

★Reading: chapter 13.

- Finish gravity, mention black holes. Escape velocity $v = \sqrt{2GM/R} = c$ gives $R_S = 2GM/c^2$. This is the place of no return. Einstein's theory of general relativity shows, among other things, that stronger gravity makes clocks run slower. Atomic clocks are super accurate and can measure this even for clocks differing in height above the earth by a meter or so. The GPS system is so tuned and accurate that it needs to account for this effect!

- Let's now start the study of oscillations. Periodic motion in time is given by $x(t) = A \cos(\omega t + \phi)$. Here A is the amplitude, ω is the angular frequency, and ϕ is an initial condition for the phase. Note that $\omega = 2\pi f = 2\pi/T$. Example: object rotating in a circle of radius $R = A$, with $x(t)$ the x-component. As we'll now discuss, this same $x(t)$ describes the position of a mass on a spring.

For a mass on a spring, $F = -kx$ (Hooke's law), so $F = ma$ gives $-kx = m \frac{d^2x}{dt^2}$. This equation is solved by

$$x(t) = A \cos(\omega t + \phi), \tag{1}$$

where $\omega = \sqrt{k/m}$. The position of a mass on a spring is just like the x-component of a particle rotating in a circle! The amplitude and phase A and ϕ are two constants of integration, specified by the initial position and velocity of the mass. These are analogous to the constants x_0 and v_0 in $x = \frac{1}{2}at^2 + v_0t + x_0$, in the case of constant acceleration. Here we have $x_0 = x(t=0) = A \cos \phi$ and $v_0 = v(t=0) = -\omega A \sin \phi$.

- We saw before that a spring's force is conservative, with $F = -\frac{d}{dx}U(x)$, with potential energy $U = \frac{1}{2}kx^2$. There is a point of stable equilibrium at $x = 0$. For a more general potential $U(x)$, expanding (in a Taylor's series) near a point of stable equilibrium will always give something that's approximately quadratic for small displacements (ignoring higher terms in the Taylor's series), so a mass on a spring behaves exactly like ANY small oscillations around a point of stable equilibrium.

Example: vibrations of molecules, $U \approx U_0((\frac{R_0}{r})^{12} - 2(\frac{R_0}{r})^6)$ has a minimum at $r = R_0$, and expanding $r = R_0 + x$ gives a SHO with $k = 72U_0/R_0^2$.

- Let's check that the energy of the oscillating mass on a spring is indeed constant (as it should be, since we haven't yet put in any friction):

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)) = \frac{1}{2}m\omega^2 A^2.$$

Where we plugged in the solution (1). So E is indeed a time-independent constant, good. As the mass oscillates, its energy goes back and forth between kinetic and potential energy. The maximum velocity is ωA .