Physics 1A

Lecture 10B

"Sometimes the world puts a spin on life. When our equilibrium returns to us, we understand more because we've seen the whole picture."

--Davis Barton

Cross Products

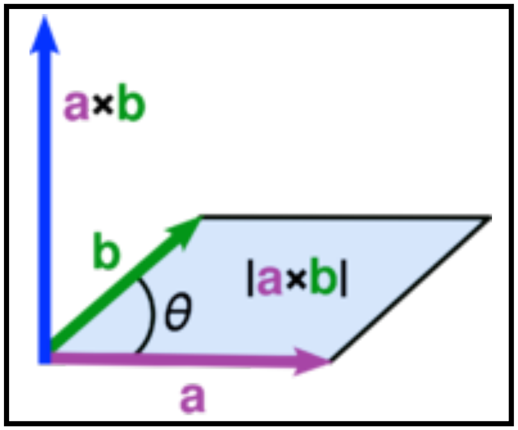
- Another way to multiply vectors is the cross product:

$$\vec{C} = \vec{A} \times \vec{B}$$

- The magnitude of C is given by:

$$C = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta$$

- where θ is the angle between vectors A and B.



- The direction of C is determined by the Right Hand Rule.
- C is perpendicular to both A and B.
- A cross product measures how perpendicular two vectors (A and B) are.
- If A and B are parallel, their cross product is zero.

Cross Products

- Thus, for unit vectors: $\boxed{\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0}$
- Also, for î and ĵ: $\boxed{\hat{i}\times\hat{j} = \hat{k}}$ $\boxed{\hat{j}\times\hat{i} = -\hat{k}}$
- For 2D vectors A and B: $\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}$ $\boxed{\vec{B} = B_x\hat{i} + B_y\hat{j}}$
- the cross product will be given by:

$$\boxed{\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right)\times\left(B_x\hat{i} + B_y\hat{j}\right)}$$

$$\boxed{\vec{A}\times\vec{B} = A_yB_x\left(\hat{j}\times\hat{i}\right) + A_xB_y\left(\hat{i}\times\hat{j}\right)}$$

$$\boxed{\vec{A}\times\vec{B} = \left(A_xB_y - A_yB_x\right)\hat{k}}$$

- Warning! The cross product is not commutative: $\boxed{\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}}$
- In fact: $\boxed{\vec{A}\times\vec{B} = -\left(\vec{B}\times\vec{A}\right)}$

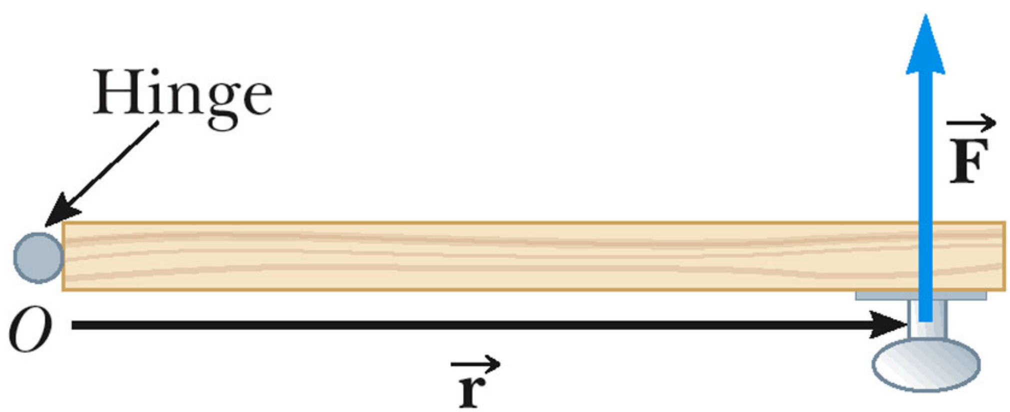
Torque

- When applying force to rotating objects, it is very important to understand where the force is being applied on the object.

- We turn to torque, or turning force, τ. The magnitude of torque is given by:

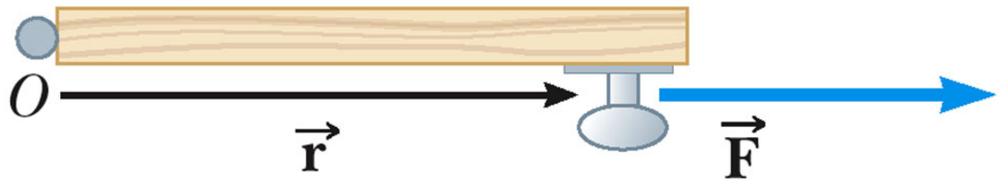
$$\tau = |\vec{r}||\vec{F}|\sin\theta = r_{\perp}F$$

- where F is the applied force, r is the radius from the axis of rotation (or pivot point) and θ is the angle between r and F

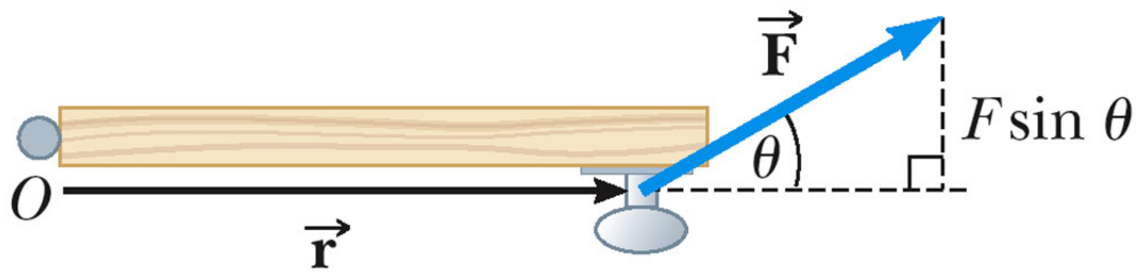


Torque

- When the force is parallel to r, then the magnitude of torque is zero.



- When the force is at some angle to r, the perpendicular component of F is what causes the rotation.

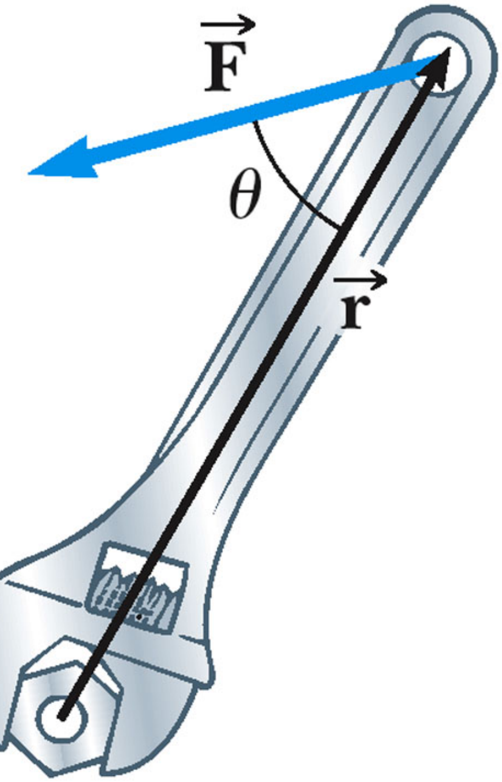


- The SI unit for torque is the [Nm] (not a Joule).
- Just as force causes linear accelerations on an object, torque will cause angular accelerations on an object.

Torque

- Torque is a vector quantity. The direction is perpendicular to the plane determined by r and F.
- Mathematically, we say: $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$
- The direction is determined by the Right Hand Rule.

- Point your fingers in the direction of r.
- Curl the fingers toward F.
- Your thumb points in the direction of torque.
- We use the same conventions as with rotational variables. A counterclockwise torque is considered positive and a clockwise torque is considered negative.



Moment of Inertia

- Sometimes two rigid bodies that have the same mass will have different rotational properties. One may be harder to rotate than another.

- When it comes to rotation, not only do you have to take into account how much mass is present, but you also have to take into account how the mass is distributed compared to the axis of rotation.

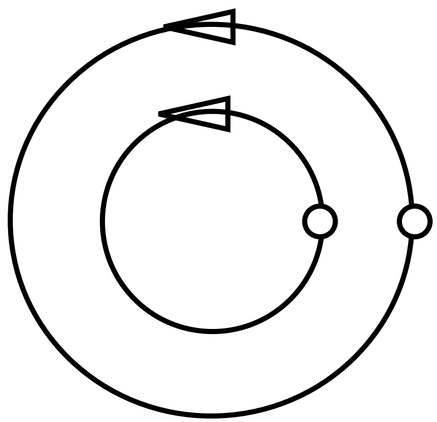
- To quantify this we introduce, I, the moment of inertia. You can calculate the moment of inertia by:

$$I = \sum mr^2$$

- where r is the perpendicular distance to the axis of rotation.

Moment of Inertia

- Example
- What is the moment of inertia of two balls (each with a mass of 1.0kg) moving around a common center? One ball is at r = 1.0m and the other is at r = 2.0m from the center of the circle.



- Answer
- $I = \Sigma mr^2 = m_1r_1^2 + m_2r_2^2$
- $I = (1.0kg)(1.0m)^2 + (1.0kg)(2.0m)^2$
- $I = 1.0(kg)m^2 + 4.0(kg)m^2 = 5.0(kg)m^2$.

Moment of Inertia

- If you have a continuous body, like a cylinder, then you can calculate the rotational inertia by:

$$\boxed{I = \int r^2 dm}$$

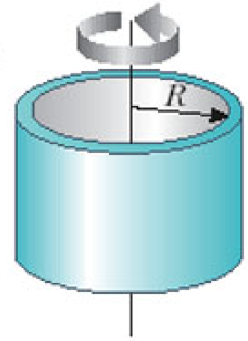
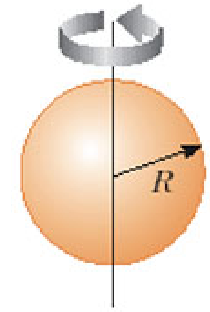
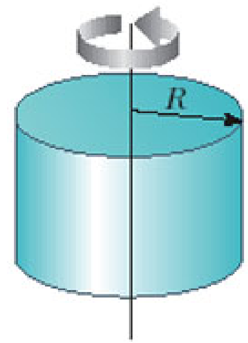
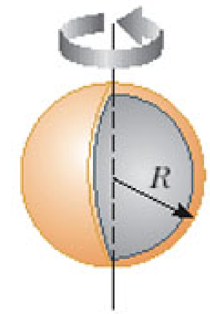
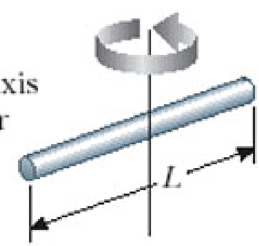
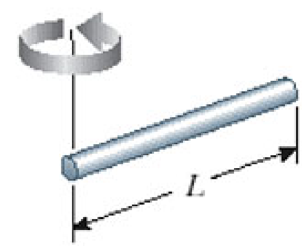
- Approach this the same way we approached center of mass (COM) calculations.
- Use facts like: $\boxed{\rho = \frac{M}{V} = \frac{dm}{dV}}$
- or mass per unit area: $\boxed{dm = \frac{M}{A} dA}$
- Example 10-4 in the text shows the proper way to find the rotational inertia of a uniform solid cylinder about an axis through its center: $\boxed{I_{disk} = \frac{1}{2} MR^2}$

Moment of Inertia

- You should become familiar with the basic moments of inertia for common shapes.
- All moments of inertia will be of the form (#)mr^2; you just need to determine (#).
- Note the moment of inertia has the SI units: (kg)m^2.

TABLE 8.1

Moments of Inertia for Various Rigid Objects of Uniform Composition

Hoop or thin cylindrical shell $I = MR^2$		Solid sphere $I = \frac{2}{5} MR^2$	
Solid cylinder or disk $I = \frac{1}{2} MR^2$		Thin spherical shell $I = \frac{2}{3} MR^2$	
Long thin rod with rotation axis through center $I = \frac{1}{12} ML^2$		Long thin rod with rotation axis through end $I = \frac{1}{3} ML^2$	

Equilibrium

- A rigid body has parts held in a fixed position with respect to other parts of the body.

- If a rigid body is in static equilibrium, it is not moving (either linearly or rotationally).

- The conditions for equilibrium to occur are that p_{com} and L_{com} are constant.

- But if we turn back to Newton's Laws this requires that:

$$\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt} = 0} \qquad \boxed{\sum \vec{\tau} = \frac{d\vec{L}}{dt} = 0}$$

- The sum of forces in the x, y, and z directions and the sum of the torques (at any given point) must be zero for equilibrium to exist.

Center of Gravity

- In order to help us solve torque problems, it is useful to define the center-of-gravity (CG).
- CG is very similar to Center Of Mass (COM). They are essentially the same thing.
- The CG of a rigid body is the point where all of the weight of the body can be considered to act upon.
- Free objects will more likely rotate about their CG than any other point.
- If I apply a force away from the CG, this will most likely cause rotation about the CG.
- If I apply a force at the CG, this will most likely just linearly move the object.

Torque

- When more than one torque is acting on an object, you must sum the torques, $\Sigma\tau$, as vectors (take direction and magnitude into account).

- The pivot point is the point where rotation is occurring or where rotation may occur.

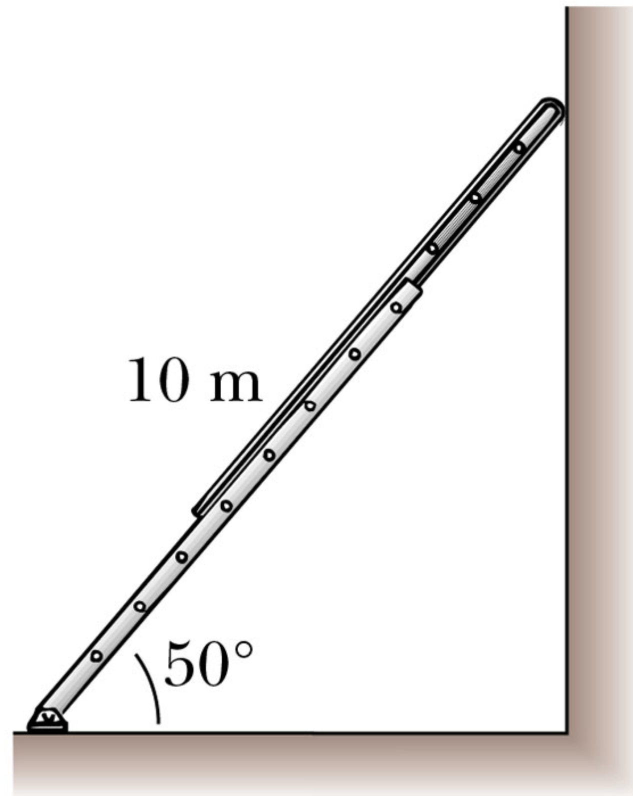
- Sometimes you are free to choose the pivot point (this ruler) and sometimes you are not (this door).

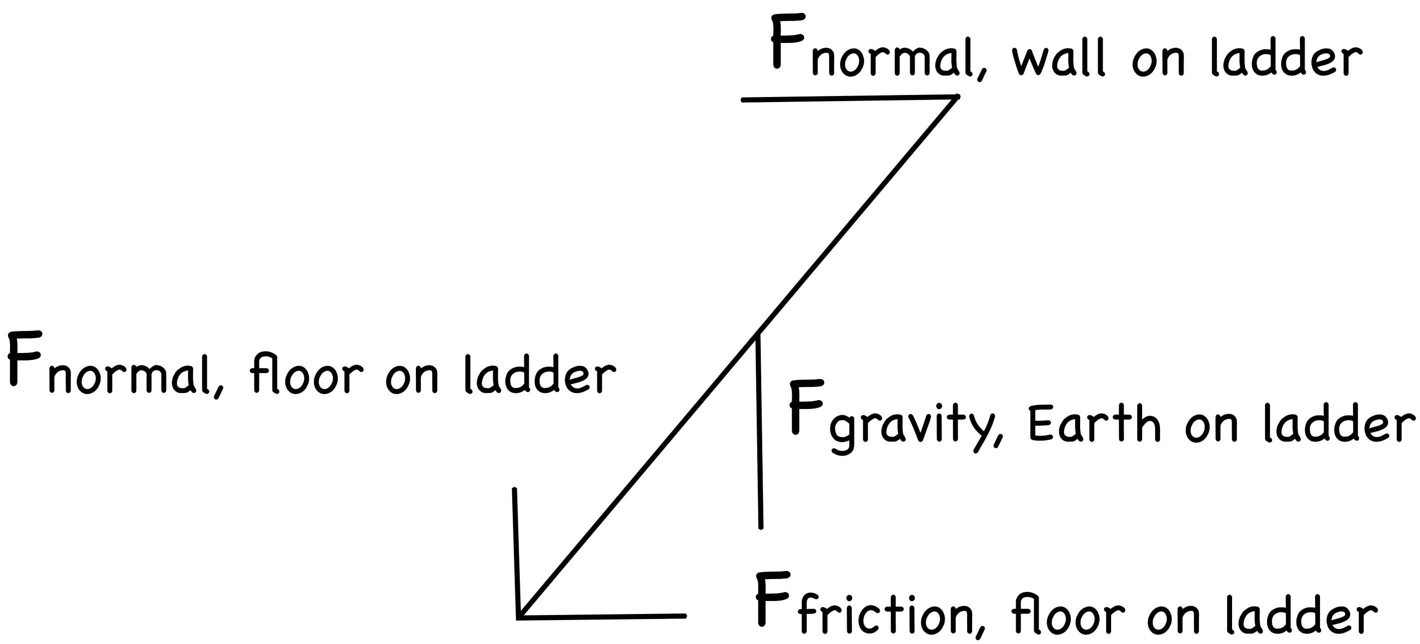
- When attempting to calculate torque values it becomes useful to draw what is called an extended force diagram.

- In this diagram you not only draw the forces that are acting on the object but also where they act.

Torque

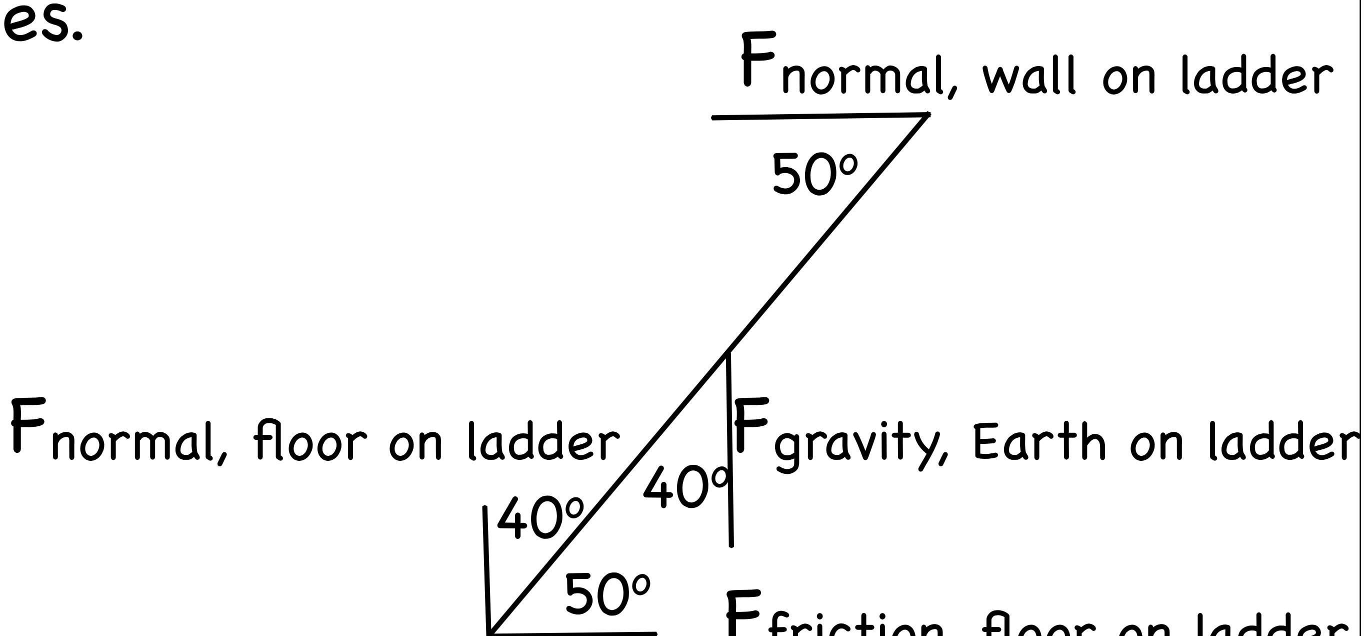
- For example, a ladder rests on a frictionless vertical wall. The floor however is not frictionless. Draw the appropriate extended force diagram for this situation.





Torque

- Note that the gravity acts in the center (CG).
- If we were to try and perform a torque calculation we could choose a pivot point at any location since it is not rotating.
- Take special care in noting what the angles are for the various forces.

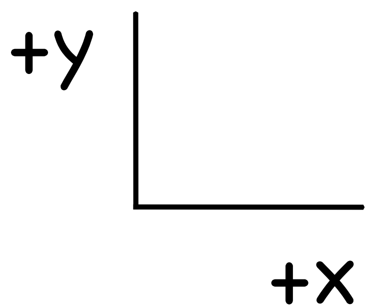


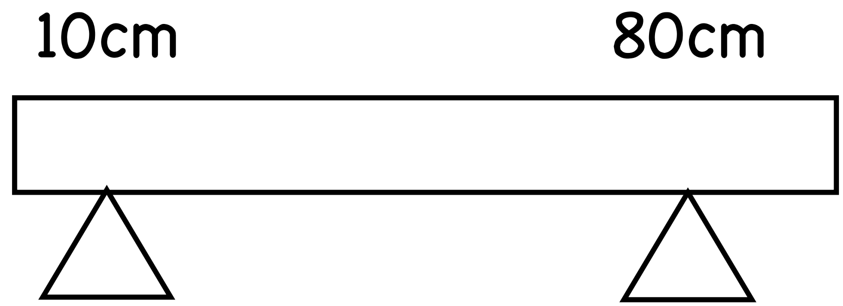
Solving Equilibrium Problems

- When performing equilibrium problems, follow similar guidelines to Newton's Laws:
- 1) Choose an appropriate coordinate system. (x,y)
- 2) Make an extended force diagram.
- 3) Choose appropriate equilibrium equations to apply.
- $\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma \tau = 0$
- 4) Choose appropriate pivot point for torque calculations.
- 5) Perform algebra or math techniques.

Static Equilibrium

- <u>Example</u>
- A uniform 0.10kg meter stick is statically held by two fulcrums (at 10cm and 80cm) as shown in the diagram. What is the upwards force exerted by the two fulcrums?

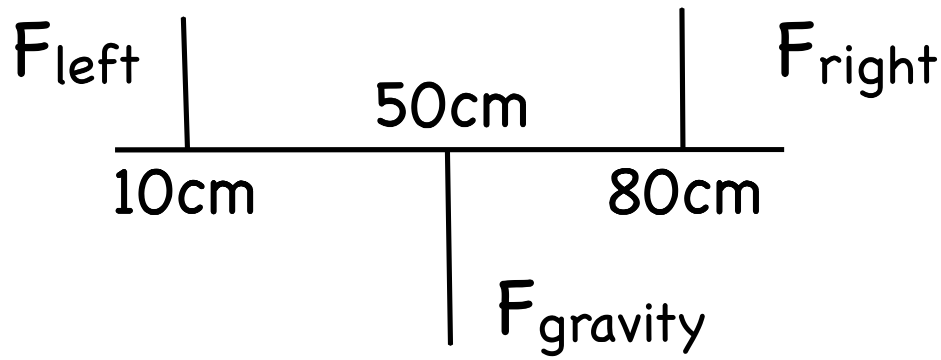




- <u>Answer</u>
- First, you must define a coordinate system.
- Choose x to the right as positive and up as the positive y-direction.

Static Equilibrium

- Answer
  - Next, draw an extended force diagram:



  - Since this meter stick is in static equilibrium we can apply the equilibrium conditions.
  - First, in the x-direction:
  - $\Sigma F_x = 0$ <- no help!
  - But we can turn to the y-direction:
  - $\Sigma F_y = 0$

Static Equilibrium

- Answer

$$\Sigma F_y = F_{left} + F_{right} - F_{gravity} = 0$$

$$F_l + F_r = F_g = mg = (0.1kg)(9.8N/kg)$$

$$F_l + F_r = 0.98N$$

- Next, we can turn to torque to find another relation.

- Choose the pivot point at the Center of Gravity (even though we can choose our pivot point anywhere) and calculate the different torques.

$$\tau_r = |\vec{r}_r||\vec{F}_r|\sin\theta = (0.3\mathrm{m})F_r\sin 90^\circ$$

$$\tau_r = +(0.3\mathrm{m})F_r$$

<-- plus sign comes from counterclockwise rotation

Static Equilibrium

- Answer

$$\tau_l = |\vec{r}_l||\vec{F}_l|\sin\theta = (0.4\text{m})F_l\sin 90^\circ$$

$$\tau_l = -(0.4\text{m})F_l$$

<-- minus sign comes from clockwise rotation

$$\tau_g = |\vec{r}_g||\vec{F}_g|\sin\theta = (0\text{m})F_g\sin\theta = 0$$

- Next, we can sum the torques and set them equal to zero since it is in equilibrium.

$$\sum\tau = \tau_r + \tau_l + \tau_g = 0$$

$$(0.3\text{m})F_r - (0.4\text{m})F_l = 0$$

$$(0.3\text{m})F_r = (0.4\text{m})F_l$$

Static Equilibrium

- <u>Answer</u>

$$F_l = \frac{(0.3\text{m})}{(0.4\text{m})} F_r = (0.75) F_r$$

- We can use this equation with our previous equation to solve (2 equations, 2 unknowns).

$$F_l + F_r = 0.98\text{N}$$

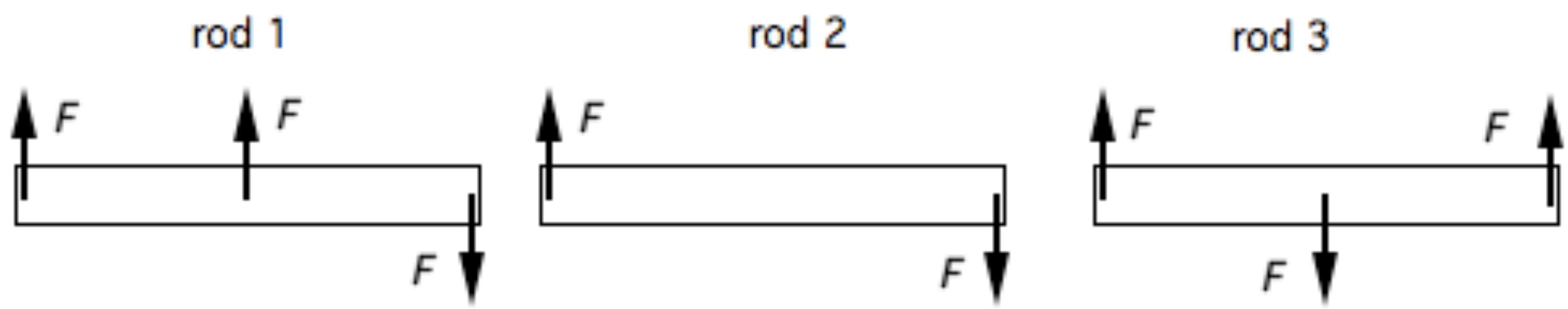
$$(0.75)F_r + F_r = (1.75)F_r = 0.98\text{N}$$

$$F_r = 0.56\text{N}$$

- F_l will then be: $F_l = (0.75)F_r = (0.75)0.56\text{N} = 0.42\text{N}$

Conceptual Question

- Three identical uniform static rods are each acted on by two or more forces, all perpendicular to the rods and all equal in magnitude (with a force of magnitude F). Which of these rods is in equilibrium?



- A) rod 1.
- B) rod 2.
- C) rod 3.
- D) None of these rods are in equilibrium.

For Next Time (FNT)

- Start the Homework for Chapter 10.
- Finish reading Chapter 10.