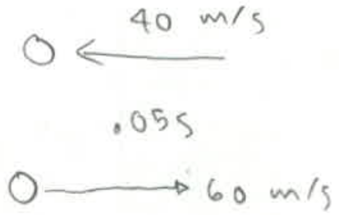
1.



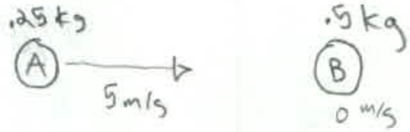
$$I = F\Delta t = \boxed{m\Delta v} = \Delta p$$

$$I = (60 - (-40))\frac{m}{s} \times .2\ kg = 20\ kg\frac{m}{s}$$

$$= 20\ N\cdot s$$

2.

Elastic collision → • total KE is same before and after collision



after they hit, what happens?

• also, momentum is conserved

• strategy: two equations, P and KE.

- solve both for one particular velocity.
- set equations equal to each other
- solve for remaining velocity.

$KE_1 = KE_2$

$\frac{1}{2} m_A V_{A1}^2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2$

$\frac{1}{2} m_A (V_{A1}^2 - V_{A2}^2) = \frac{1}{2} m_B V_{B2}^2$

$V_{B2} = \sqrt{\frac{m_A}{m_B}(V_{A1}^2 - V_{A2}^2)}$

$\Sigma P_1 = \Sigma P_2$

$m_A V_{A1} = m_A V_{A2} + m_B V_{B2}$

$V_{B2} = \frac{m_A}{m_B}(V_{A1} - V_{A2})$

$\frac{m_A}{m_B}(V_{A1} - V_{A2}) = \sqrt{\frac{m_A}{m_B}(V_{A1}^2 - V_{A2}^2)}$

$\left(\frac{m_A}{m_B}\right)^2 (V_{A1} - V_{A2})^2 = \frac{m_A}{m_B}(V_{A1}^2 - V_{A2}^2)$ ← factor this

$\left(\frac{m_A}{m_B}\right)^2 (V_{A1} - V_{A2})^2 = \frac{m_A}{m_B}(V_{A1} - V_{A2})(V_{A1} + V_{A2})$

$\left(\frac{m_A}{m_B}\right)(V_{A1} - V_{A2}) = (V_{A1} + V_{A2})$ → $\frac{m_A}{m_B} = \frac{.25\ kg}{.5\ kg} = \frac{1}{2}$

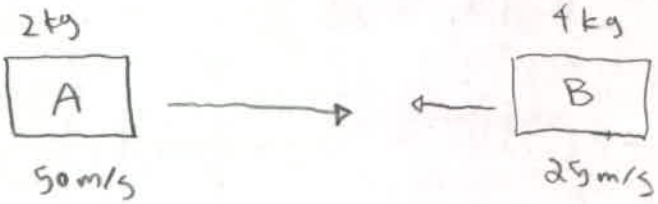
$\frac{1}{2} V_{A1} - \frac{1}{2} V_{A2} = V_{A1} + V_{A2}$ → we want V_{A2}, and $V_{A1} = +5\ m/s$

$-\frac{1}{2} V_{A1} = \frac{3}{2} V_{A2}$

$V_{A2} = -\frac{1}{3} V_{A1} = -\frac{1}{3}(5\ m/s)$

$V_{A2} = -1.7\ m/s$

3.



KE is not conserved (inelastic collision) but $\vec{P}$ is conserved.
("plus" velocity means travelling to the right, "minus" velocity to the left)

$$P_1 = (m_A V_{A1}) + (m_B V_{B1})$$
$$P_1 = (2\,kg\,(50\,m/s)) + (4\,kg\,(-25\,m/s)) = 0\ kg\,m/s$$

they stop $\left((m_A + m_B)V_f = 0 \rightarrow V_f = 0\right)$

therefore they have no final kinetic energy and

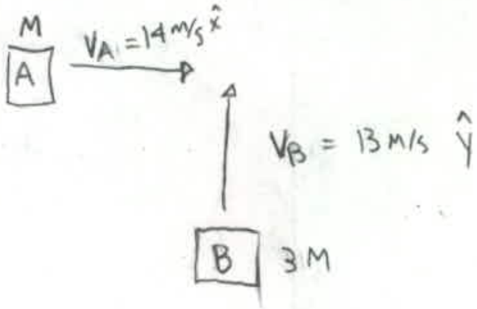
$$\Delta KE = KE_1 - KE_f = KE_1$$ (they lose all KE)

$$\Delta KE = KE_1 = \tfrac{1}{2} m_A V_{A1}^2 + \tfrac{1}{2} m_B V_{B1}^2$$

(note that kinetic energies, unlike momentums are always positive, due to squaring the velocity.)

$$\Delta KE = \tfrac{1}{2}(2\,kg)(50\,m/s)^2 + \tfrac{1}{2}(4\,kg)(25\,m/s)^2$$
$$\Delta KE = 3750\ J$$

4.



momentum is conserved in each direction.

$P_{1y} = P_{2y}$ and $P_{1x} = P_{2x}$

(total mass = 4M)
(after collision they stick)

$V_{B2} = V_{A2} = V_2$

(V_{2x} and V_{2y} components)

$\vec{P}_{1y} = 3M\,\vec{V}_B$

$\vec{P}_{2y} = 4M\,\vec{V}_{2y}$

$3M\,\vec{V}_B = 4M\,\vec{V}_{2y}$

$\vec{V}_{2y} = \frac{3}{4}\vec{V}_B$ $\quad 9.75\ m/s\ \hat{y}$

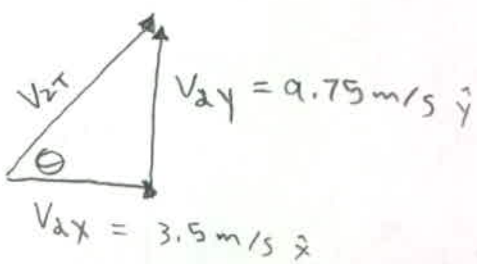
$\vec{P}_1 x = M\,\vec{V}_A$

$\vec{P}_2 x = 4\,\vec{V}_{2x}$

$4M\vec{V}_{2x} = M\vec{V}_A$

$\vec{V}_{2x} = \frac{1}{4}\vec{V}_A = 3.5\ m/s\ \hat{x}$

after collision



magnitude of V_{2T} =

$|V_{2T}| = \sqrt{V_{2y}^2 + V_{2x}^2}$

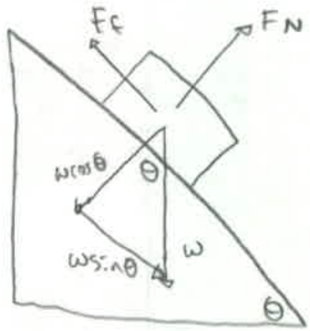
$|V_{2T}| = \sqrt{(9.75)^2 + (3.5)^2}\ m/s = 10.36$

$\theta = \tan^{-1}\left(\frac{V_{2y}}{V_{2x}}\right) = \tan^{-1}\left(\frac{9.75}{3.5}\right)$

$\theta = 70.3°$

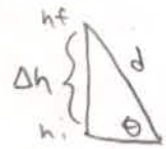
5.

w = weight = 20 N
= mg



$$W_f = \Delta KE + \Delta U$$

$$-20\,J = \tfrac{1}{2}m(V_f^2 - V_i^2) - w(h_f - h_i)$$



$$\Delta h = d\sin\theta$$

$$m = w/g \quad , \quad V_i = 0$$

$$-20\,J = \frac{w}{2g}(V_f^2) - w\,d\sin\theta$$

$w = 20\,N$, $g = 9.8\ m/s^2$, $d = 5\ m$, $\theta = 25°$

$$(20\,N)(5\,m)(\sin 25) - 20\,J = \frac{20\,N}{(2\times 9.8)\,m/s^2} V_f^2$$

$$22.26\,J = \frac{20\,N}{19.6\ m/s^2} V_f^2$$

$$V_f = 4.7\ m/s$$

6. $\frac{-dU}{dx} = F(x)$

$$U(x) = -\int F(x)\,dx$$

$$U(x) = -\int (\alpha - \beta x^3)\,dx$$

$$U(x) = -\left(\alpha x - \tfrac{1}{4}\beta x^4\right) + c$$ (c is a constant)

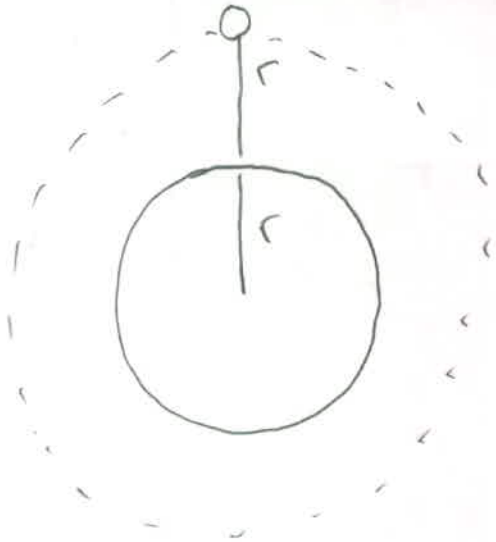
$$U(x) = \tfrac{1}{4}\beta x^4 - \alpha x + c$$

$$U(0) = 0 = \tfrac{1}{4}\cdot 0 - \alpha\cdot 0 + c$$

$$\Rightarrow c = 0$$

$$U(x) = -\alpha x + \tfrac{1}{4}\beta x^4$$

7.



$$U_g = -\frac{G m_1 m_2}{r} = -\frac{G M_E \, m_s}{2 r_E} \quad \text{DONE}$$

* can plug in units directly, or,

with $g = \frac{G M_E}{r_E^2} = 9.8\ m/s^2$

$$U_g = \frac{-g \times r_E}{2} \times m_s$$ in our specific case

$$U_g = -\frac{(9.8\ m/s^2)(6.37 \times 10^6\ m)}{2} (200\ kg)$$

estimation can be quicker than a calculator and more reliable (fewer typos)

$$U_g \approx -10^1 (6 \times 10^6)(10^2)\ J$$

$$U_g \approx -6 \times 10^9\ J$$ (good enough to get right answer)

or more exactly

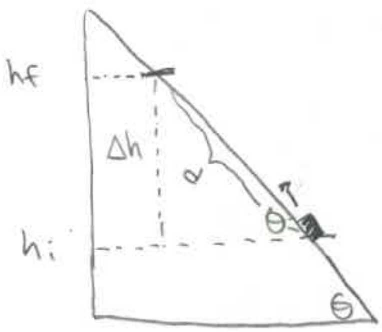
$$U_g = -\frac{(9.8\ m/s^2)(6.37 \times 10^6\ m)(200\ kg)}{2}$$

$$U_g = -6.24 \times 10^9\ J$$

(actually, this discrepancy comes from $g = \frac{G M_E}{r_E^2}$ being rounded out to 2 decimal places. Here it is good enough, but for stricter accuracy, plug in at the very top.)

$$U_g = -6.27 \times 10^9\ J$$

8.



$$\Delta U = mgh_f - mgh_i$$
$$= mg\,\Delta h$$

$$\Delta h = d \sin\theta$$

$$\Delta U = mgd\sin\theta$$

$$\Delta U = (49\,kg)(9.8\,m/s^2)(17.4\,m)(\sin 37)$$

$$\Delta U = 4600\,J$$