Combinations of lenses

- When two lenses are used in combination, the image of the first lens is the object for the second lens.
- The total magnification is the product of the magnifications of the first and second lens.

Find the image formed by two lenses in combination

Image formed by lens 1

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30)(10)}{30-10} = 15\text{cm}$$

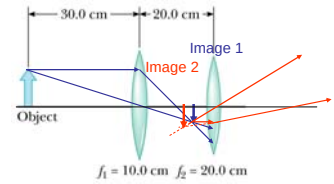
from lens 1

Image formed by lens 2

$p_2 = 20 - q_1 = 20 - 15 = 5$ cm

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(5)(20)}{5-20} = -6.7\text{cm}$$

from lens 2



Magnification

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{15}{30}\right)\left(-\frac{-6.7}{5}\right) = -0.67$$

Inverted
Reduced

Lens Power

The power of a lens is defined as

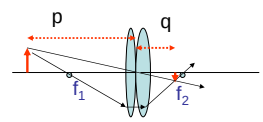
$$P = \frac{1}{f}$$ Units of diopter (m^{-1})

where f is the focal length of the lens in meter.

A lens with a focal length of -20 cm has a power of -5.0 diopters.

Powers are often used by optometrists to describe eye glass lenses

For two lenses in contact the total power is the sum of powers of the individual lenses



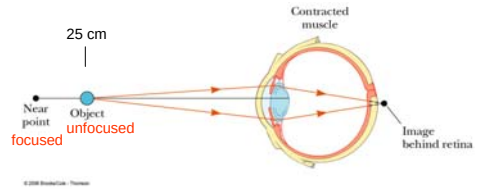
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

Defects in vision

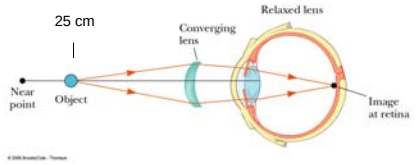
Nearsightedness and farsightedness – due to the mismatch between the focal length of the eye and the distance between lens and retina.

Farsightedness
Lack of near vision



Farsighted eye cannot focus on an object at the near point of a normal eye.
The lens-retina distance is too short and/ or the lens is not convergent enough.

Correcting farsightedness



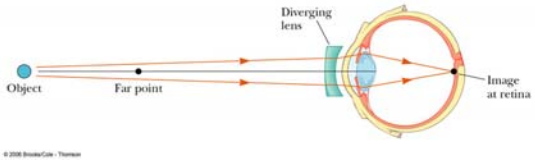
The light is made to converge more by using a converging lens.

Nearsightedness lack of far vision

Nearsighted eye cannot focus on objects far away (further than the far point < infinity)

The lens retina distance is too long and/or the lens is too converging.

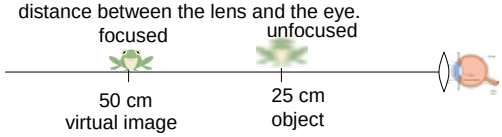
Correcting nearsightedness



Nearsighted vision can be corrected by using a diverging lens (making the light less convergent).

Example

A farsighted person has a near point of 50 cm. What power lens will correct this to normal vision. (ignore the distance between the lens and the eye.



Use a lens that can take an object at 25 cm and form a virtual image at a distance of 50 cm.

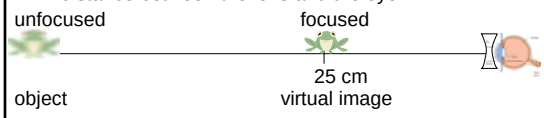
$$f = \frac{pq}{p+q} = \frac{(25)(-50)}{25-50} = +50\text{cm} = 0.50\text{m}$$

P= 1/0.5m =2.0 diopters

A converging lens

Example

A nearsighted person has a far point of 25 cm. What power lens will correct this to normal vision. (ignore the distance between the lens and the eye.



Use a lens that can take an object at infinity and form a virtual image at 25 cm.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{\infty}+\frac{1}{-25}=\frac{1}{f}$$

A diverging lens

f=-25 cm=-0.25m P= $\frac{1}{-0.25} = -4.0\text{diopters}$

5.2 Optical Instruments

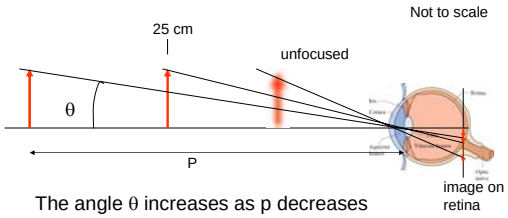
- Simple magnifier
- Compound microscope
- Telescope

Magnifiers

How do we image small objects?

- We can image a small object by bringing it close to our eye.
- But we cannot bring it closer than the near point. (we can't focus on it).
- A magnifier can produce a larger image of the object at the near point (or farther away) that can be focused on by the eye.
- Key concept - Angular magnification

Angular size

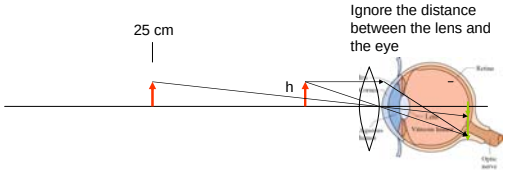


The angle θ increases as p decreases
The image size increases

Objects closer than the near point are not in focus.

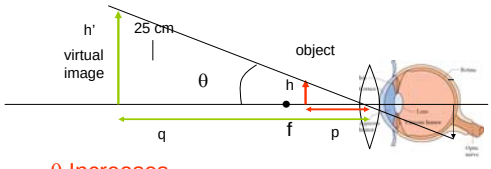
Simple Magnifier

A converging lens in combination with the lens of the eye forms an image on the retina from an object closer than the near point of the eye.



How does it work?

Produces an enlarged virtual image at a distance from the eye (from 25 cm to infinity) that the eye can focus on.



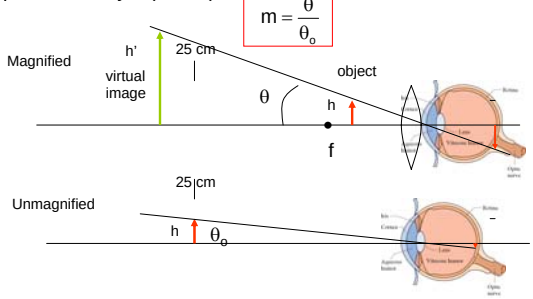
θ Increases

Simple Magnifier

Angular Magnification

The angular magnification is the ratio of θ for the magnified image compared to value of θ_o for the object at the near point of the eye. (25 cm)

$$m = \frac{\theta}{\theta_o}$$

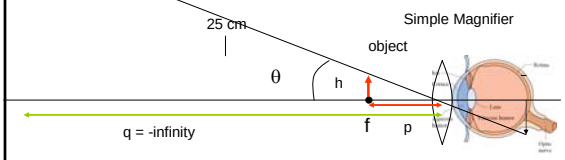


Angular magnification

The angular magnification for the simple magnifier can have a range of values because the focal length of the eye can vary due to accommodation.

The simplest case is the magnification for the relaxed eye. (focused at infinity)

θ increases at short object distances

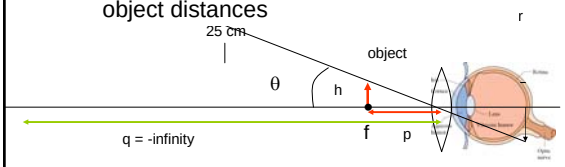


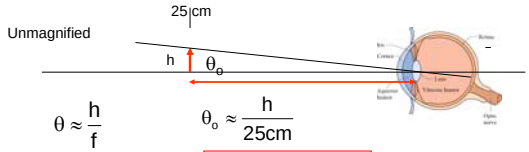
Object at the focal point. p = f
Image at - infinity q = - infinity

The linear magnification is infinite (not useful) but the angular size is well defined.

$\tan\theta = \frac{h}{f}$ for small angles $\tan\theta \sim \theta$ $\Rightarrow$ $\theta \approx \frac{h}{f}$

Angular magnification increases at short object distances





$\theta \approx \frac{h}{f}$ $\theta_o \approx \frac{h}{25cm}$

Angular magnification $m = \frac{\theta}{\theta_0} = \frac{25cm}{f}$ f is the focal length of the lens in cm.

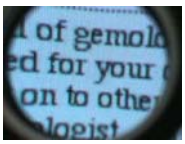
Simple magnifier.

A simple magnifier with a focal length of 5.0 cm is used to view an insect. What is the angular magnification for a relaxed eye?

$$m = \frac{25cm}{f} = \frac{25cm}{5.0cm} = 5.0$$

Simple magnifiers.

The angular magnification for a single lens is limited by aberration to about 4. Combination lenses can have magnification to about 20.



Compound Microscopes.

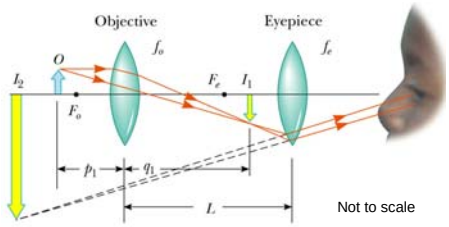
Magnification by 2 lenses.

- Objective lens – Produces an enlarged real image of the object.
- Eyepiece – Used like a simple magnifier to view the image.

The net angular magnification of the product of the two magnifications.

Compound microscope

Compound microscope

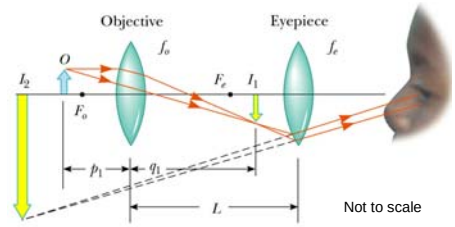


Not to scale

The objective lens produces a magnified real image I_1
The image is viewed through the eyepiece.

M
a

Compound microscope

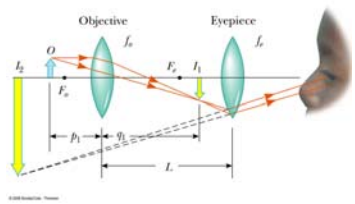


Not to scale

The objective lens produces a magnified real image I_1
The image is viewed through the eyepiece.

$$M_o = -\frac{q_1}{p_1} \approx -\frac{L}{f_o} \qquad m_e = \frac{25\text{cm}}{f_e}$$ For relaxed eye

Magnification



Overall angular magnification

$$m = M_1 m_e = -\frac{L}{f_o}\frac{(25\text{cm})}{f_e}$$

Magnification increases when f_e and f_e get smaller.

Magnification

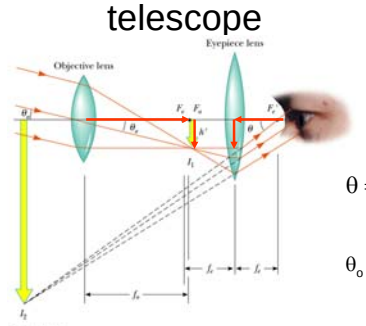
A compound microscope has an objective lens and eyepiece with a focal lengths of 1.5 cm and 2.0 cm respectively. The microscope is 20 cm long. Find the angular magnification

$$m = -\frac{L}{f_o}\frac{(25\text{cm})}{f_e} = -\frac{20}{1.5}\frac{(25\text{cm})}{2.0} = -167$$

Refracting Telescope

Two lenses
Objective lens – produces a reduced image of a distant object near the focal point.
Eyepiece – used to magnify the image.

telescope



$$\theta = \frac{h'}{f_e}$$

$$\theta_o = \frac{h'}{f_o}$$

Angular magnification $$m = \frac{\theta}{\theta_o} = \frac{f_o}{f_e}$$

focus at infinity

Telescope

The Hubble space telescope has an objective mirror with a focal length of 57.8 m viewed with optics equivalent to an eyepiece with a focal length of $7.2\text{x}10^{-3}$m What is the angular magnification?

$$m = \frac{f_o}{f_e} = \frac{57.8}{7.2\text{x}10^{-3}} = 8.0\text{x}10^{3}$$

Hubble Telescope Image of M100 Spiral Galaxy (NASA)

Limits to magnification

Why can't we use light microscopes to see atoms?

- For refracting optics there are problems of chromatic and spherical aberration.
- Problems in precision in constructing the refracting and reflecting surfaces.
- Diffraction – A basic problems having to do with the wave nature of light (discussed next week)