Physics 1B Spring 2010

Quiz 3 version A

Solution

1.(c)

The resistance is determined by the following equation:

$$R = \rho\frac{L}{A} \tag{1}$$

From this we can express the cross-sectional area, it's given by

$$A = \frac{\rho L}{R} \tag{2}$$

And

$$\begin{aligned} d &= \sqrt{A} \\ &= \sqrt{\frac{\rho L}{R}} \\ &= \sqrt{\frac{2.5\Omega \cdot m \times 1.5 \times 10^{-2} m}{200\Omega}} \\ &= 1.37cm \end{aligned} \tag{3}$$

2.(b)

The time constant of a RC circuit is given by:

$$\begin{aligned} \tau &= RC \\ &= 2 \times 10^{-6}\Omega \times 6 \times 10^{-6} F \\ &= 12s \end{aligned} \tag{4}$$

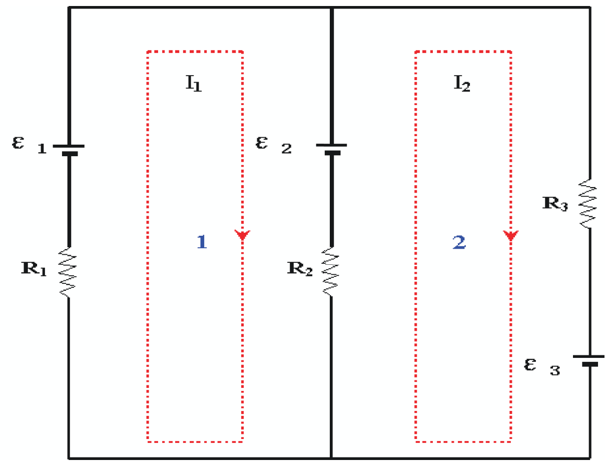

Figure 1:

3.(c)

This circuit has 2 independent loops. Thus, we can write down equation for each. Please refer to Figure 1.

For loop 1, applying Kirchhoff's loop rule, we get:

$$\varepsilon_1 - \varepsilon_2 - (I_1 - I_2)R_2 - I_1 R_1 = 0 \tag{5}$$

It reduces to:

$$\begin{aligned} (R_1 + R_2)I_1 - R_2 I_2 &= \varepsilon_1 - \varepsilon_2 \\ 5000I_1 - 3000I_2 &= 11 \end{aligned} \tag{6}$$

For loop 2, we can also get:

$$\varepsilon_2 - I_2 R_3 - \varepsilon_3 - (I_2 - I_1)R_2 = 0 \tag{7}$$

It reduces to:

$$\begin{aligned} R_2 I_1 - (R_2 + R_3)I_2 &= \varepsilon_3 - \varepsilon_2 \\ 3000I_1 - 7000I_2 &= 17 \end{aligned} \tag{8}$$

Then we are going to solve equations 6 and 8. Ultimately we get:

$$\begin{aligned} I_1 &= 1mA \\ I_2 &= -2mA \end{aligned} \tag{9}$$

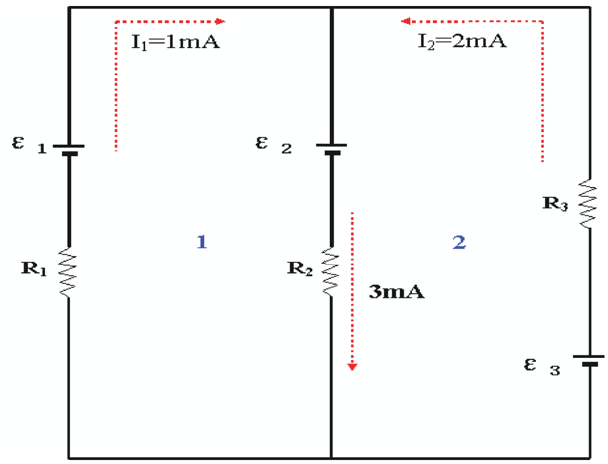

Figure 2:

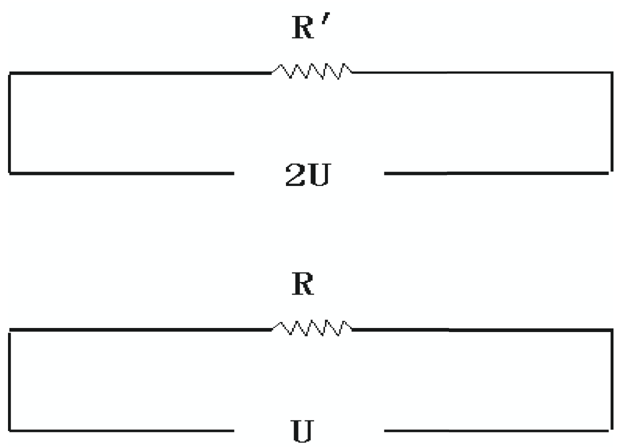

Figure 3:

Figure 2 will show the actual currents in this circuit. The potential difference in R_3 is:

$$\begin{aligned} \Delta V &= I_2 R_3 \\ &= 2 \times 10^{-3} A \times 4 \times 10^{3} \Omega \\ &= 8V \end{aligned} \tag{10}$$

4.(d)

Like Figure 3, we have 2 resistors, which are R' and R as indicated in the figure. The voltages for these two are $2U$ and U. And the power is the same. Knowing $P = \frac{U^2}{R}$, we can get the power

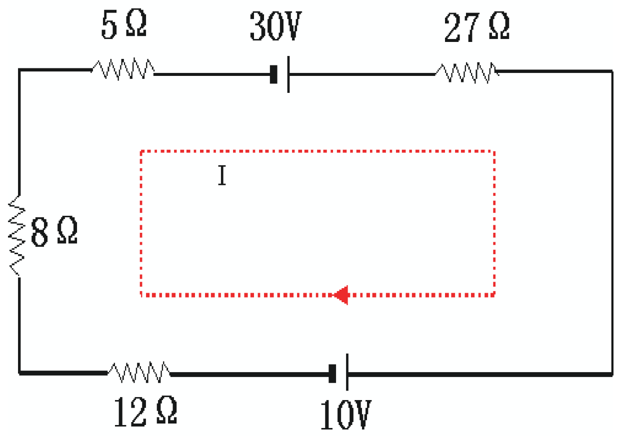

Figure 4:

for these two resistors:

$$
\begin{aligned}
P_1 &= \frac{(2U)^2}{R'} \\
P_2 &= \frac{U^2}{R} \\
P_1 &= P_2 \qquad (11)
\end{aligned}
$$

So,

$$\frac{(2U)^2}{R'} = \frac{U^2}{R} \qquad (12)$$

The value of R' is easily obtained: $R' = 4R$.

5.(a)

Please refer to Figure 4. We get 1 loop for this simple circuit. Now let's write down Kirchhoff's loop rule.

$$-5I + 30 - 27I - 10 - 12I - 8I = 0 \qquad (13)$$

From this equation, we get the current $I = \frac{(30-10)V}{(5+27+12+8)\Omega} = 0.385A$. Then the power for 12Ω is given by:

$$
\begin{aligned}
P &= I^2 \cdot 12\Omega \\
&= 0.385^2 A^2 \cdot 12\Omega \\
&= 1.775W \qquad (14)
\end{aligned}
$$

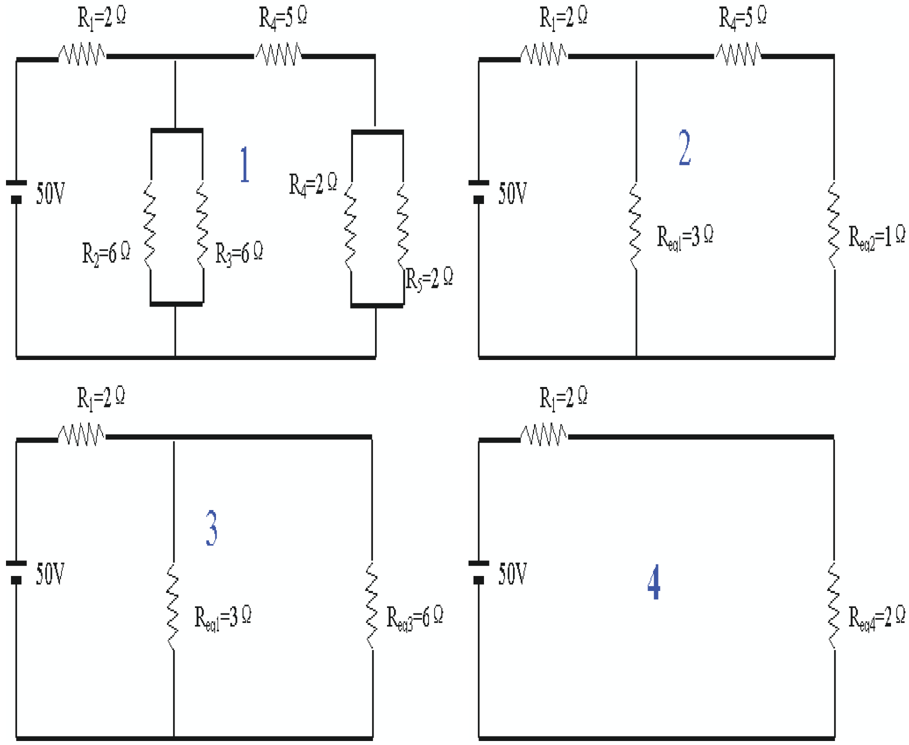

Figure 5:

6.(a)

Please refer to Figure 5. We simplify the circuit step by step.

R_2 and R_3 are connected in parallel. R_{eq1} is given by:

$$\frac{1}{R_{eq1}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{6} \tag{15}$$

Then $R_{eq1} = 3\Omega$.

R_5 and R_6 are connected in parallel. R_{eq2} is given by:

$$\frac{1}{R_{eq2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{2} + \frac{1}{2} \tag{16}$$

Then $R_{eq2} = 1\Omega$.

This step is shown in Figure 5-(2).

From Figure 5-(2), R_4 and R_{eq2} are connected in series, so,

$$R_{eq3} = R_4 + R_{eq2} = 5 + 1 = 6 \tag{17}$$

This step is shown in Figure 5-(3).

From Figure 5-(3), R_{eq1} and R_{eq3} are connected in parallel,

$$\frac{1}{R_{eq4}} = \frac{1}{R_{eq1}} + \frac{1}{R_{eq3}} = \frac{1}{3} + \frac{1}{6} \tag{18}$$

Then $R_{eq4} = 2\Omega$. This step is shown in Figure 5-(4).

Finally, R_1 and R_{eq4} are connected in series. The entire equivalent resistance is $R_{eq} = R_1 + R_{eq4} = 2 + 2 = 4$.

7.(a)

The charge in the capacitor varies with time. The explicit relation is

$$q(t) = CU(1 - e^{-\frac{t}{\tau}}) = CU(1 - e^{-\frac{t}{RC}}) \tag{19}$$

The magnitude of the current is:

$$\begin{aligned} I(t) &= |\frac{dq}{dt}| \\ &= \frac{U}{R} e^{-\frac{t}{RC}} \end{aligned} \tag{20}$$

This equation gives the maximum value of the current when $t = 0$. Thus,

$$I_{max} = I(0) = \frac{U}{R} \tag{21}$$

We can get the resistance $R = \frac{U}{I_{max}} = \frac{50V}{0.5A} = 100\Omega$.

The time constant $\tau = 1s$. We can get the capacitance $C = \frac{\tau}{R} = \frac{1.0s}{100\Omega} = 0.01F$.

With these results, we are able to evaluate the time-dependent charge at $t = 2s$,

$$\begin{aligned} q(2) &= CU(1 - e^{-\frac{t}{\tau}}) \\ &= 0.01F \times 50V(1 - e^{-\frac{2.0s}{1.0s}}) \\ &= 0.5 \times 0.865C \\ &= 0.432C \end{aligned} \tag{22}$$

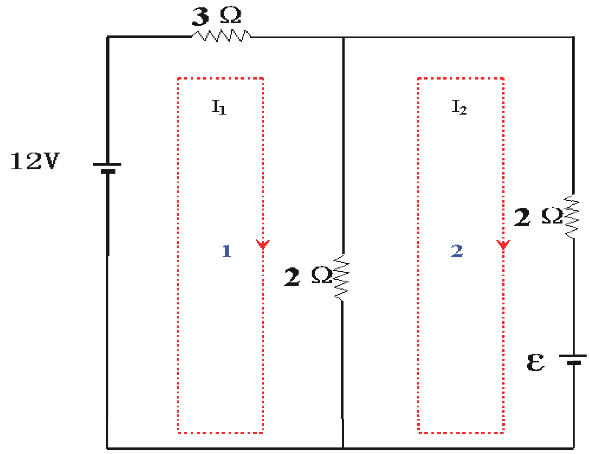

Figure 6:

8.(e)

Using Kirchhoff's law, we write down equations for these 2 loops. Please refer to Figure 6. For loop 1, we get:

$$-3I_1 - 2(I_1 - I_2) + 12 = 0 \tag{23}$$

For loop 2, we get:

$$-2(I_2 - I_1) - 2I_2 - \varepsilon = 0 \tag{24}$$

In Eq.23, $I_1 = 2A$. Then $I_2 = -1A$. Putting these values in Eq.24, we get $\varepsilon = -2(I_2 - I_1) - 2I_2 = 8V$.