Physics 1A

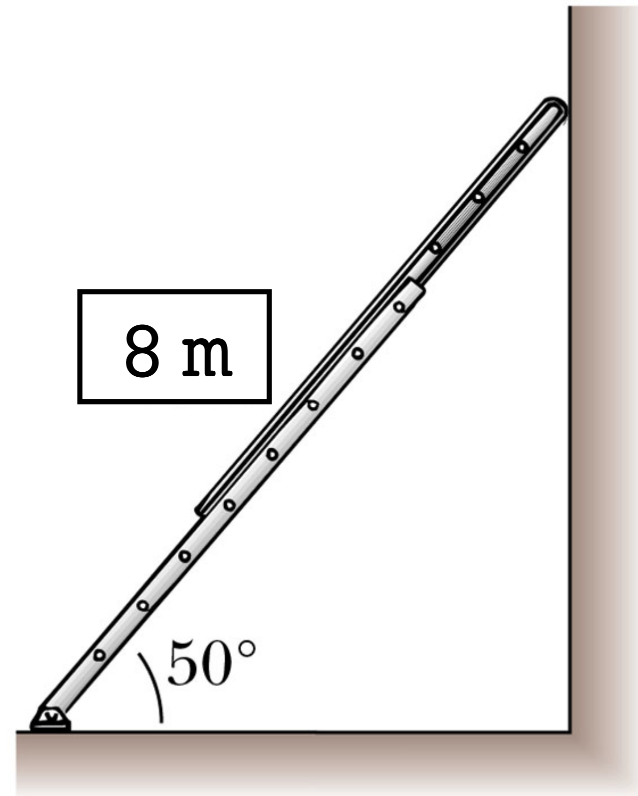
Lecture 10C

"If you neglect to recharge a battery, it dies. And if you run full speed ahead without stopping for water, you lose momentum to finish the race."

--Oprah Winfrey

Static Equilibrium

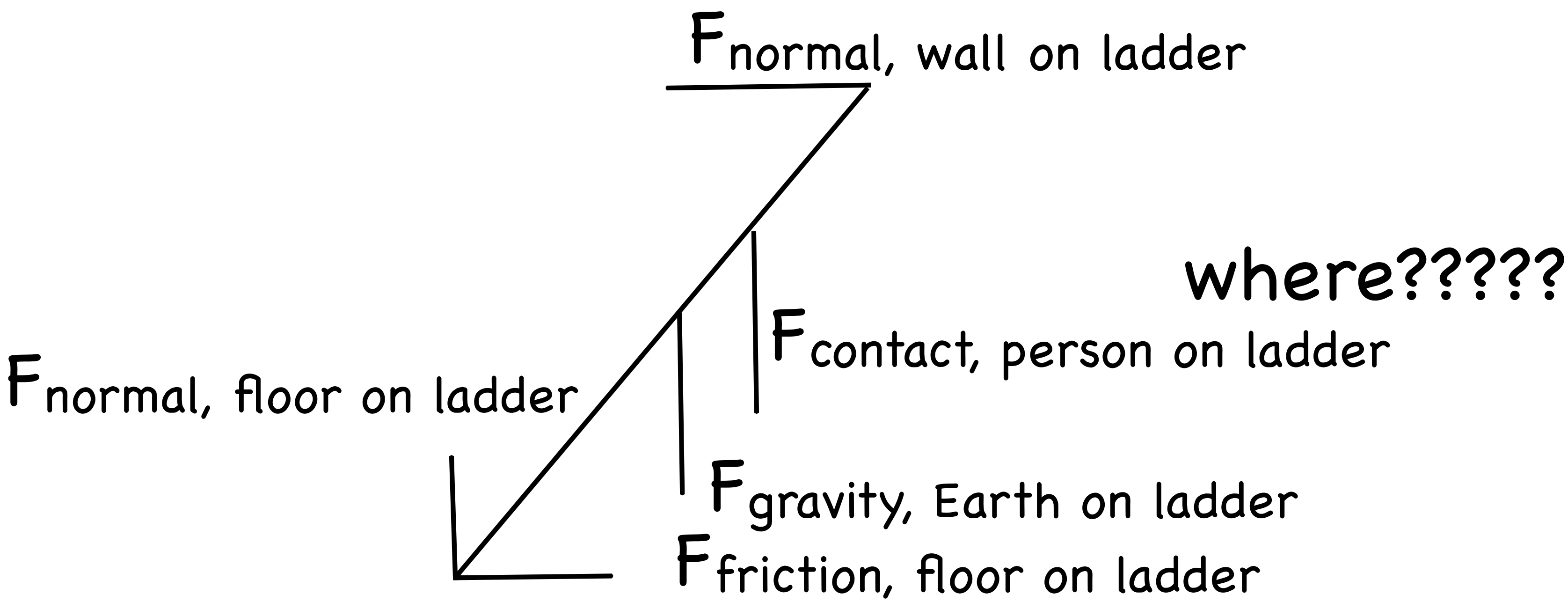
- <u>Example</u>
- An 8.00m, 200N uniform ladder rests against a smooth wall. The coefficient of static friction between the ladder and the ground is 0.600, and the ladder makes a 50.0° angle with the ground. How far up the ladder can an 800N person climb before the ladder begins to slip?



- <u>Answer</u>
- First, you must define a coordinate system.
- Choose x to the right as positive and up as the positive y-direction.

Static Equilibrium

- Answer
  - Next, draw an extended force diagram:



  - Since the wall is smooth it has no friction.
  - If the ladder is on the verge of slipping, then the force due to friction from the floor will be a maximum:

$$F_{friction} = \mu_s F_N$$

Static Equilibrium

<u>Answer</u>

Try summing the forces in the y-direction:

$$\Sigma F_y = 0$$

$$\Sigma F_y = F_N - F_g - F_{person} = 0$$

$$F_N = F_g + F_{person}$$

$$F_N = 200N + 800N = 1{,}000N$$

Thus, we can go back and calculate the frictional force:

$$F_{friction} = \mu_s F_N$$

$$F_{friction} = (0.600)(1{,}000\text{N}) = 600\text{N}$$

Try summing the forces in the x-direction:

$$\Sigma F_x = 0$$

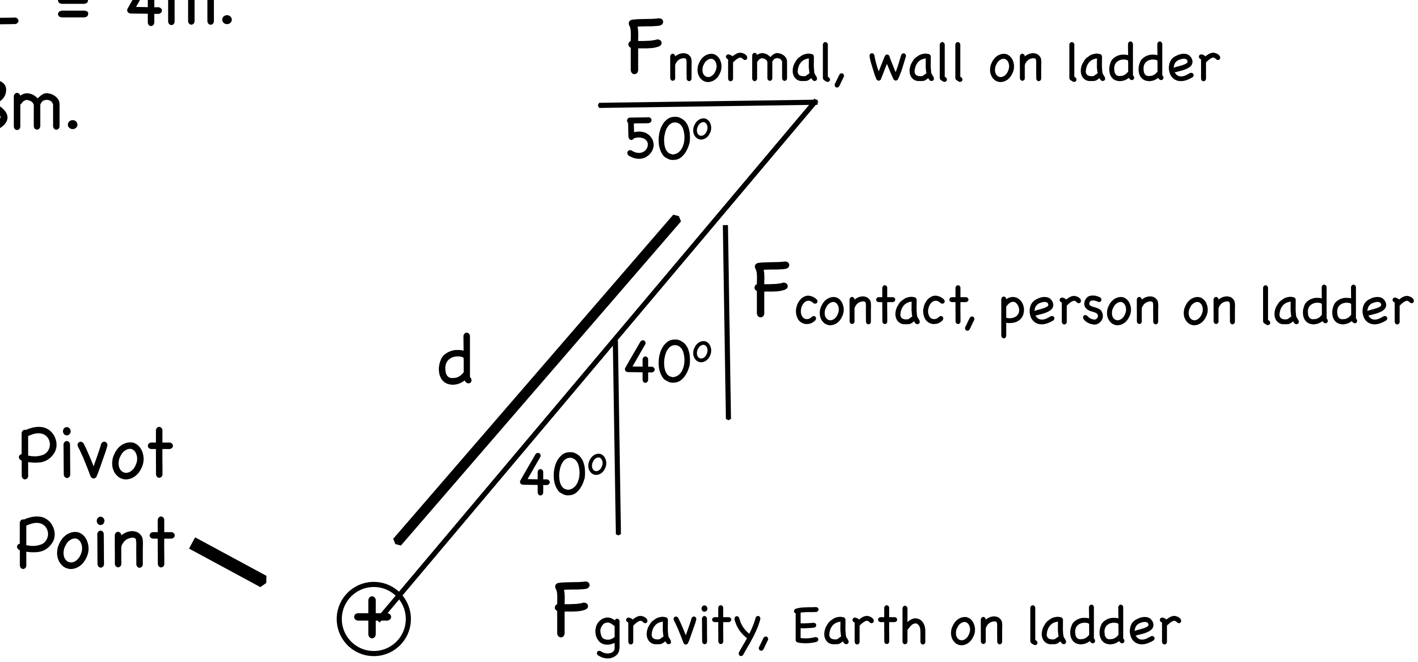
$$\Sigma F_x = F_{friction} - F_{wall} = 0$$

$$F_{wall} = F_{friction} = 600N$$

Static Equilibrium

- <u>Answer</u>
- Next, we can turn to calculating the net torque.

- Choose the pivot point at the bottom of the ladder (this choice eliminates F_N and $F_{friction}$).
  - r_{grav} = (0.5)L = 4m.
  - r_{wall} = L = 8m.
  - r_{person} = d.



- Calculate the individual torques for the three remaining forces.

Static Equilibrium

<u>Answer</u>

$$\tau_{wall} = (r_{wall})(F_{wall})\sin\theta$$

$$\tau_{wall} = (8\text{m})(600\text{N})\sin 50°$$

$$\tau_{wall} = +3{,}667\text{N}\cdot\text{m}$$

<-- plus sign comes from counterclockwise rotation

$$\tau_{grav} = (r_{grav})(F_{grav})\sin\theta$$

$$\tau_{grav} = (4\text{m})(200\text{N})\sin 40°$$

$$\tau_{grav} = -514\text{N}\cdot\text{m}$$

<-- minus sign comes from clockwise rotation

$$\tau_{person} = (r_{person})(F_{person})\sin\theta$$

$$\tau_{person} = (d)(800\text{N})\sin 40°$$

$$\tau_{person} = -514\text{N}(d)$$

<-- minus sign comes from clockwise rotation

Static Equilibrium

- Answer
- Next, we can sum the torques and set them equal to zero since it is in equilibrium.

$$\sum \tau = \tau_{wall} + \tau_{grav} + \tau_{person} = 0$$

$$\sum \tau = 3{,}667\text{N} \cdot \text{m} - 514\text{N} \cdot \text{m} - 514\text{N}(d) = 0$$

$$514\text{N}(d) = 3{,}667\text{N} \cdot \text{m} - 514\text{N} \cdot \text{m} = 3{,}163\text{N} \cdot \text{m}$$

$$d = \frac{3{,}163\text{N} \cdot \text{m}}{514\text{N}} = 6.15\text{m}$$

- This the distance that the person climbed up the ladder.

Newton's Laws (Rotationally)

- We can relate Newton's Laws with rotational motion with the following (if rotational inertia is constant):

$$\sum \vec{\tau} = I\vec{\alpha}$$

- This is very similar to $\Sigma F = ma$:
- $\Sigma\tau$ is like a rotational force.
- I is like a rotational mass (inertia)
- α is angular acceleration.
- This means that a net torque on a rigid object will lead to an angular acceleration.

Newton's Laws (Rotationally)

- We can also turn to Newton's 3rd Law rotationally:

$$\vec{\tau}_{1on2} = -\vec{\tau}_{2on1}$$

- This means that if I exert a torque on an object, then it will exert the same torque right back at me but opposite in direction.
- This demonstrates the vector nature of torques and angular motion.
- This stool is an excellent example.

Work

- Just like a force can perform work over a distance, torque can perform work over an angle.
- For a constant torque: $$W = \tau(\Delta\theta)$$
- For a variable torque: $$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$
- By the work-energy theorem, we can say that:

$$W = \Delta KE$$

- Making something spin around an axis is another place to put energy.

Rotational Kinetic Energy

- So we can define a new type of kinetic energy; rotational kinetic energy, KE_{rot}:

$$KE_{rot} = \frac{1}{2} I\omega^2$$

- Rotational kinetic energy is similar to linear kinetic energy (just switch from linear variables to rotational variables).

- The units of rotational kinetic energy are still Joules.

- Rotational kinetic energy is just a measure of how much energy is going into rotating an object.

Conceptual Question

- A solid disk and a hoop are rolled down an inclined plane (without slipping). Both have the same mass and the same radius. Which one will reach the bottom of the incline first?

- A) The solid disk.
- B) The hoop.
- C) They will reach the bottom at the same time.

Rotational Kinetic Energy

- Example
- A solid disk and a hoop are rolled down an inclined plane (without slipping). Both have a mass of 1.0kg and a radius of 50cm. They are both originally placed at a height of 1.5m. What is the ratio of their velocities (v_{disk}/v_{hoop}) at the bottom of the inclined plane?

- Answer
- Choose up as the positive y-direction (y = 0 at bottom of the ramp).
- We also know that: $I_{disk} = (1/2)mr^2$ $I_{hoop} = mr^2$

Rotational Kinetic Energy

- <u>Answer</u>
- Use conservation of energy.
- At the top of the inclined plane, there is no KE such that:

$$E_{top} = KE + PE = 0 + PE$$

$$E_{top} = mgh = (1.0\text{kg})(9.8\,\text{N}/\text{kg})(1.5\text{m})$$

$$E_{top} = 14.7\text{J}$$

- At the bottom of the inclined plane, there is no PE such that:

$$E_{bot} = KE + PE = KE + 0 = KE_{trans} + KE_{rot}$$

$$E_{bot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Rotational Kinetic Energy

- Answer
  - Since v = rω (no slipping), we can write:

$$E_{bot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$$

- Due to conservation of energy we can say:

$$E_{top} = E_{bot}$$

$$14.7\text{J} = \tfrac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$$

$$v^2 = \frac{2(14.7\text{J})}{\left(m + \frac{I}{r^2}\right)} = \frac{(29.4\text{J})}{\left(m + \frac{I}{r^2}\right)}$$

- This equation is true for either shape.

Rotational Kinetic Energy

<u>Answer</u>

- For the solid disk, I = (1/2)mr²

- So, for the disk we can say that:

$$v^2 = \frac{(29.4\text{J})}{\left(m + \frac{\left(\frac{1}{2}mr^2\right)}{r^2}\right)} = \frac{(29.4\text{J})}{\left(m + \frac{1}{2}m\right)} = \frac{(29.4\text{J})}{\frac{3}{2}(1\text{kg})}$$

$$v^2 = 19.6 \,{}^{\text{m}^2}\!/_{\text{s}^2} \qquad v_{disk} = 4.4 \,{}^{\text{m}}\!/_{\text{s}}$$

- This is the disk's velocity at the bottom of the incline.

Rotational Kinetic Energy

Answer

For the hoop, $I = mr^2$

So, for the hoop we can say that:

$$v^2 = \frac{(29.4\text{J})}{\left(m + \frac{(mr^2)}{r^2}\right)} = \frac{(29.4\text{J})}{(m+m)} = \frac{(29.4\text{J})}{2(1\text{kg})}$$

$$v^2 = 14.7\ {}^{\text{m}^2}\!/_{\text{s}^2} \qquad v_{hoop} = 3.8\ {}^{\text{m}}\!/_{\text{s}}$$

This is the hoop's velocity at the bottom of the incline.

Ratio is:

$$\frac{v_{disk}}{v_{hoop}} = \frac{4.4\ {}^{\text{m}}\!/_{\text{s}}}{3.8\ {}^{\text{m}}\!/_{\text{s}}} = 1.2$$

Angular Momentum

- We are aware of linear momentum. There is a rotational equivalent known as angular momentum.
- Angular momentum, L, is given by: $$\vec{L} = \vec{r} \times \vec{p}$$
- It is a measure of how perpendicular p and r are.
- The units for angular momentum are: $kg(m^2/s)$.
- Take the time derivative of angular momentum to find:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{L}}{dt} = m\frac{d}{dt}(\vec{r} \times \vec{v})$$

- Apply the product rule to get:

$$\frac{d\vec{L}}{dt} = m\left(\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}\right)$$

Angular Momentum

$$\frac{d\vec{L}}{dt} = m(\vec{v} \times \vec{v} + \vec{r} \times \vec{a})$$

$$\frac{d\vec{L}}{dt} = m(\vec{r} \times \vec{a}) = (\vec{r} \times m\vec{a})$$

$$\frac{d\vec{L}}{dt} = \left(\vec{r} \times \sum \vec{F}\right) = \sum \vec{\tau}$$

- If the net external torque on an object is zero, then angular momentum is conserved.
- Angular momentum creates an "axis of stability" which takes some effort to remove.
- This axis is caused by the angular momentum which would prefer to be conserved.

Angular Momentum

- The "real" way to define net torque is:

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$L = \int I\alpha dt = \int I \frac{d\omega}{dt} dt$$

$$L = I\omega$$

- This is another equation to find the magnitude of angular momentum.

- If you decrease your moment of inertia, then your angular velocity will increase.

Angular Momentum

- Example
- A merry-go-round (m = 100kg, r = 2.00m) spins with an angular velocity of 2.50(rad/s). A monkey (m = 25.0kg) hanging from a nearby tree, drops straight down onto the merry-go-round at a point 0.500m from the edge. What is the new angular velocity of the merry-go-round?

  - Answer
  - We can assume that the merry-go-round rotates in the positive direction (ccw).

Angular Momentum

- Answer
- Use conservation of angular momentum; there is no net external torque.

- Before the monkey jumps on, L is:

$$L_i = I_i\omega_i = I_{disk}\omega_i = \left(\tfrac{1}{2}m_d r_d^2\right)\omega_i$$

$$L_i = (0.5)100\text{kg}(2.00\text{m})^2(2.50\ {}^{\text{rad}}\!/\!_{\text{s}}) = 500\text{kg}\ {}^{\text{m}^2}\!/\!_{\text{s}}$$

- After the monkey jumps on (at r = 1.50m), L is:

$$L_f = I_f\omega_f = \left(I_{disk} + I_{monkey}\right)\omega_f = \left(\tfrac{1}{2}m_d r_d^2 + m_m r_m^2\right)\omega_f$$

$$L_f = \left[(0.5)100\text{kg}(2.00\text{m})^2 + 25.0\text{kg}(1.50\text{m})^2\right]\omega_f$$

Angular Momentum

- Answer

$$L_f = \left(256\text{kg}\cdot\text{m}^2\right)\omega_f$$

- By conservation of angular momentum:

$$L_i = L_f$$

$$500\text{kg}\,\text{m}^2/\text{s} = \left(256\text{kg}\cdot\text{m}^2\right)\omega_f$$

$$\omega_f = \frac{500\text{kg}\,\text{m}^2/\text{s}}{256\text{kg}\cdot\text{m}^2} = 1.95\ \text{rad}/\text{s}$$

- As the monkey jumps on the merry-go-round, the moment of inertia of the system increases.

- The angular velocity will decrease due to conservation of angular momentum.

For Next Time (FNT)

- Finish the Homework for Chapter 10.
- Start reading Chapter 15.