness of Bulb 2?

It goes out.

What happens to the brightness of Bulb 1?

$\Delta V = I_{orig}(R_1+R_2)$

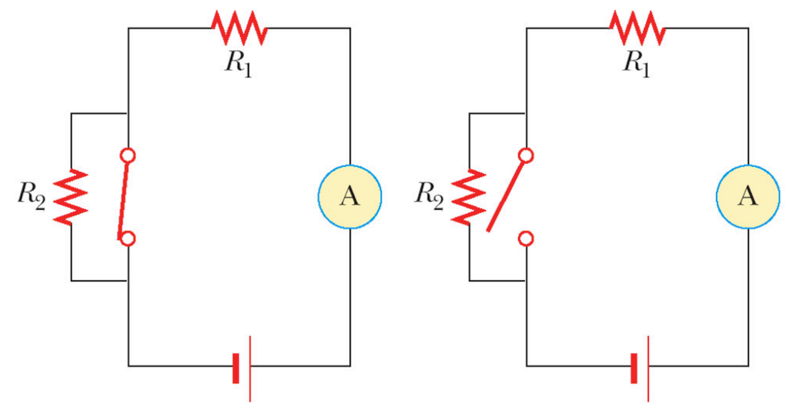
$\Delta V = I_{new}(R_1)$

$I_{new} > I_{orig}$

Brightness of Bulb 1 increases due to increased power due to increased current.

Quiz 18.2:

Current I_{orig} is measured in the ammeter with the switch closed. When the switch is opened, what happens to the reading on the ammeter?



Initially, all current flows through switch, bypassing R_2.
$\Delta V = I_{orig}R_1$

When switch is opened, all current is forced through R_2; we have a circuit with two resistors in series.

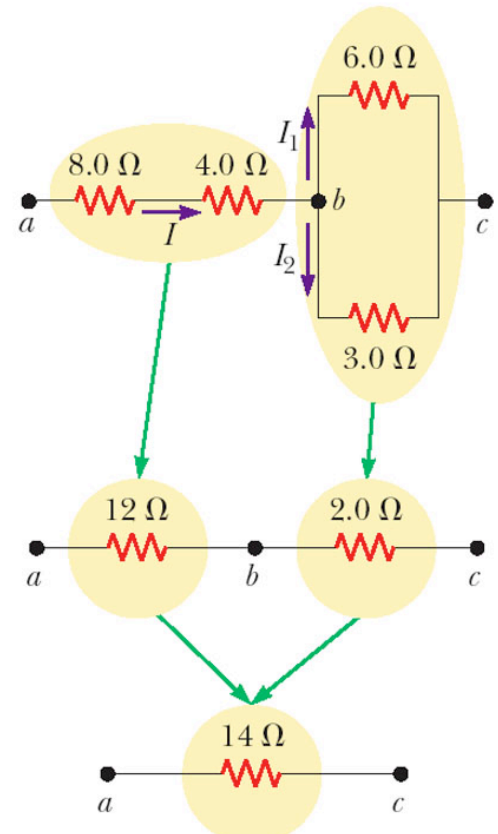
$\Delta V = I_{new}(R_1+R_2) = I_{new}(R_{eq})$

$R_{eq} > R_1$ and ΔV remains fixed, so $I_{new} < I_{orig}$. (current decreases)

What happens when you have resistors in series AND in parallel?

Find R_{eq} through multiple steps:

1. Connect in series
2. Then connect in parallel
3. Repeat #1-2 as needed



Kirchhoff's Rules for Complex DC circuits

Used in analyzing relatively more complex DC circuits

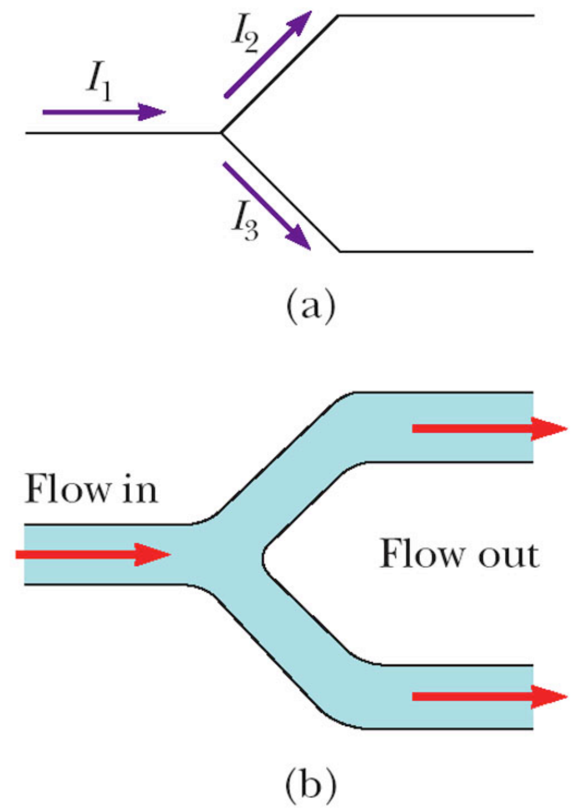
1. Junction rule
2. Loop rule

Junction Rule

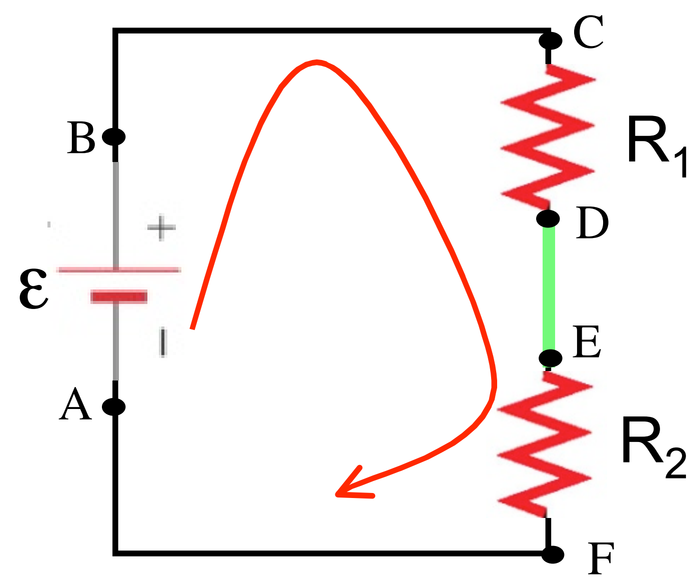
Sum of currents entering any junction must equal the sum of the currents leaving that junction:

$I_1 = I_2 + I_3$

A consequence of conservation of charge (charge can't disappear/appear at a point)



Loop Rule



"The sum of voltage differences in going around a closed current loop is equal to zero"

Stems from conservation of energy

$+\mathcal{E} - IR_1 - IR_2 = 0$

$\mathcal{E} = IR_1 + IR_2$

Application of Loop Rule

Choose a current direction (a to b)

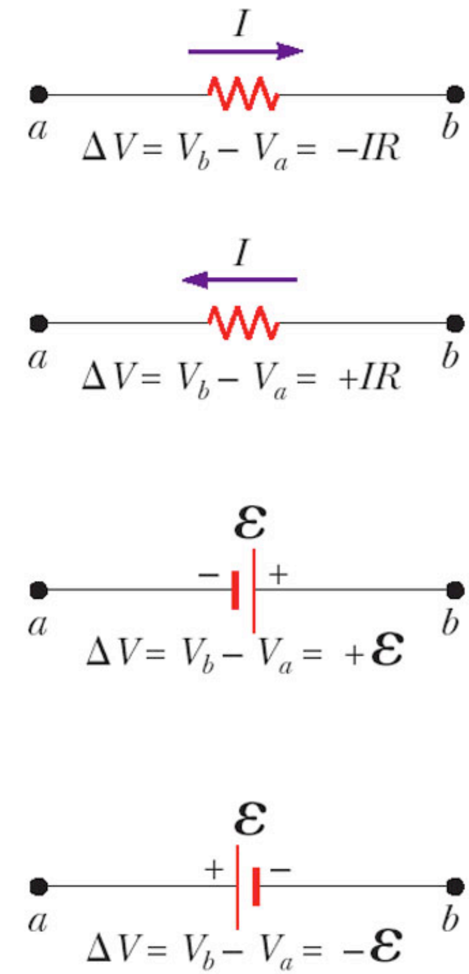
When crossing a resistor: $\Delta V = -IR$ in traversal direction
When crossing a resistor: $\Delta V = +IR$ in opposing direction

When crossing a battery: – to + terminals:

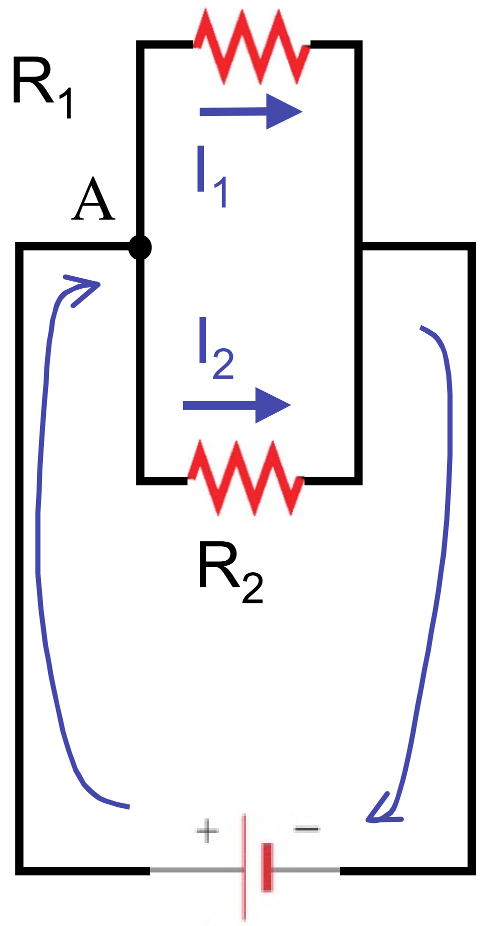
$\Delta V = +\mathcal{E}$

When crossing a battery: + to – terminals:

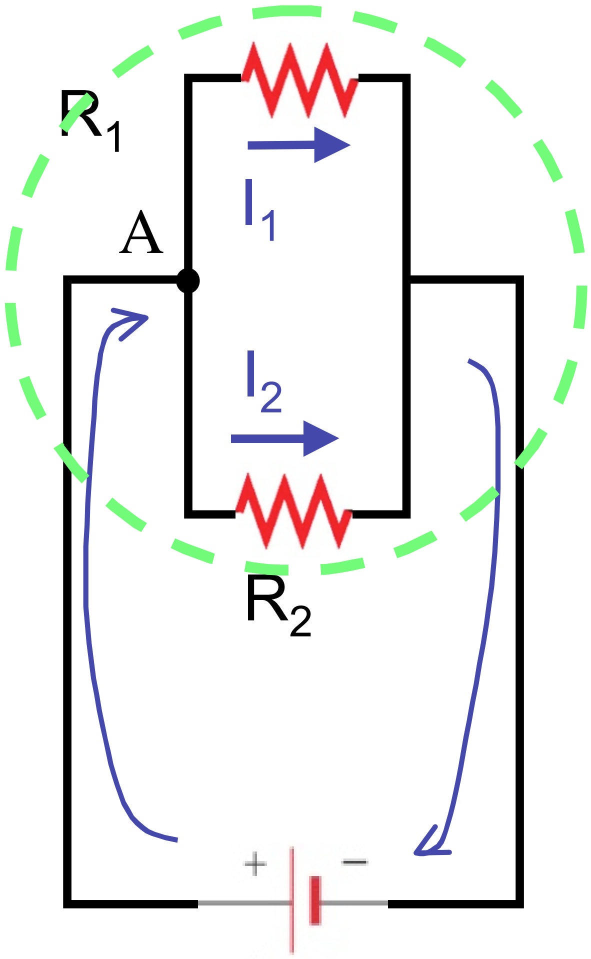
$\Delta V = -\mathcal{E}$



Example of loop/junction rules



Example of loop/junction rules



Loop rule:

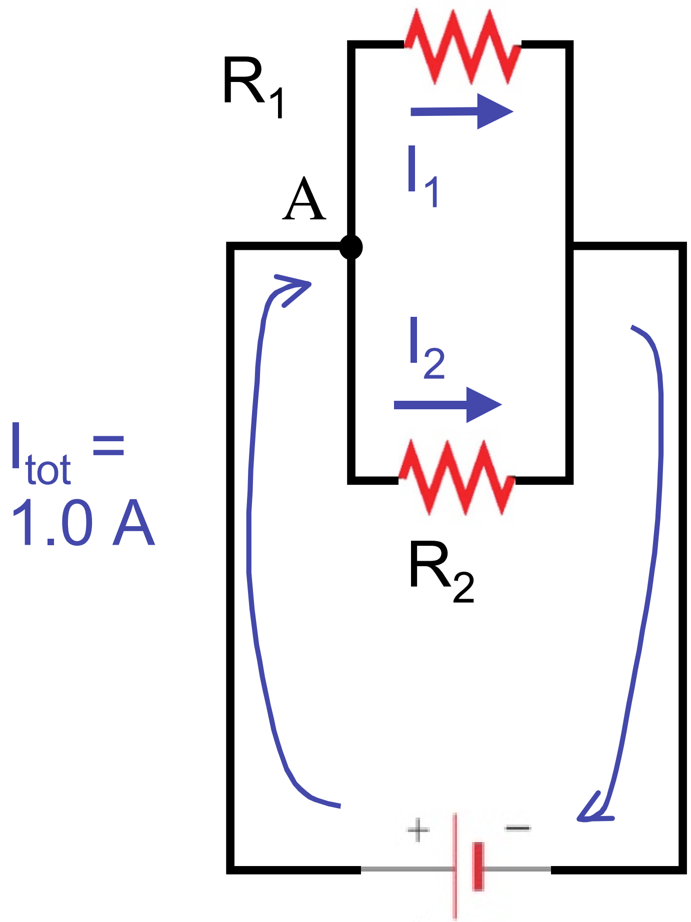
Start at point A, go in CW direction:

$-I_1R_1 + I_2R_2 = 0$

$I_1R_1 = I_2R_2$

$I_1/I_2 = R_2/R_1$

Example of loop/junction rules



Suppose I_{tot} = 1.0 A, R_1 = 3 Ω and R_2 = 6Ω.

Find I_1 & I_2.

$I_1/I_2 = R_2/R_1 = 2$

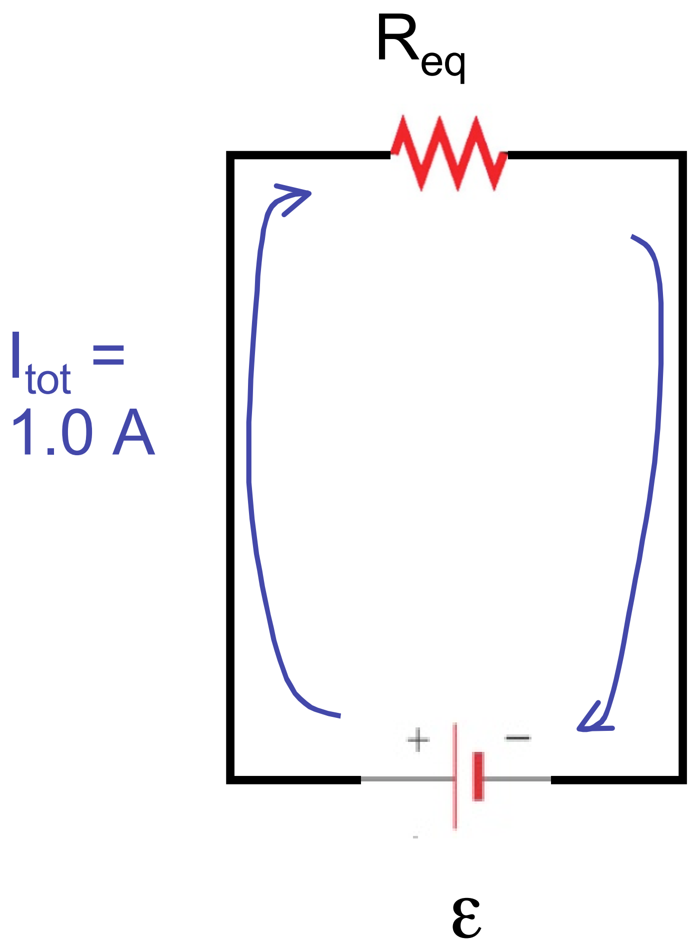
or, $I_1 = 2I_2$

But $I_1 + I_2 = I_{tot}$ = 1.0A.

$2I_2 + I_2$ = 1.0 A

So I_2 = 0.33 A, and I_1 = 0.67 A.

Example of loop/junction rules



Now, calculate ε of the battery.

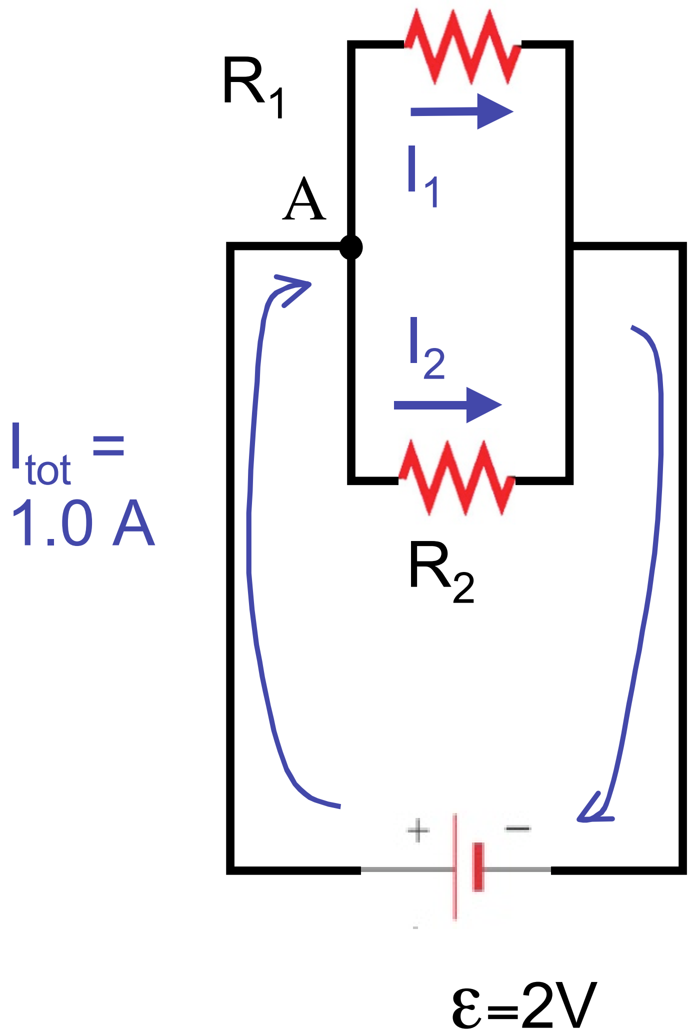
$1/R_{eq} = 1/(3\Omega) + 1/(6\Omega) = 1/(2\Omega)$

$R_{eq} = 2\Omega$

Loop rule for simplified circuit:

$\varepsilon = I_{tot}\, R_{eq} = 1.0\text{ A } 2\Omega = 2.0\text{ V}$

Example of loop/junction rules

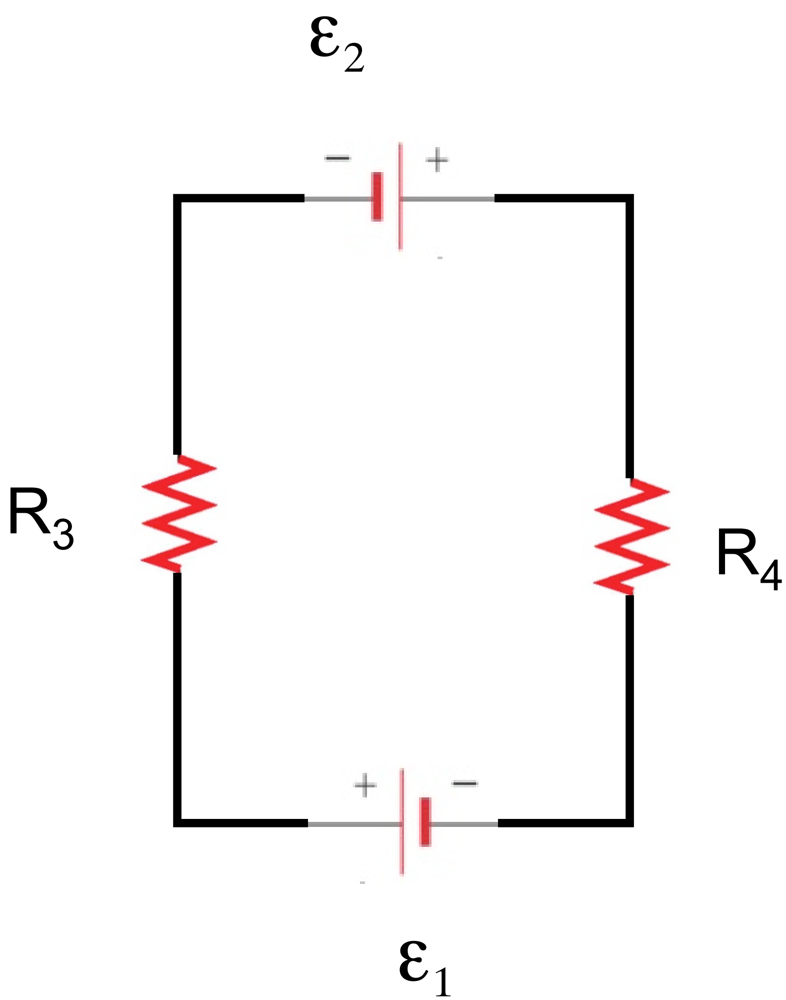


Confirm that the amount of the voltage drop across each resistor is 2V:

$\Delta V_1 = I_1 R_1 = (0.67A)(3\Omega) = 2V$

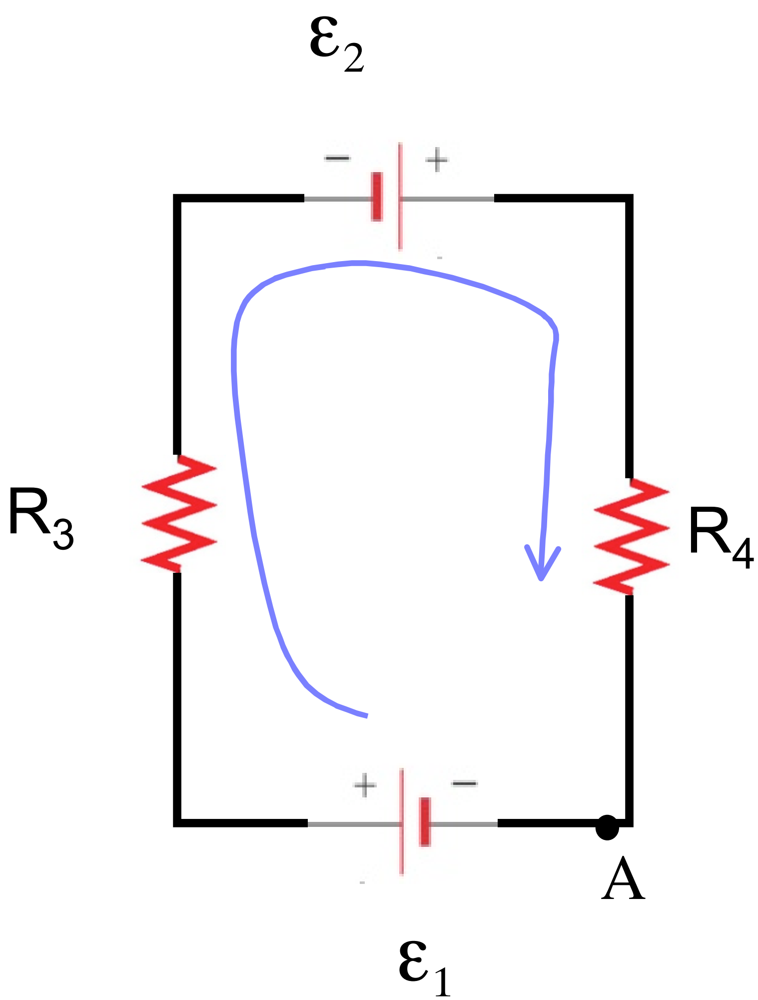
$\Delta V_2 = I_2 R_2 = (0.33A)(6\Omega) = 2V.$

more loop rule



which way will current flow?

more loop rule



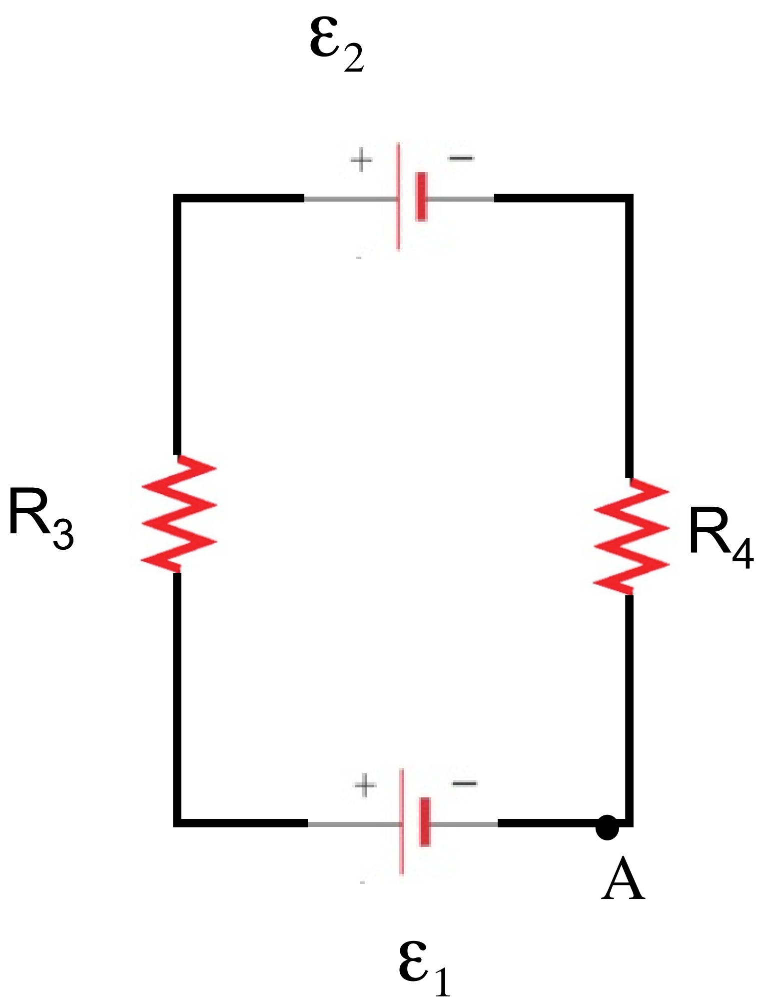
Starting at point A, and going with the current:

$$+\varepsilon_1 - IR_3 + \varepsilon_2 - IR_4 = 0$$

$$+\varepsilon_1 + \varepsilon_2 - IR_4 - IR_3 = 0$$

$$+\varepsilon_1 + \varepsilon_2 = IR_4 + IR_3$$

more loop rule



But watch the direction of EMF in batteries:

Starting at point A, and going with the current:

$$+\varepsilon_1 - IR_3 - \varepsilon_2 - IR_4 = 0$$

$$+\varepsilon_1 - \varepsilon_2 - IR_4 - IR_3 = 0$$

$$+\varepsilon_1 - \varepsilon_2 = IR_4 + IR_3$$

How to use Kirchhoff's Rules

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities
- Assign directions to currents.
- Apply the junction rule to any junction in the circuit
- Apply the loop rule to as many loops as are needed to solve for the unknowns
- Solve the equations simultaneously for the unknown quantities
- <u>Check your answers -- substitute them back into the original equations!</u>