Physics 2B

Lecture 27A

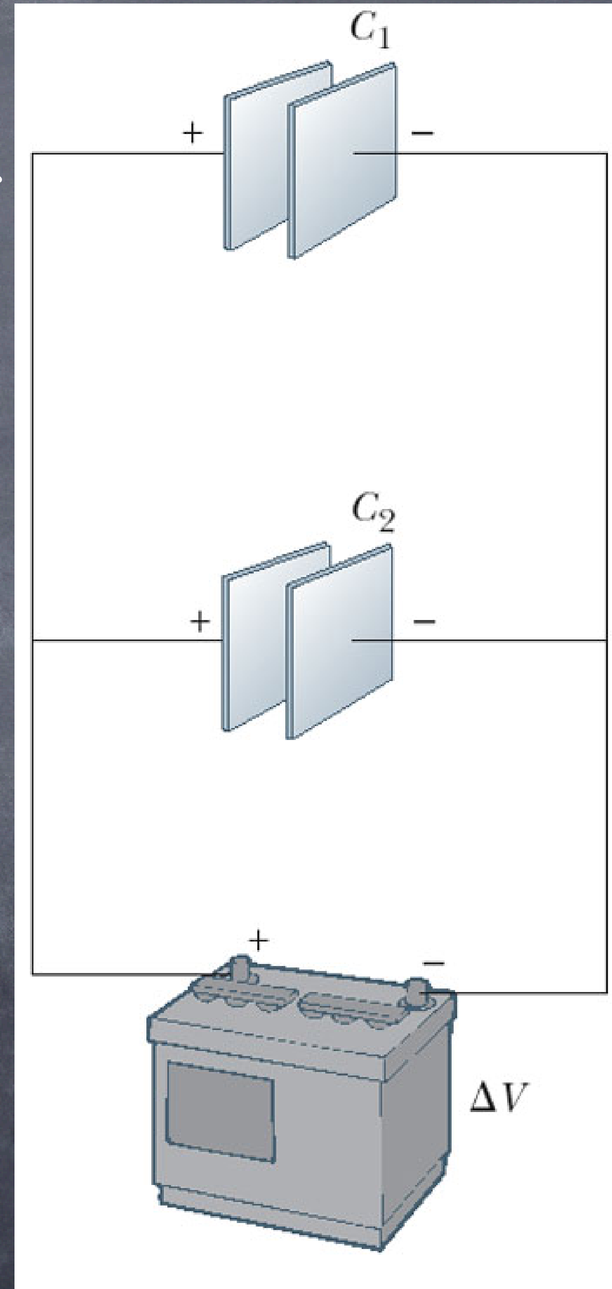
"Success is the ability to go from one failure to another with no loss of enthusiasm."
--Winston Churchill

Capacitors

- Capacitors are usually used in circuits.
- A circuit is a collection of objects usually containing a source of electrical energy (like a battery).
- This energy source is connected to elements (like capacitors) that convert the electrical energy to other forms.
- We usually create a circuit diagram to represent all of the elements at work in the real circuit.

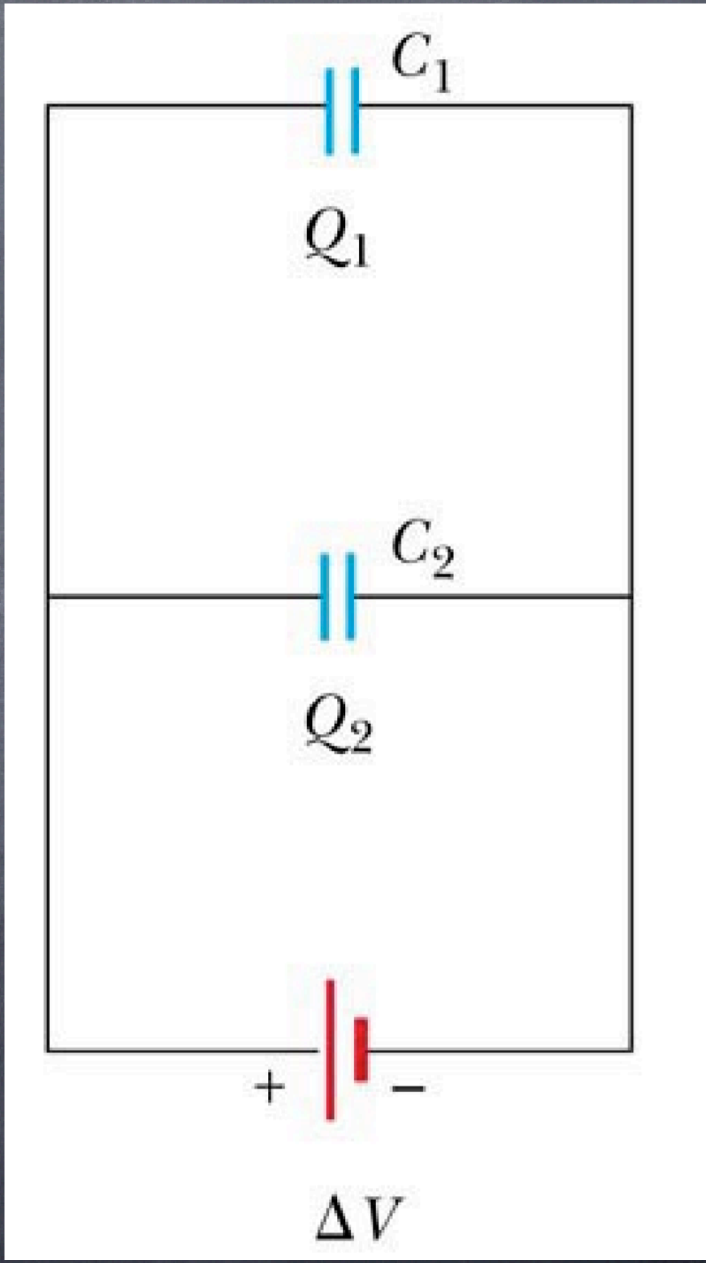
Circuits

- For example, let's say that we had two capacitors connected in parallel to a battery.
- In the circuit diagram we would represent a capacitor with a parallel line symbol: ||
- Also, in the circuit diagram we would represent a battery with a short line and a long line: ı|
- The short line representing the negative terminal and the long line representing the positive terminal.



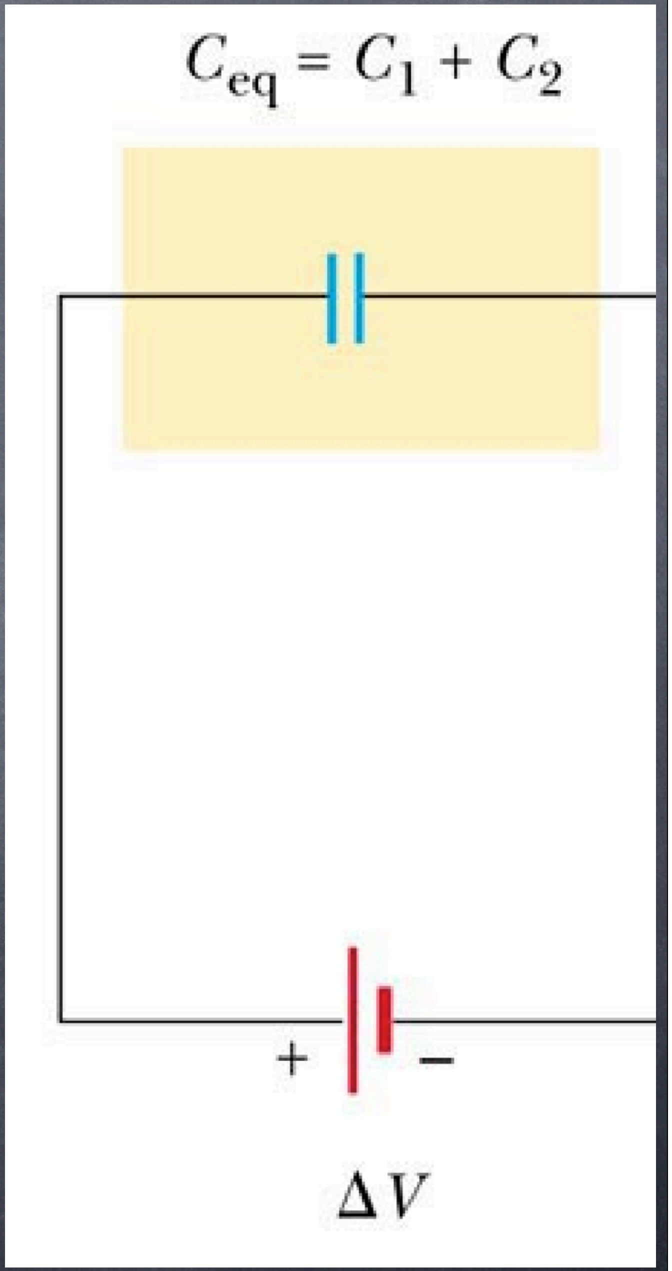
Capacitors

- The previous real circuit can then be drawn as a circuit diagram as follows:
- In this circuit, both capacitors would have the same potential difference as the battery.
- $$\Delta V_{bat} = \Delta V_1 = \Delta V_2$$
- Plus, we can say that the charges on either plate are equal to the total that passes through the battery.
- $$Q_{Tot} = Q_1 + Q_2$$



Capacitors

- We can essentially replace the two capacitors in parallel with one equivalent capacitor (this may make our life easier).
- The equivalent capacitor must have exactly the same external effect on the circuit as the original capacitors.
- The battery sees Q_{Tot} passing through it and believes that the one equivalent capacitor has a potential difference of ΔV.



Capacitors

- So, to the battery the equivalent capacitance is:

$$C_{eq} = \frac{Q_{Tot}}{\Delta V_{bat}}$$

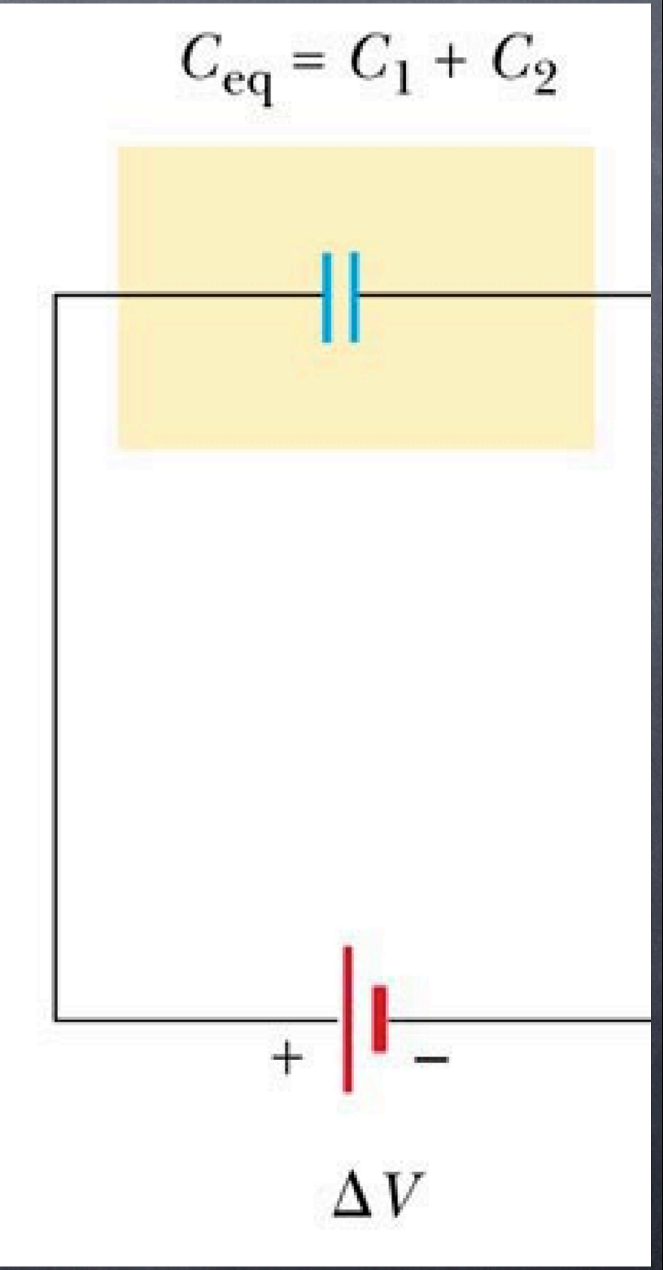
$$C_{eq} = \frac{Q_1 + Q_2}{\Delta V_{bat}} = \frac{Q_1}{\Delta V_1} + \frac{Q_2}{\Delta V_2}$$

$$C_{eq} = C_1 + C_2$$

The equivalent capacitance of capacitors in parallel is greater than the individual capacitors.

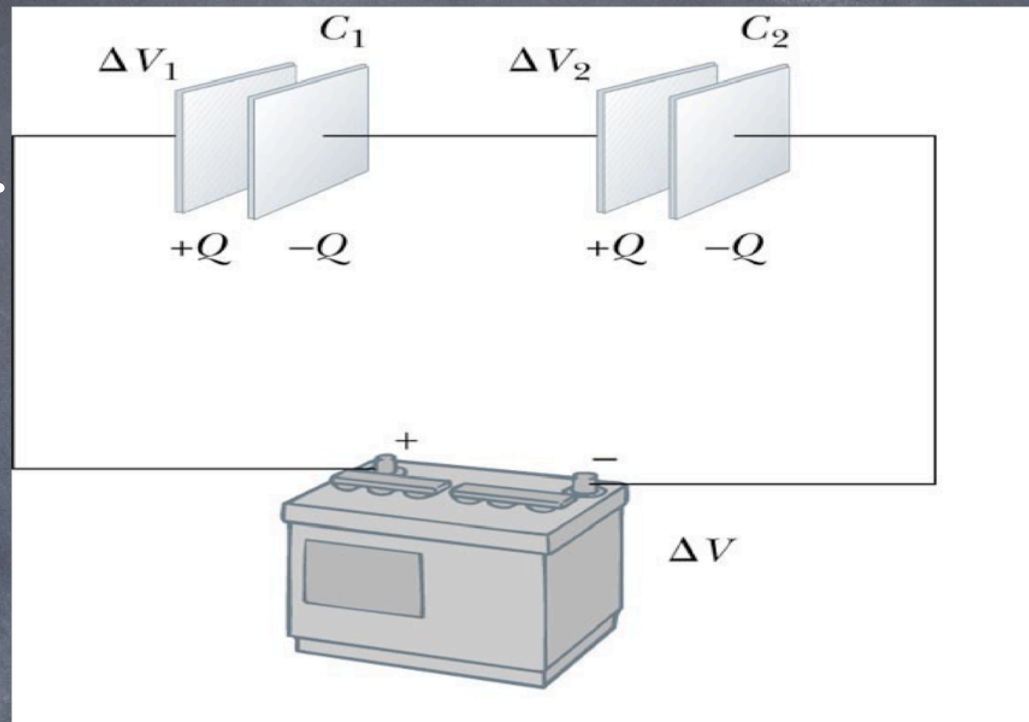
In general, for parallel capacitors:

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$



Capacitors

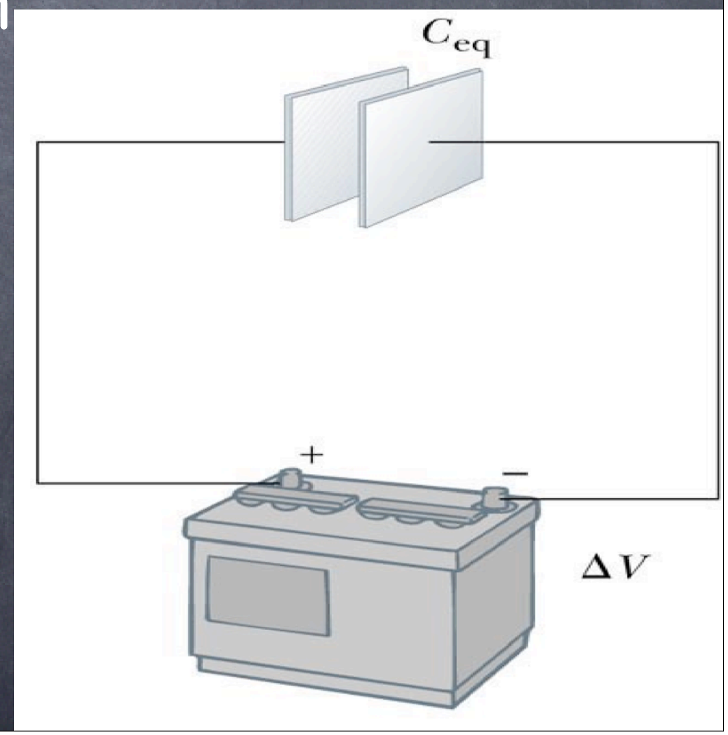
- We can also have two capacitors hooked up in series to a battery.



- In this case the potential difference across the battery is no longer equivalent to the potential difference across either capacitor.
- But since the right C_1 plate gets its negative charges from the left C_2 plate, we can say that the charge on either capacitor must be the same.

Capacitors

- This means that the charge passing through the battery is equivalent to the charge on either capacitor.
- $$Q_{Tot} = Q_1 = Q_2$$
- Also, the potential difference across either capacitor will sum to be the potential difference across the battery.
- $$\Delta V_{bat} = \Delta V_1 + \Delta V_2$$
- We can essentially replace the two capacitors in series with one equivalent capacitor.



Capacitors

- The battery sees Q_{Tot} passing through it and believes that the one equivalent capacitor has a potential difference of ΔV_{bat}.

- So, to the battery the equivalent capacitance is:

$$C_{eq} = \frac{Q_{Tot}}{\Delta V_{bat}}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V_{bat}}{Q_{Tot}}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V_1 + \Delta V_2}{Q_{Tot}} = \frac{\Delta V_1}{Q_1} + \frac{\Delta V_2}{Q_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

- In general, for series capacitors:

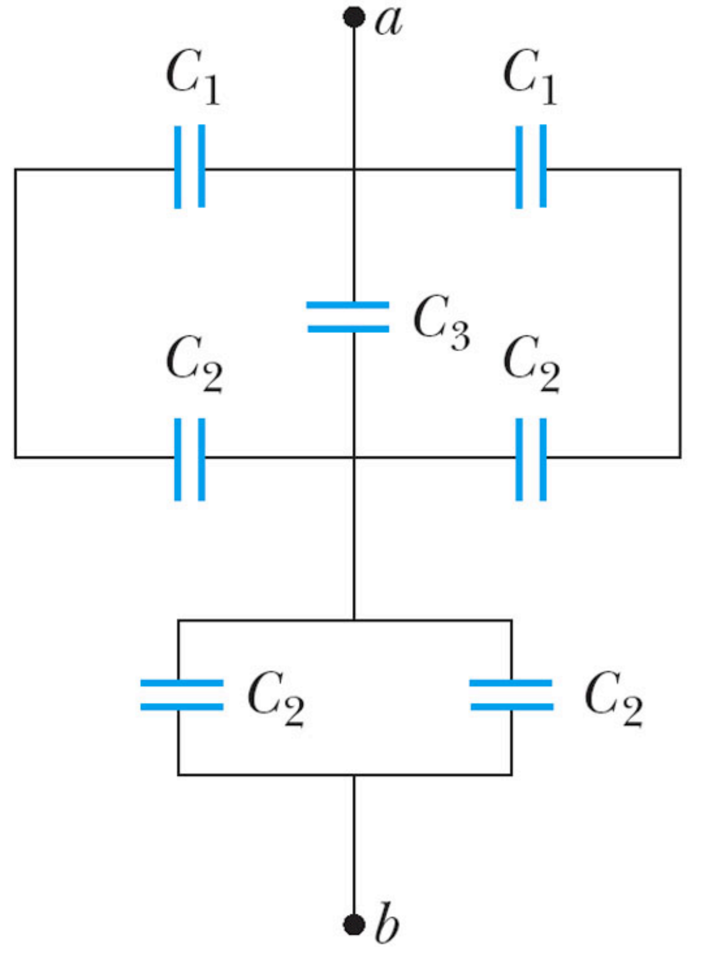
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Capacitors

- Remember when you have multiple capacitors in a circuit that:
- Capacitors in parallel all have the same potential differences.
- The equivalent capacitance of the parallel capacitors also will have the same potential difference.
- Capacitors in series all have the same charge.
- The equivalent capacitor of the series capacitors also will have the same charge.

Capacitance

- Example
- Find the equivalent capacitance between points a and b in the group of capacitors connected in series as shown in the figure to the right (take C_1=5.00μF, C_2=10.0μF, and C_3=2.00μF). If the potential between points a and b is 60.0V, what is the charge stored on C_3?



- Answer
- Reduce the circuit by equivalent capacitance to find Q_3.
- Start with the C_1 and C_2 in series on the top.

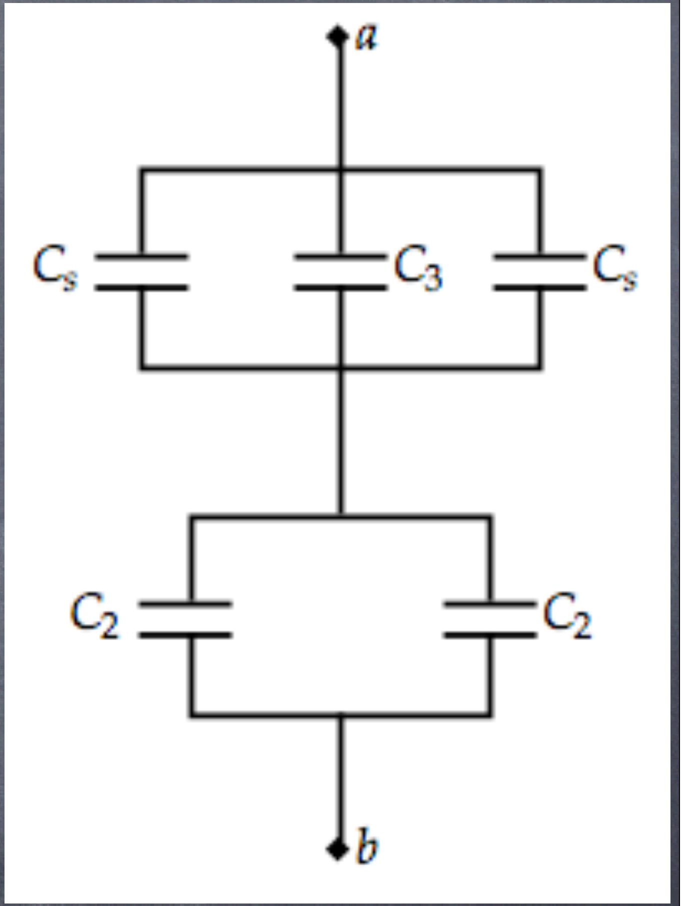
Capacitance

- Answer
- Combine the two top capacitors that are in series (left and right are the same) to form an equivalent C_S:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5\mu\text{F}} + \frac{1}{10\mu\text{F}}$$

$$\frac{1}{C_S} = \frac{2}{10\mu\text{F}} + \frac{1}{10\mu\text{F}} = \frac{3}{10\mu\text{F}}$$

$$C_S = \frac{10\mu\text{F}}{3} = 3.33\mu\text{F}$$



- Next, combine the top three that are in parallel:

$$C_{\text{P1}} = C_S + C_3 + C_S$$

$$C_{\text{P1}} = 3.33\mu\text{F} + 2.00\mu\text{F} + 3.33\mu\text{F} = 8.66\mu\text{F}$$

Capacitance

- Answer
- Next, combine the bottom two that are in parallel:

$$C_{P2} = C_2 + C_2$$

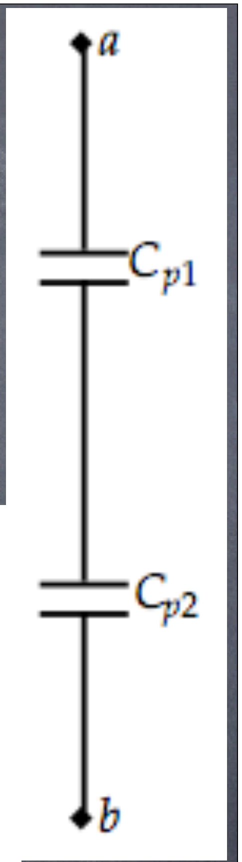
$$C_{P2} = 10.0\mu\text{F} + 10.0\mu\text{F} = 20.0\mu\text{F}$$

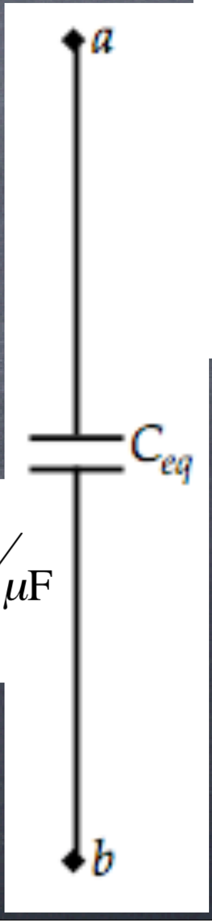
- Finally, combine the remaining two capacitors that are in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_{P1}} + \frac{1}{C_{P2}} = \frac{1}{8.66\mu\text{F}} + \frac{1}{20\mu\text{F}}$$

$$\frac{1}{C_{eq}} = (0.1155)\ {}^{1}\!/_{\mu\text{F}} + (0.0500)\ {}^{1}\!/_{\mu\text{F}} = (0.1655)\ {}^{1}\!/_{\mu\text{F}}$$

$$C_{eq} = 6.04\mu\text{F}$$





Capacitance

- <u>Answer</u>
- Next, we need to find the total charge stored on the equivalent capacitor:

$$Q_{eq} = C_{eq}(\Delta V_{ab})$$

$$Q_{eq} = 6.04\,\mu\text{F}(60\text{V}) = 362\,\mu\text{C}$$

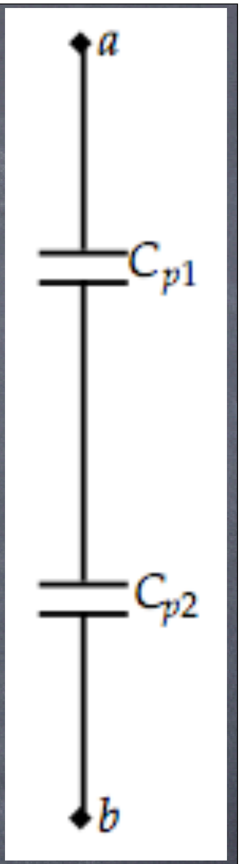
- Looking back at the two equivalent capacitors in series (C_{p1} and C_{p2}), we see that they must have the same amount of charge:

$$Q_{eq} = Q_{P1} = Q_{P2} = 362\,\mu\text{C}$$

- This means that they potential difference across C_{p1} is:

$$\Delta V_{P1} = \frac{Q_{P1}}{C_{P1}}$$

$$\Delta V_{P1} = \frac{362\,\mu\text{C}}{8.66\,\mu\text{F}} = 41.8\text{V}$$



Capacitance

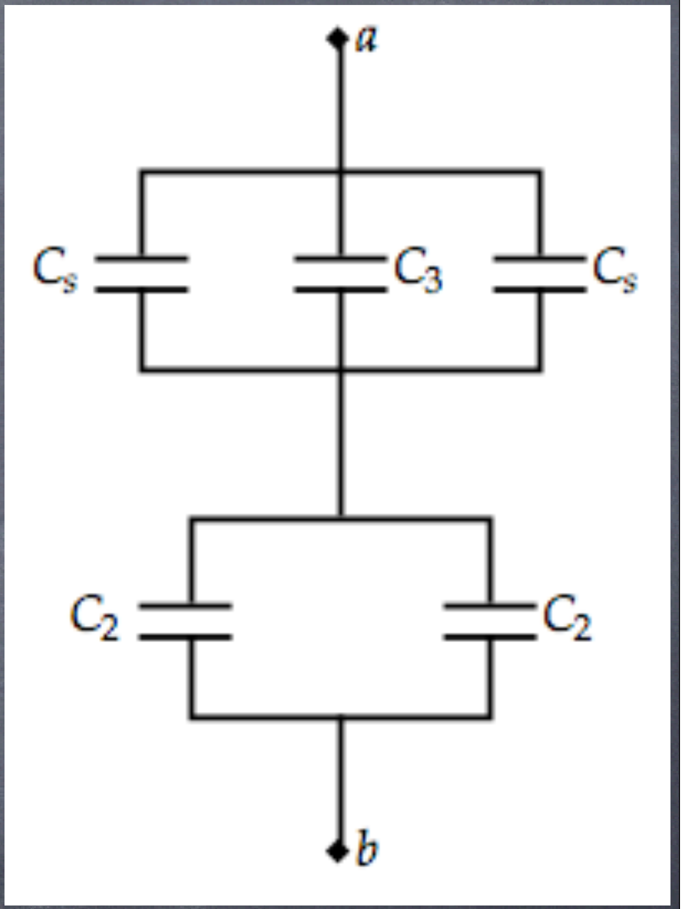
- Answer
- We turn back to the three top capacitors in parallel (C_S, C_3, and C_S).
- We note that all three of these capacitors must have the same potential difference as each other (and C_{p1}).

$$\Delta V_{C3} = 41.8\text{V}$$

- This means that the charge on C_3 is:

$$Q_3 = C_3(\Delta V_{C3})$$

$$Q_3 = 2.00\mu\text{F}(41.8\text{V}) = 83.6\mu\text{C}$$



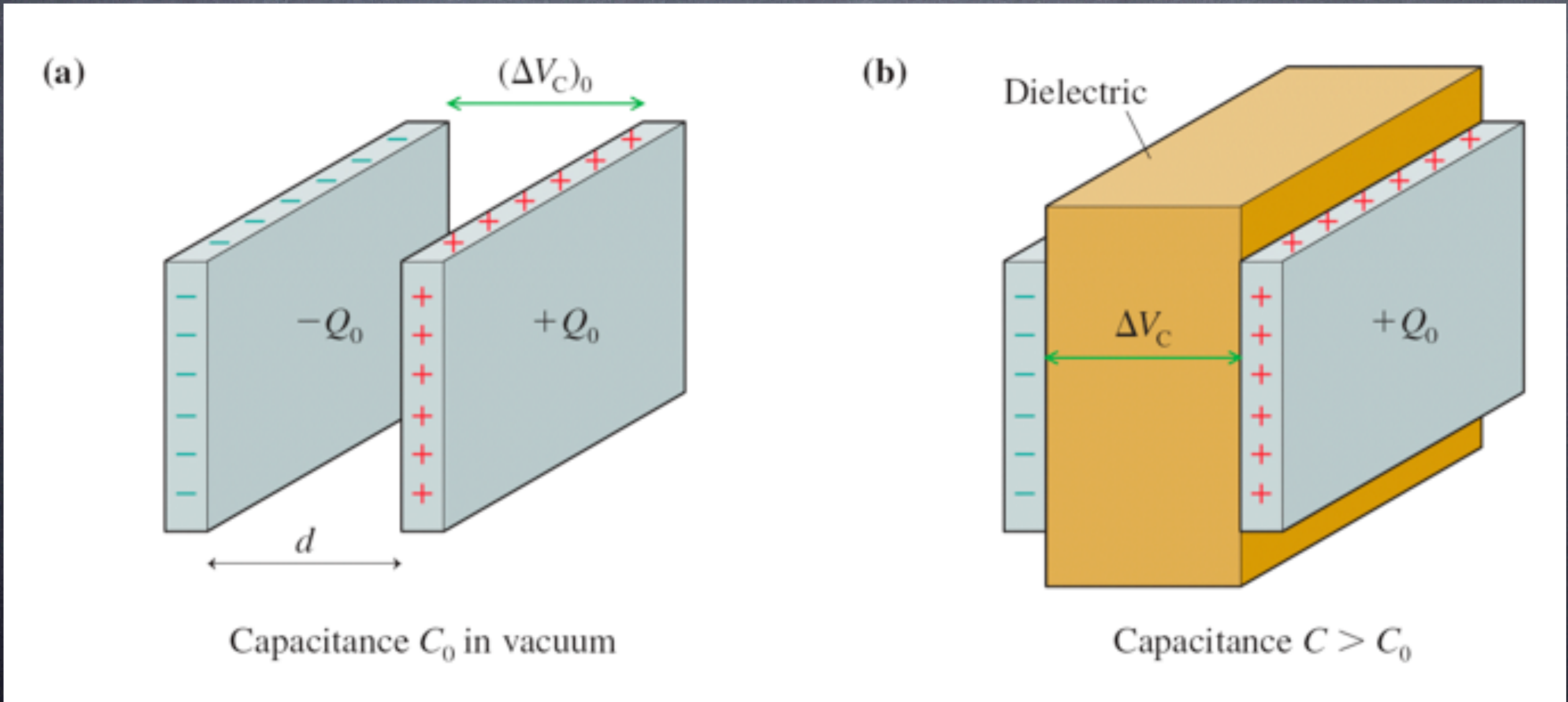
Dielectrics

- The problem with capacitors is that they need to have huge dimensions to carry a significant amount of charge.
- Cost of material and manufacturing become a problem.
- The solution is to substitute an electrically insulating material between the parallel-plates instead of air or a vacuum.
- This is known as a dielectric.
- When inserted into the capacitor the dielectric will increase the overall capacitance.

Dielectrics

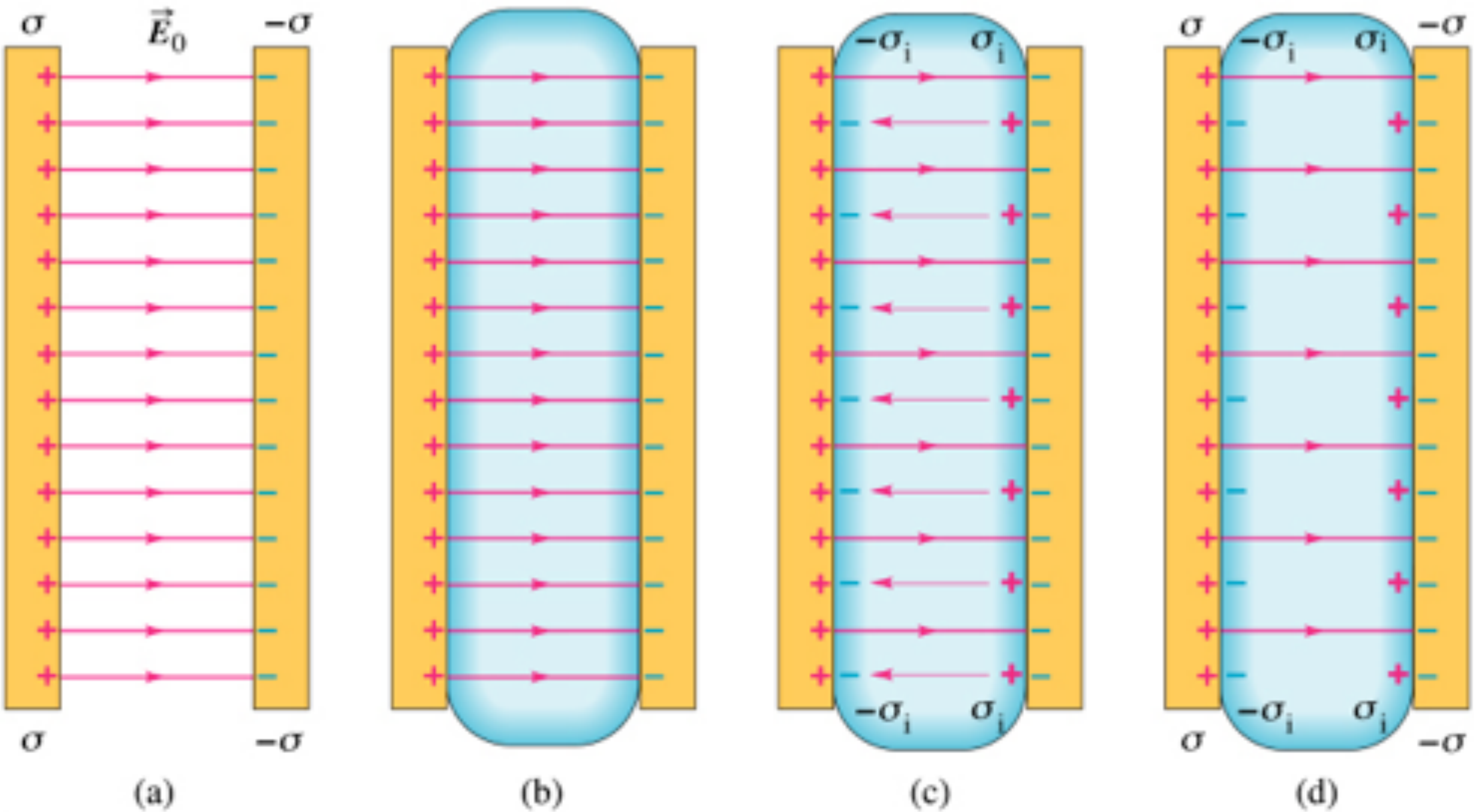
- The dielectric constant, κ, is the ratio of the new capacitance to the capacitance in a vacuum:

$$\kappa = \frac{C_{dielectric}}{C_{vacuum}}$$



Dielectrics

- Easily polarized materials have larger dielectric constants than materials not easily polarized.



Dielectrics

- Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant.
- The capacitance for a parallel-plate capacitor changes to:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

  - Common dielectric values:
  - $K_{vacuum} = 1$
  - $K_{air} = 1.0006$
  - $K_{glass} \approx 7$
- Note that the dielectric constant is a unitless variable.

For Next Time (FNT)

- Start reading Chapter 27
- Keep working on the homework for Chapter 26