Physics 1B Spring 2010

Final version A

Solution

1.(c)

As shown in the Figure 1, the electric field (at position B) along x axis cancels. The net E field (at position B) along y should be zero. So,

$$2 \times \frac{k_e q}{L^2} \cos 45^\circ = \frac{k_e \times 1C}{(\sqrt{2}L)^2} \tag{1}$$

From this equation, we solve for the magnitude of the unknown charge.

$$q = \frac{1}{2\sqrt{2}} C \simeq 0.3536C \tag{2}$$

Form the directions shown in the figure, we know they are negative charges.

2.(d)

The uniformly charged sphere contributes no E field to the interior. Only the charge at the center is counted. Thus it's a problem of a point charge creating E filed.

$$\begin{aligned} E &= \frac{k_e q}{r^2} \\ &= \frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} \times 4 \times 10^{-6} C}{1.5^2 m^2} \\ &= 1.6 \times 10^4 N/C \end{aligned} \tag{3}$$

3.(c)

The time dependant charge is

$$Q(t) = CU(1 - e^{-\frac{t}{\tau}}) \tag{4}$$

At $t = 0.2s$, it's

$$6.85 \times 10^{-3} = C \times 100(1 - e^{-\frac{0.2}{0.334}}) \tag{5}$$

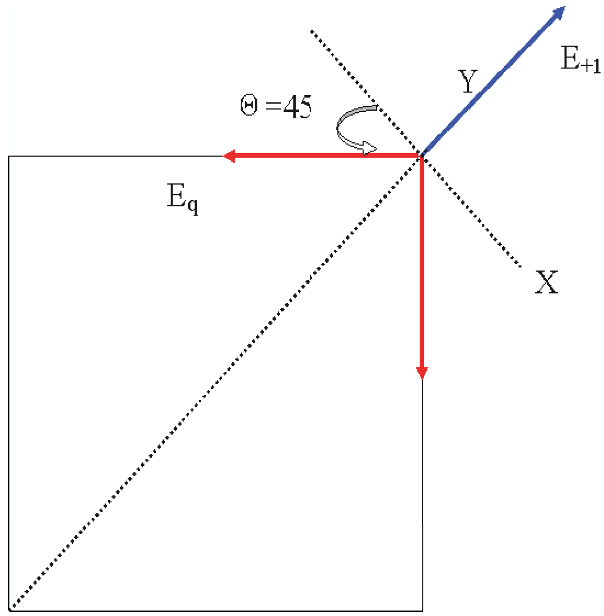

Figure 1:

From this equation, we get,

$$C = \frac{6.85 \times 10^{-3}}{100(1 - e^{-\frac{0.2}{0.334}})} = 1.52 \times 10^{-4}F = 152\mu F \tag{6}$$

4.(b)

Please refer to Figure 2. It's an equilibrium state. The net force is 0.

$$\tan\theta = \frac{F_E}{mg} \tag{7}$$

And

$$F_E = \frac{k_e q^2}{d^2} \tag{8}$$

Here d is the distance between the 2 charges. $d = 2L\sin\theta$.

Thus,

$$\tan\theta = \frac{\frac{k_e q^2}{(2L\sin\theta)^2}}{mg} \tag{9}$$

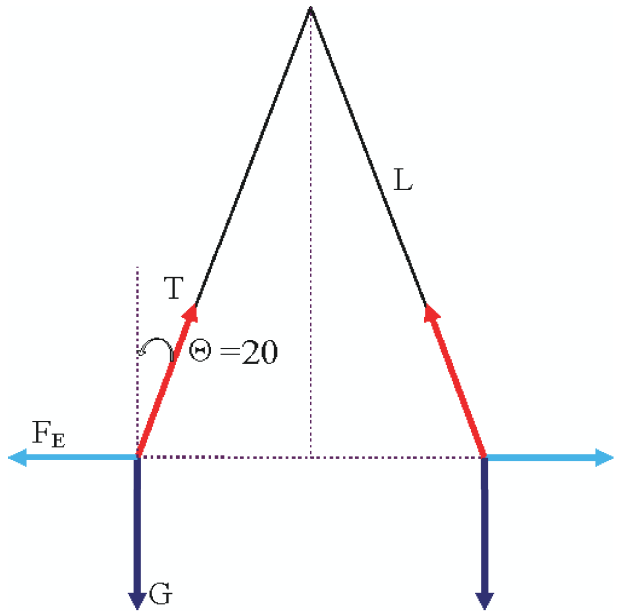

Figure 2:

And

$$
\begin{aligned}
q &= \sqrt{\frac{4L^2 \sin^2\theta \tan\theta mg}{k_e}} \\
&= \sqrt{\frac{4(1.2)^2 m^2 \sin^2 20^\circ \tan 20^\circ 0.02kg \times 10N/kg}{9\times 10^9 \frac{N\cdot m^2}{C^2}}} \\
&= 2.33\times 10^{-6}C \qquad (10)
\end{aligned}
$$

5.(b)

The $12\mu F$ and $24\mu F$ are in series.

$$C_{eq1} = \frac{12\times 24(\mu F)^2}{(12+24)\mu F} = 8\mu F \qquad (11)$$

C_{eq1} and 12μF are im parallel, so, $C_{eq2} = C_{eq1} + 12 = 20\mu F$.

C_{eq2} and $20\mu F$ are in series. The total equivalent capacitance is

$$C_{eq} = \frac{20\times 20(\mu F)^2}{(20+20)\mu F} = 10\mu F \qquad (12)$$

6.(a)

The potential is algebraically added.

$$\begin{aligned} V &= V_1 + V_2 \\ &= \frac{k_e q_1}{L/2} + \frac{k_e q_2}{L/2} \\ &= \frac{2k_e}{L}(q_1 + q_2) \\ &= \frac{2 \times 9 \times 10^9 \frac{N \cdot m}{C^2}}{0.1m}(10 \times 10^{-6}C) \\ &= 1.8 \times 10^6 V \end{aligned} \tag{13}$$

7.(e)

The capacitance for a parallel plate capacitor is

$$\begin{aligned} C &= \frac{\varepsilon_0 A}{d} \\ &= \frac{8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \times (0.2)^2 m^2}{1 \times 10^{-2} m} \\ &= 3.54 \times 10^{-11} F \end{aligned} \tag{14}$$

The energy stored is

$$\begin{aligned} E &= \frac{1}{2} C U^2 \\ &= \frac{1}{2} \times 3.54 \times 10^{-11} F \times 50^2 V^2 \\ &= 4.425 \times 10^{-8} J \end{aligned} \tag{15}$$

8.(b)

This circuit has 2 independent loops. Thus, we can write down equation for each. Please refer to Figure 3.

For loop 1, applying Kirchhoff's loop rule, we get:

$$\varepsilon_1 - \varepsilon_2 - (I_1 - I_2)R_2 - I_1 R_1 = 0 \tag{16}$$

It reduces to:

$$\begin{aligned} (R_1 + R_2)I_1 - R_2 I_2 &= \varepsilon_1 - \varepsilon_2 \\ 5000 I_1 - 3000 I_2 &= 22 \end{aligned} \tag{17}$$

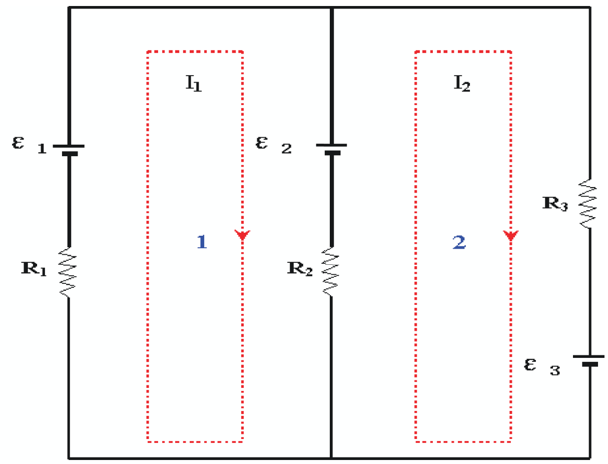

Figure 3:

For loop 2, we can also get:

$$\varepsilon_2 - I_2R_3 - \varepsilon_3 - (I_2 - I_1)R_2 = 0 \tag{18}$$

It reduces to:

$$\begin{aligned} R_2I_1 - (R_2 + R_3)I_2 &= \varepsilon_3 - \varepsilon_2 \\ 3000I_1 - 7000I_2 &= 34 \end{aligned} \tag{19}$$

Then we are going to solve equations 17 and 19. Ultimately we get:

$$\begin{aligned} I_1 &= 2mA \\ I_2 &= -4mA \end{aligned} \tag{20}$$

The current through R_2 is $I_1 - I_2 = 6mA$.

9.(b)

The radius is expressed as,

$$r = \frac{mv}{qB} \tag{21}$$

So, the mass is

$$\begin{aligned} m &= \frac{qBr}{v} \\ &= \frac{2.25 \times 10^{-15}C \times 0.5T \times 0.25/2m \times}{5 \times 10^5 m/s} \\ &= 2.81 \times 10^{-22}kg \end{aligned} \tag{22}$$

10.(b)

R_1 is short-circuited. R_2 and R_3 are essentially in parallel.

11.(c)

The potential energy is converted in to kinetic energy.

$$qU = \frac{1}{2}mv^2 \tag{23}$$

So, the velocity is expressed as,

$$\begin{aligned} v &= \sqrt{\frac{2qU}{m}} \\ &= \sqrt{\frac{2 \times 1.6 \times 10^{-19}C \times 1200V}{9.11 \times 10^{-31}kg}} \\ &= 2.053 \times 10^7 m/s \\ &\simeq 2.1 \times 10^7 m/s \end{aligned} \tag{24}$$

12.(c)

The capacitive reactance is

$$\begin{aligned} X_c &= \frac{1}{2\pi f C} \\ &= \frac{1}{2\pi 60H_z \times 12 \times 10^{-6}F} \\ &= 221.049\Omega \end{aligned} \tag{25}$$

The rms current is

$$\begin{aligned} I_{rms} &= \frac{\frac{V_{max}}{\sqrt{2}}}{X_c} \\ &= \frac{\frac{170V}{\sqrt{2}}}{221.049\Omega} \\ &= 0.543A \end{aligned} \tag{26}$$

13.(c)

We choose upward is positive. Current I_1 creates B_1 which is upward at $x = 2cm$.

$$B_1 = +\frac{\mu I}{2\pi r_1} \tag{27}$$

Current 2 creates B_2 which is also upward,

$$B_2 = +\frac{\mu I}{2\pi r_2} \tag{28}$$

Current 3 creates B_3 which is positive too.

$$B_3 = +\frac{\mu I}{2\pi r_3} \tag{29}$$

Thus,

$$\begin{aligned} B &= B_1 + B_2 + B_3 \\ &= +\frac{\mu I}{2\pi r_1} + \frac{\mu I}{2\pi r_2} + \frac{\mu I}{2\pi r_3} \\ &= \frac{\mu I}{2\pi}(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}) \\ &= 2 \times 10^{-7}\frac{N}{A^2} \times 10A \times (\frac{1}{0.02-(-0.04)} + \frac{1}{0.02-0} + \frac{1}{0.04-0.02})/m \\ &= 2.33 \times 10^{-4}T = 233\mu T \end{aligned} \tag{30}$$

14.(c)

The change of the magnetic flux is,

$$\Delta\Phi = N\Delta BA \tag{31}$$

According to Faraday's law, the emf is then

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -NA\frac{\Delta B}{\Delta t} \tag{32}$$

According to Ohm's law, the current is,

$$I = \frac{\varepsilon}{R} = -\frac{NA}{R}\frac{\Delta B}{\Delta t} \tag{33}$$

The problem tells us $\frac{\Delta B}{\Delta t} = \frac{6.0T-2.0T}{2.0s} = 2T/s$. With this value, we get the magnitude of the current,

$$\begin{aligned} I &= \frac{NA}{R}\frac{\Delta B}{\Delta t} \\ &= \frac{20 \times 50 \times 10^{-4}m^2}{0.40\Omega} \times 2T/s \\ &= 0.5A = 500mA \end{aligned} \tag{34}$$

15.(e)

The magnetic field inside a solenoid is,

$$B = \mu \frac{N}{L} I \tag{35}$$

So, the current is,

$$\begin{aligned} I &= \frac{BL}{\mu N} \\ &= \frac{2.0 \times 10^{-4} T \times 0.2m}{4\pi \times 10^{-7} \frac{N}{A^2} \times 500} \\ &= 63.7mA \end{aligned} \tag{36}$$

16.(a)

The force between two same charges is,

$$F = \frac{k_e Q^2}{L^2} \tag{37}$$

So, the distance is

$$\begin{aligned} L &= \sqrt{\frac{k_e Q^2}{F}} \\ &= \sqrt{\frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} (1.0 \times 10^{-6} C)^2}{4N}} \\ &= 4.74cm \end{aligned} \tag{38}$$

17.(b)

Please refer to Figure 4.

The magnetic force should be equal to the gravitational force acted on the conductor.

$$\begin{aligned} F_B &= IBl \\ &= mg \end{aligned} \tag{39}$$

Then

$$\begin{aligned} I &= \frac{mg}{Bl} \\ &= \frac{(\frac{m}{l})g}{B} \\ &= \frac{0.040kg/m \times 10N/kg}{3.6T} \\ &= 0.11A \end{aligned} \tag{40}$$

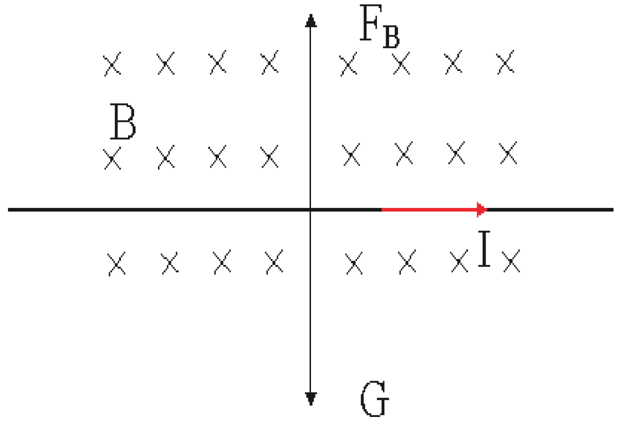

Figure 4:

The direction is indicated in the figure.

18.(b)

The resistance is

$$R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2} \tag{41}$$

The resistivity is

$$\begin{aligned} \rho &= \frac{\pi r^2 R}{l} \\ &= \frac{\pi \times 10^{-4} m^2 \times 3.2 \times 10^{-5} \Omega}{0.20 m} \\ &= 5.03 \times 10^{-8} \Omega \cdot m \end{aligned} \tag{42}$$

19.(c)

The magnetic flux is

$$\begin{aligned} \Phi &= BA \cos\theta \\ &= 4.5T \times 0.1 m^2 \times \cos 30^\circ \\ &= 0.390 T \cdot m^2 \end{aligned} \tag{43}$$

20.(a)

The time constant is $\tau = L/R$. From this, we get L,

$$\begin{aligned} L &= \tau R \\ &= 4.0 \times 10^{-4} s \times 6.0\Omega \\ &= 2.4 \times 10^{-3} H \end{aligned} \tag{44}$$

When current is $2.0A$, the energy stored is

$$\begin{aligned} PE &= \frac{1}{2} L I^2 \\ &= \frac{1}{2} \times 2.4 \times 10^{-3} H \times 4A^2 \\ &= 4.8 \times 10^{-3} J \end{aligned} \tag{45}$$

21.(c)

When it's resonating, the frequency is $f = \frac{1}{2\pi\sqrt{LC}}$. Then inductor is

$$\begin{aligned} L &= \frac{1}{4\pi^2 f^2 C} \\ &= \frac{1}{4\pi^2 \times (105 \times 10^6)^2 H_Z^2 \times 2 \times 10^{-12} F} \\ &= 1.15 \times 10^{-6} H = 1.15 \mu H \end{aligned} \tag{46}$$

And X_L is equal to X_c. $Z = R = 50\Omega$.

22.(a)

The inductive reactance is given by $X_L = 2\pi f L$, so

$$\begin{aligned} L &= \frac{X_L}{2\pi f} \\ &= \frac{50\Omega}{2\pi \times 100Hz} \\ &= 79.6mH \simeq 80mH \end{aligned} \tag{47}$$