Physics 1B Spring 2010

Quiz 2 version A

Solution

1.(e)

The electric potential in the middle by charge $+Q$ is :

$$V_{+} = \frac{k(+Q)}{(\frac{d}{2})} \tag{1}$$

For charge $-Q$, it's

$$V_{-} = \frac{k(-Q)}{(\frac{d}{2})} \tag{2}$$

So the total is $V = V_{+} + V_{-} = 0$.

2.(b)

We can solve this problem separately. Please refer to Figure 1. In this figure, $C_1 = 10\mu F, C_2 = 6\mu F, C_3 = 12\mu F, C_4 = 6\mu F$.

(1)Calculate C_{eq1}. C_2 and C_3 are connected in series.

$$\frac{1}{C_{eq1}} = \frac{1}{C_2} + \frac{1}{C_3} \tag{3}$$

This gives $C_{eq1} = 4\mu F$.

(2)Calculate C_{eq2}. C_{eq1} and C_4 are connected in parallel.

$$C_{eq2} = C_{eq1} + C_4 \tag{4}$$

So we get $C_{eq2} = 10\mu F$.

(3)Calculate C_{tot}. C_1 and C_{eq2} are connected in series.

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_{eq2}} \tag{5}$$

So, the entire capacitance is $C_{tot} = 5\mu F$.

3.(d)

Problem 2 and 3 have the same combination of capacitors. Please refer to Figure 1. We'll use this figure from the right to the left.

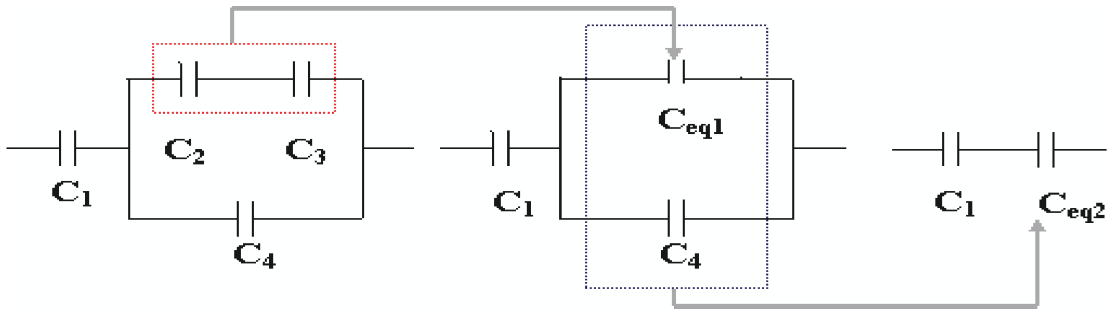

Figure 1: Problem 2 and 3.

C_1 and C_{eq2} are connected in series, thus they have the same charges.

$$\begin{aligned}
\frac{U_1}{U_{eq2}} &= \frac{\frac{Q_1}{C_1}}{\frac{Q_{eq2}}{C_{eq2}}} \\
&= \frac{C_{eq2}}{C_1} \\
&= \frac{10\mu F}{10\mu F} \\
&= 1 \\
U_1 + U_{eq2} &= 18V \qquad (6)
\end{aligned}$$

From these 2 equations we get $U_1 = U_{eq2} = 9V$.

As we can see, C_{eq1} and C_4 are connected in parallel. The potential difference between them is $9V$.

C_2 and C_3 are connected in series. Then $Q_2 = Q_3$.

$$\begin{aligned}
\frac{U_2}{U_3} &= \frac{\frac{Q_2}{C_2}}{\frac{Q_3}{C_3}} \\
&= \frac{C_3}{C_2} \\
&= \frac{12\mu F}{6\mu F} \\
&= 2 \\
U_2 + U_3 &= 9V \qquad (7)
\end{aligned}$$

We get U_2=6V, U_3=3V.

The charge in C_3 is given by $Q_3 = C_3 U_3 = 12 \times 10^{-6} F \times 3V = 36 \times 10^{-6} C = 36\mu C$.

4.(b)

These 2 capacitors are connected in series, and it means $Q_1 = Q_2$. Here $C_1 = 3.0\mu F$, $C_2 = 6.0\mu F$.

$$
\begin{aligned}
\frac{U_1}{U_2} &= \frac{\frac{Q_1}{C_1}}{\frac{Q_2}{C_2}} \\
&= \frac{C_2}{C_1} \\
&= \frac{6\mu F}{3\mu F} \\
&= 2 \\
U_1 + U_2 &= 90V \qquad (8)
\end{aligned}
$$

We can get $U_1 = 60V$, $U_2 = 30V$. So,

$$
\begin{aligned}
Energy\ stored &= \frac{1}{2}C\Delta V^2 \\
&= \frac{1}{2} \times 3 \times 10^{-6}F \times 60^2V^2 \\
&= 5.4 \times 10^{-3}J \qquad (9)
\end{aligned}
$$

5.(e)

The capacitance of a parallel-plate capacitor is

$$
\begin{aligned}
C &= \epsilon_0 \frac{A}{d} \\
&= 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \times \frac{(20 \times 10^{-2}m)^2}{1.0 \times 10^{-2}m} \\
&= 3.54 \times 10^{-11}F \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
Energy\ stored &= \frac{1}{2}C\Delta V^2 \\
&= \frac{1}{2} \times 3.54 \times 10^{-11}F \times 50^2V^2 \\
&= 4.4 \times 10^{-8}J \qquad (11)
\end{aligned}
$$

6.(c)

The electric potential energy is converted into electron's kinetic energy. $|\Delta PE| = \frac{1}{2}m_e v_{final}^2 - \frac{1}{2}m_e v_0^2$. Here v_0 is 0 since electron is initially at rest. The change of PE is $|\Delta PE| = eU$. From these 2 equations, the v_{final} is expressed as

$$\begin{aligned} v_{final} &= \sqrt{\frac{2eU}{m_e}} \\ &= \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 4800V}{9.11 \times 10^{-31} kg}} \\ &= 4.2 \times 10^7 m/s \end{aligned} \tag{12}$$

7.(d)

The $+q$ charge creates a potential $V_1 = \frac{k(+q)}{d}$. The $+2q$ charge creates a potential $V_2 = \frac{k(+2q)}{\sqrt{2}d}$. The $+5q$ charge creates a potential $V_5 = \frac{k(+5q)}{d}$. The total potential at site of $-3q$ is $V_1 + V_2 + V_5$,

$$\begin{aligned} V_1 + V_2 + V_5 &= \frac{k(+q)}{d} + \frac{k(+2q)}{\sqrt{2}d} + \frac{k(+5q)}{d} \\ &= \frac{kq}{d}(1 + \sqrt{2} + 5) \\ &= \frac{899 \times 10^9 \frac{N \cdot m^2}{C^2} \times 10 \times 10^{-9} C}{2m} \times 7.414 \\ &= 333V \end{aligned} \tag{13}$$

8.(a)

These 2 capacitors are connected in parallel. They have the same potential difference. This gives $U_1 = U_2$. Here $C_1 = 2.0\mu F$, $C_2 = 1.0\mu F$.

$$\frac{Q_1}{Q_2} = \frac{C_1 U_1}{C_2 U_2} = \frac{C_1}{C_2} = \frac{2.0\mu F}{1.0\mu F} = 2 \tag{14}$$