Physics 1A

Lecture 3A

"I believe there are

15,747,724,136,275,002,577,605,653,961,181,555,468,044,717,9

14,527,116,709,366,231,425,076,185,631,031,296

protons in the universe and the same number of electrons."

--Sir Arthur Eddington

Kinematics Equations

- The kinematics equations that we learned for one dimensional motion (when a = const) still hold when we increase to two dimensions.

- But the equations apply separately to each component of two dimensional motion.

- BIG PHYSICS CONCEPT!!!!!!

- Horizontal and vertical motion are independent of one another.

- Neither motion affects the other.

2D Motion

- For example, if acceleration and velocity are colinear, then the velocity changes only in magnitude and not in direction.
- Thus, acceleration in the y-direction cannot affect velocity in the x-direction only in the y-direction.
- Also, acceleration that is always perpendicular to velocity changes only the direction and not the magnitude.

Kinematics Equations

- If there is an acceleration in the y-direction; it will not affect motion in the x-direction at all.

- So for motion in two dimensions you can use the following equations as long as a_x and a_y are constants.

$$v_x = v_{ox} + a_x t \qquad v_y = v_{oy} + a_y t$$

$$\Delta x = \tfrac{1}{2}\left(v_x + v_{ox}\right)t \qquad \Delta y = \tfrac{1}{2}\left(v_y + v_{oy}\right)t$$

$$\Delta x = v_{ox} t + \tfrac{1}{2} a_x t^2 \qquad \Delta y = v_{oy} t + \tfrac{1}{2} a_y t^2$$

$$v_x^2 = v_{ox}^2 + 2a_x \Delta x \qquad v_y^2 = v_{oy}^2 + 2a_y \Delta y$$

Kinematics Equations

- Note that we have subscripts now denoting variables that are in the x and y directions.

- Also note that time has no subscript. It is the one link between the two sets of equations.

$$v_x = v_{ox} + a_x t \qquad v_y = v_{oy} + a_y t$$

$$\Delta x = \tfrac{1}{2}\left(v_x + v_{ox}\right)t \qquad \Delta y = \tfrac{1}{2}\left(v_y + v_{oy}\right)t$$

$$\Delta x = v_{ox} t + \tfrac{1}{2} a_x t^2 \qquad \Delta y = v_{oy} t + \tfrac{1}{2} a_y t^2$$

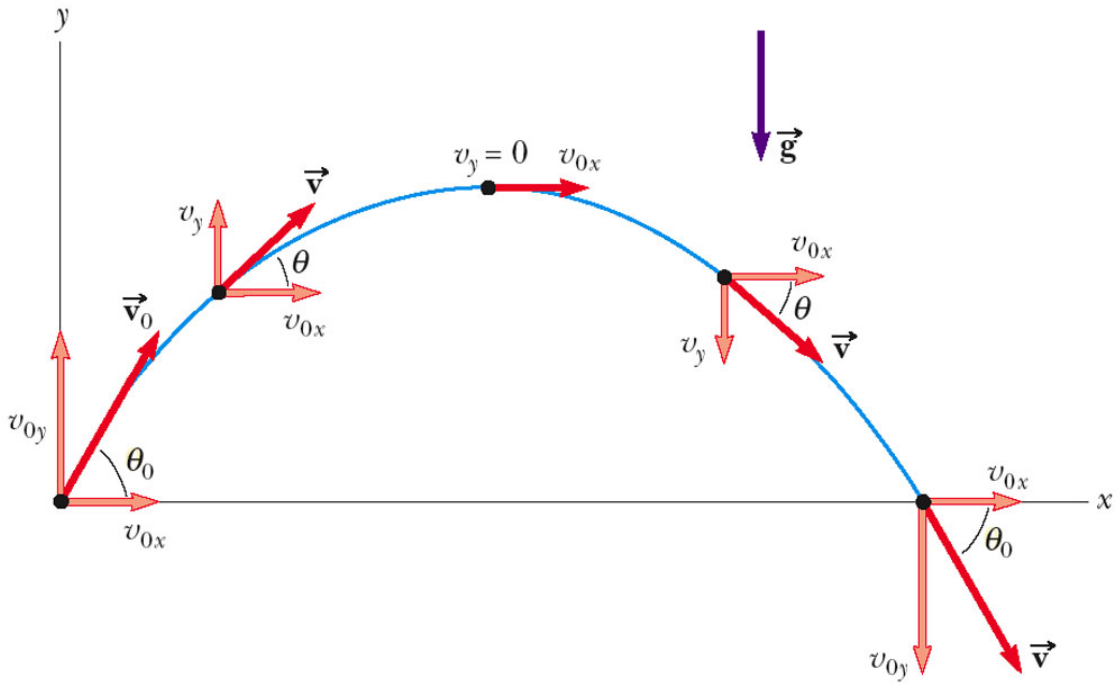
$$v_x^2 = v_{ox}^2 + 2a_x \Delta x \qquad v_y^2 = v_{oy}^2 + 2a_y \Delta y$$

Projectile Motion

- An object that moves in both the x and y directions simultaneously is said to have projectile motion.

- With projectile motion we are assuming that air friction is negligible.

- An object following projectile motion will follow a parabolic path.



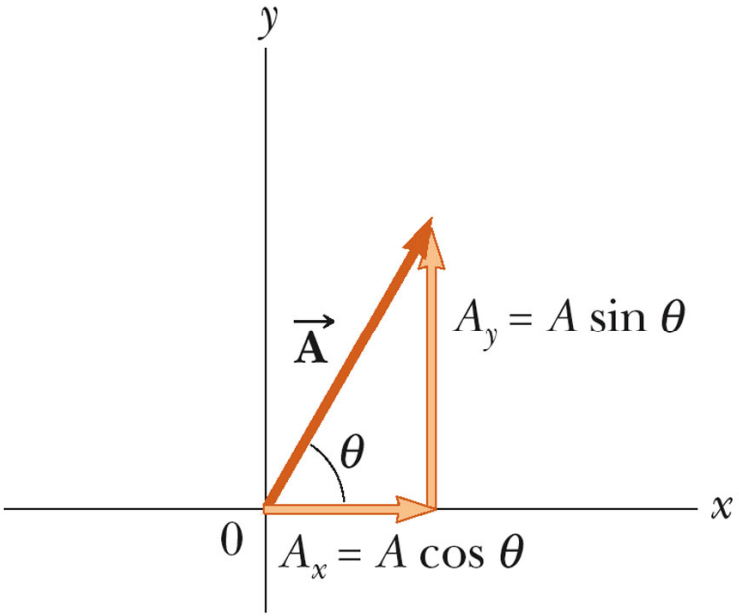
Projectile Motion

- Typical Projectile Motion Problem
- A cannonball is shot at a given angle (30.0°) with a certain muzzle velocity (100m/s). How far from the cannon does the cannonball land?

- Answer
- First, you must define a coordinate system.
- Let's choose the upward direction as positive in the y-direction, and the horizontal direction the cannonball travels as the positive x-direction.

Projectile Motion

- Answer
- To handle, separate into two problems (x and y).
- First, break velocity into components.



$$\sin\theta = \frac{v_y}{|\vec{v}|} \qquad \cos\theta = \frac{v_x}{|\vec{v}|}$$

$$v_y = |\vec{v}|\sin\theta = \left(100\,{}^{m}\!/_{s}\right)\sin 30^\circ \qquad v_x = |\vec{v}|\cos\theta = \left(100\,{}^{m}\!/_{s}\right)\cos 30^\circ$$

$$v_y = \left(100\,{}^{m}\!/_{s}\right)\frac{1}{2} = 50.0\,{}^{m}\!/_{s} \qquad v_x = \left(100\,{}^{m}\!/_{s}\right)\frac{\sqrt{3}}{2} = 86.6\,{}^{m}\!/_{s}$$

Projectile Motion

- Answer
- Now, use the kinematics equations separately in each direction.
- First, let's try the x-direction.
- Let's list the quantities we know:
- v_{ox} = +86.6m/s
- a_x = 0m/s^2 = constant <-- no acc. horizontally
- v_x <-- don't know
- Δx <-- finding
- t <-- don't know

- Let's try the third equation:

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad \Delta x = v_{ox}t \cdot$$

Projectile Motion

- Answer
- That didn't help us much, one equation with two unknowns.

$$\Delta x = v_{ox} t \cdot$$

- What about the other three kinematics equations for the x-direction (with $a_x = 0$)?

$$v_x = v_{ox} + a_x t \qquad v_x = v_{ox}$$

$$v_x^2 = v_{ox}^2 + 2 a_x \Delta x \qquad v_x^2 = v_{ox}^2$$

$$\Delta x = \tfrac{1}{2}\left(v_x + v_{ox}\right) t \qquad \Delta x = v_{ox} t \cdot$$

- We should turn to the y-direction.

Projectile Motion

- Answer
- Now, let's try the y-direction.
- Let's list the quantities we know:
- v_{oy} = +50.0m/s
- a_y = -9.80m/s^2 = constant
- v_y <-- don't know
- $\Delta y = 0$ <-- falls to same height
- t <-- don't know

- What is the one variable we can solve for in the y-direction and place in the x-direction equations?
- time, t: so we are know finding time.

Projectile Motion

- Answer
- Looks like it's the third equation for us.

$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$\Delta y = \left(50.0\,{}^{m}\!/_{s}\right)t + \tfrac{1}{2}\left(-9.80\,{}^{m}\!/_{s^2}\right)t^2 = \left(50.0\,{}^{m}\!/_{s}\right)t - \left(4.90\,{}^{m}\!/_{s^2}\right)t^2$$

$$\Delta y = t\left[\left(50.0\,{}^{m}\!/_{s}\right) - \left(4.90\,{}^{m}\!/_{s^2}\right)t\right] = 0$$

$$t = 0 \qquad t = \frac{\left(50.0\,{}^{m}\!/_{s}\right)}{4.90\,{}^{m}\!/_{s^2}} = 10.2\ \text{sec}$$

- t = 0 represents when the ball was first shot.
- t = 10.2 sec is the time is takes to hit the ground.

Projectile Motion

- Answer
- We input this time into the x-direction equations:

$$\Delta x = v_{ox} t \cdot$$

$$\Delta x = v_{ox} t = \left(86.6\,{}^{m}/_{s}\right)(10.2\,\text{sec}) = 883\,\text{m}$$

- This is the horizontal distance travelled by the cannonball.

- Other typical questions asked are:
- How high does it go?
- How long is it in the air?

Conceptual Question

- A cannonball is shot from a cannon at an angle of 30° with respect to the horizontal. Which of the following choices is correct concerning the resulting projectile motion?

- A) The speed is zero at the top of its path.
- B) The speed is a maximum at the top of its path.
- C) The speed is a minimum at the top of its path.
- D) The acceleration is zero at the top of its path.
- E) The speed is 9.8 m/s at the top of its path.

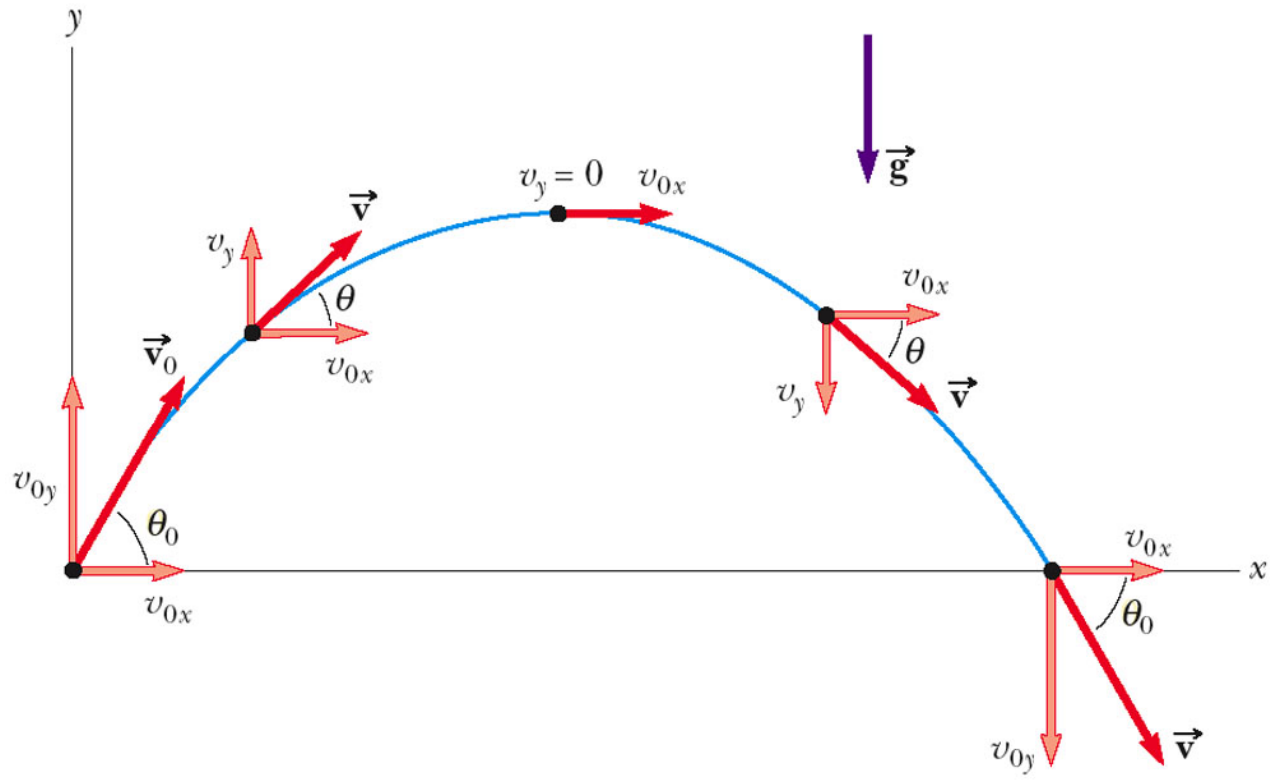
Projectile Motion

- At any point in the path of a projectile, the projectile's velocity (magnitude) will be:

$$v = \sqrt{v_x^2 + v_y^2}$$

- And the angle it makes with the horizontal will be:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



Projectile Motion

- Key facts about projectile motion:

- When a moving object (such as a plane) releases an object (like a care package), the object will move with the same velocity as the moving object (until air resistance overcomes).

- Giving an object velocity or acceleration in the horizontal direction does not affect how fast it falls to the ground.

- Horizontal motion does not affect vertical motion.

Projectile Motion

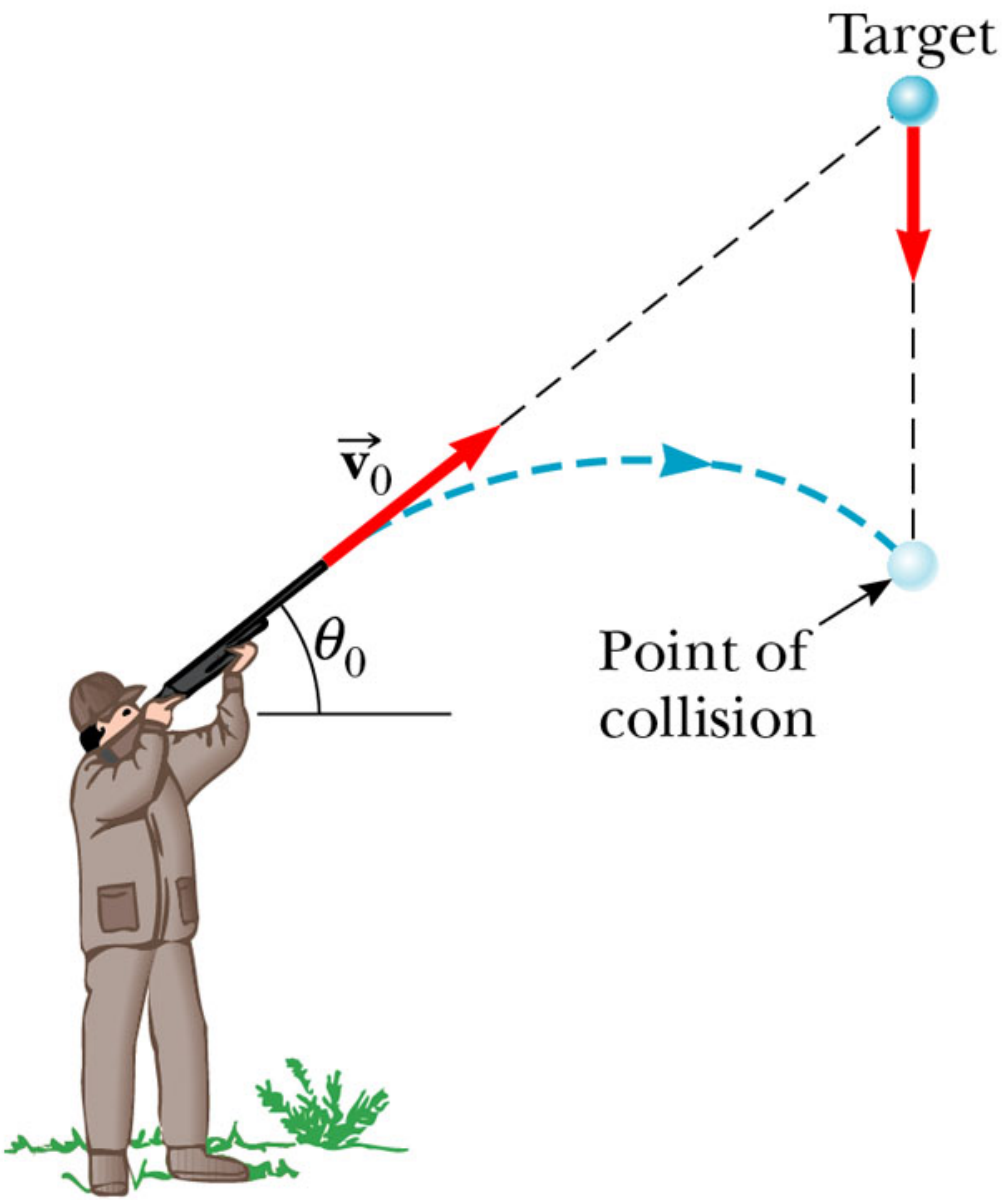
- Example
- A hunter tries to shoot a rare monkey hanging from a tree with a dart gun. The hunter has the monkey in his sights. But the monkey notices the hunter and drops from the branch exactly when he hears the hunter blow. Will the hunter hit the monkey or miss the target?

Clicker Question 4B-1

- A) Will the dart hit the monkey?
- B) Will the dart miss above the monkey?
- C) Will the dart miss below the monkey?

Projectile Motion

- Answer
- Monkey and the dart fell at the same rate, under the same influence of gravity.
- So the dart will hit the monkey.



For Next Time (FNT)

- Work on Chapter 3 homework