PHYSICS 4E QUIZ 3 SPRING QUARTER 2010
PROF. HIRSCH MAY 7

Formulas and constants:

$hc = 12{,}400\ eV\,A$; $k_B = 1/11{,}600\ eV/K$; $ke^2 = 14.4\,eVA$; $m_e c^2 = 0.511\times10^6 eV$; $m_p/m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3\times10^8 m/s$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Integrals : $I_n \equiv \int_0^\infty x^n e^{-\lambda x^2} dx$; $\frac{dI_n}{d\lambda} = -I_{n+2}$; $I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$; $I_1 = \frac{1}{2\lambda}$; $\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{\pi^4}{15}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67\times10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = hc/4.96k_B$

Compton scattering : $\lambda' - \lambda = \frac{\mathrm{h}}{\mathrm{m_e}c}(1-\cos\theta)$; $\lambda_c \equiv \frac{\mathrm{h}}{\mathrm{m_e}c} = 0.0243A$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $U = q_0V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0\frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $E_0 = \frac{ke^2}{2a_0} = 13.6eV$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2Z}{r^2} = m\frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass : $\mu = \frac{mM}{m+M}$; X - ray spectra : $f^{1/2} = A_n(Z-b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973\ eV\,A$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $<A> = \int_{-\infty}^{\infty} dx\psi^* A_{op}\psi$

Eigenvalues and eigenfunctions : $A_{op}\Psi = a\Psi$ (a is a constant) ; uncertainty : $\Delta A = \sqrt{<A^2> - <A>^2}$

Harmonic oscillator : $\Psi_n(x) = C_n H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2$; $\Delta n = \pm1$

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Step potential: $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\dfrac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x)dx}$; $\alpha(x) = \sqrt{\dfrac{2m[V(x) - E]}{\hbar^2}}$

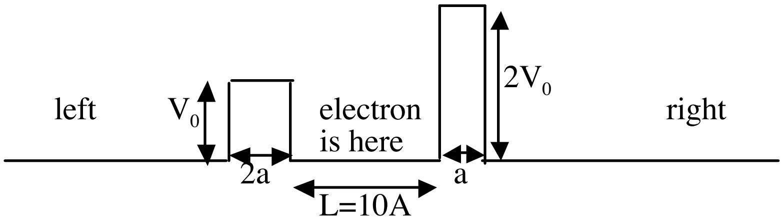
3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \dfrac{\pi^2\hbar^2}{2m}\left(\dfrac{n_1^2}{L_1^2} + \dfrac{n_2^2}{L_2^2} + \dfrac{n_3^2}{L_3^2}\right)$

Justify all your answers to all problems. Write clearly.

Problem 1 (10 pts)
An electron is in the ground state of an infinite one-dimensional well of width 8A.
(a) Estimate the probability that the electron is in a region within 1A of the center of the well, and compare with the classical answer.
(b) If you were to calculate this probability exactly, would it be larger or smaller than the answer you gave in (a)? Justify.
(c) Give the uncertainty in the momentum of this electron, Δp, in units eV/c.
(d) For extra credit, do after you finish the rest of the quiz: calculate the probability in (a) exactly.

Problem 2 (10 pts)
The ground state energy of an electron in a harmonic oscillator potential is 3eV.
(a) What is the wavelength of photons emitted and absorbed by this system, in A?
(b) What is the classical amplitude of oscillation (=classical turning point) when the electron is in the ground state, in Angstrom? Use $\hbar^2/2m_e = 3.81 eV A^2$
(c) How much more likely is it to find the electron in the ground state at position x=0 than at a classical turning point?

Problem 3 (10 pts)



An electron is in the one-dimensional well of length L=10A shown above, bounded by the two barriers of height V_0 and $2V_0$ and widths 2a and a respectively.
(a) For what energy range of this electron is it more likely to escape the well through the right barrier than through the left barrier? Give your answer in terms of V_0.
(b) Assuming you can approximate the energy levels of the well by the infinite well energy levels, for what quantum number n is the electron equally likely to escape through the right as through the left barrier, for V_0=127eV?
(c) If you now took into account that the energy levels in a finite well are not exactly the same as for the infinite well, would the particle in the level n found in (b) be more likely to escape through the right or the left barrier? Justify your answer. You can answer this (with justification) even if you didn't find the answer to (b).

Justify all your answers to all problems. Write clearly.