CHAPTER 36 IMAGE FORMATION AND OPTICAL INSTRUMENTS

ActivPhysics can help with these problems:
All activities in Section 15

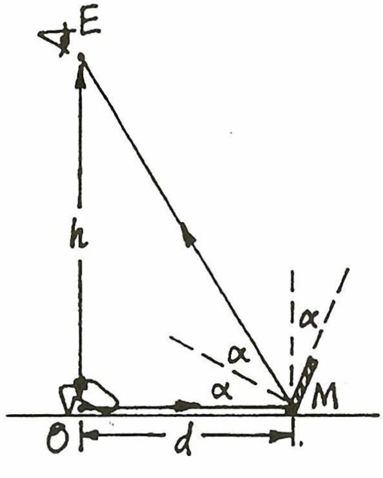
Sections 36-1 and 36-2: Plane and Curved Mirrors

Problem

1. A shoe store uses small floor-level mirrors to let customers view prospective purchases. At what angle should such a mirror be inclined so that a person standing 50 cm from the mirror with eyes 140 cm off the floor can see her feet?

Solution

A small mirror (M) on the floor intercepts rays coming from a customer's shoes (O), which are traveling nearly parallel to the floor. The angle to the customer's eye (E) from the mirror is twice the angle of reflection, so $\tan 2\alpha = h/d$, or $\alpha = \frac{1}{2}\tan^{-1} \times (140/50) = 35.2°$, for the given distances. Therefore, the plane of the mirror should be tilted by 35.2° from the vertical to provide the customer with a floor-level view of her shoes.



Problem 1 Solution.

Problem

2. Two plane mirrors occupy the first four meters of the positive x- and y-axes, as shown in Fig. 36-44. Find the locations of all images of an object at $x = 2$ m, $y = 1$ m.

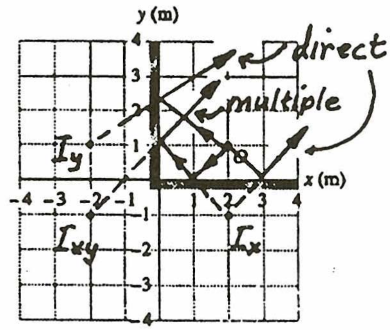


FIGURE 36-44 Problem 2 Solution.

Solution

In addition to the two direct images in each mirror (one reflection) at $I_y = (-2\text{ m}, 1\text{ m})$ and $I_x = (2\text{ m}, -1\text{ m})$, a multiple image (two reflections) also appears at $I_{xy} = (-2\text{ m}, -1\text{ m})$. (The latter is the image in one mirror of the direct image in the other.) Ray tracing confirms this, as shown on Fig. 36-44. No more than two reflections are possible for perpendicular mirror planes, so these are all the images.

Problem

3. (a) What is the focal length of a concave mirror if an object placed 50 cm in front of the mirror has a real image 75 cm from the mirror? (b) Where and what type will the image be if the object is moved to a point 20 cm from the mirror?

Solution

(a) The mirror equation relates the given distances (both positive for a real object and image) to the focal length: $f^{-1} = (50\text{ cm})^{-1} + (75\text{ cm})^{-1}$, or $f = 30$ cm. (See Equation 36-2.) (b) A second application of the mirror equation yields $(\ell')^{-1} = (30\text{ cm})^{-1} - (20\text{ cm})^{-1}$, or $\ell' = -60$ cm. A negative distance indicates a virtual, erect image located behind the mirror. (Fig. 36-8c and Table 36-1 confirm these results.)

Problem

7. A virtual image is located 40 cm behind a concave mirror with focal length 18 cm. (a) Where is the object? (b) By how much is the image magnified?

Solution

(a) The mirror equation (Equation 36-2) gives $\ell = f\ell'/(\ell' - f) = (18\text{ cm})(-40\text{ cm})/(-58\text{ cm}) = 12.4\text{ cm}$ (positive distances are in front of the mirror, negative distances behind). (b) Equation 36-1 gives $M = -\ell'/\ell = +40\text{ cm}/12.4\text{ cm} = 3.22$.

Problem

11. An object's image in a 27-cm-focal-length concave mirror is upright and magnified by a factor of 3. Where is the object?

Solution

An upright image in a concave mirror must be virtual, so $M = +3 = -\ell'/\ell$. The mirror equation gives $(1/\ell) + (1/\ell') = (1/\ell) - (1/3\ell) = 1/f$, or $\ell = (\frac{2}{3})f = (\frac{2}{3})(27\text{ cm}) = 18\text{ cm}$ (positive in front of the mirror).

Problem

15. You look into a reflecting sphere 80 cm in diameter and see an image of your face at one-third its normal size (Fig. 36-46). How far are you from the sphere's surface?

Solution

The sphere reflects like a convex mirror of focal length $f = R/2 = -40\text{ cm}/2 = -20\text{ cm}$. (Only a convex mirror produces a reduced, upright, virtual image.) The equation in the solution to Problem 5(a) can be used to find the object distance in terms of f and the magnification, $\ell = f(1 - h/h') = f(1 - 1/M) = -20\text{ cm}(1 - 3) = 40\text{ cm}$.

Section 36-3: Lenses

Problem

17. A light bulb is 56 cm from a convex lens, and its image appears on a screen located 31 cm on the other side of the lens. (a) What is the focal length of the lens? (b) By how much is the image enlarged or reduced?

Solution

(a) The object and image distances are both positive, for a real image formed by a single lens (recall that only a real image can appear on a screen), so the lens equation gives $f^{-1} = (56\text{ cm})^{-1} + (31\text{ cm})^{-1} = (20.0\text{ cm})^{-1}$. (b) Equation 36-4 gives a magnification of $M = -31/56 = -0.554$, so the inverted image is reduced to nearly 55% of the actual size of the bulb.

Problem

21. A simple camera uses a single converging lens to focus an image on its film. If the focal length of the lens is 45 mm, what should be the lens-to-film distance for the camera to focus on an object 80 cm from the lens?

Solution

Set $\ell = 80$ cm and $f = 45$ mm in the lens equation and solve for ℓ'. The result is $\ell' = \ell f/(\ell - f) = (80\times 45\text{ cm})/(800 - 45) = 4.77\text{ cm}$.

Problem

27. A lens has focal length $f = 35$ cm. Find the type and height of the image produced when a 2.2-cm-high object is placed at distances (a) $f + 10$ cm and (b) $f - 10$ cm.

Solution

The lens equation and magnification for a thin (converging, i.e., positive f) lens, Equations 36-4 and 5, give $M = -\ell'/\ell = -f/(\ell - f)$, so $h' = Mh = -fh \div (\ell - f)$. (a) If $f = 35$ cm and $\ell = f + 10$ cm, then $h' = -(35\text{ cm})(2.2/10) = -7.7$ cm. A negative image height signifies a real, inverted image. (b) If $\ell = f - 10$ cm, then $h' = -(35\text{ cm})(2.2)/(-10) = +7.7$ cm, which represents a virtual, erect image of the same size.

Problem

41. An object is 28 cm from a double convex lens with $n = 1.5$ and curvature radii 35 cm and 55 cm. Where and what type is the image?

Solution

The focal length of the lens, $f^{-1} = (n-1)(R_1^{-1} - R_2^{-1}) = (1.5-1)[(35\text{ cm})^{-1} + (55\text{ cm})^{-1}] = (42.8\text{ cm})^{-1}$ (from Equation 36-8) is greater than the object distance, so the lens equation gives a virtual, erect image located at $\ell' = \ell f/(\ell - f) = (28\times 42.8\text{ cm})\div(28 - 42.8) = -81.1$ cm (negative ℓ' being on the same side of the lens as the object).

Problem

49. Grandma's new reading glasses have 3.8-diopter lenses to provide full correction of her farsightedness. Her old glasses were 2.5 diopters. (a) Where is the near point for her unaided eyes? (b) Where will be the near point if she wears her old glasses?

Solution

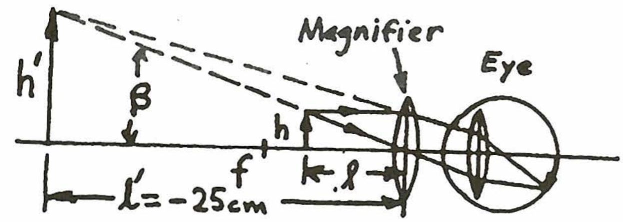
(a) The new correction is designed to make an object placed at the standard near point, $\ell = 25$ cm, appear (as a virtual image) to be at the near point for unaided vision, $\ell' = -$near point. Therefore, $(f_{\text{cor}})^{-1} = (25\text{ cm})^{-1} + (-\text{near point})^{-1}$, or $(\text{near point})^{-1} = (0.25\text{ m})^{-1} - 3.8\text{ diopters} = 0.2\text{ diopters} = (5\text{ m})^{-1}$.
(b) The old correction, $(f_{\text{cor}})^{-1} = 2.5$ diopters, would not bring an object, placed at the standard 25 cm, to a virtual image at the near point of 5 m, but only from $\ell^{-1} = (f_{\text{cor}})^{-1} - \ell'^{-1} = 2.5\text{ diopters} - (-5\text{ m})^{-1} = (37.0\text{ cm})^{-1}$, which is the near point when wearing the old correction.

Problem

53. The maximum magnification of a simple magnifier occurs with the image at the 25-cm near point. Show that the angular magnification is then given by $m = 1 + \frac{25\text{ cm}}{f}$, where f is the focal length.

Solution

Another way of using a magnifier is to arrange for the virtual image to be at the near point, as shown (instead of at ∞, as in Fig. 36-38*b*). From the diagram and the lens equation, $\beta \approx \tan\beta = h'/(-\ell') = h/\ell = h[(1/f) - (1/\ell')] = h[(1/f) + (1/25\text{ cm})]$. Thus, with the same definition of angular magnification as before (Equation 36-9), $m = \beta/\alpha = h[(1/f) + (1/25\text{ cm})]\div(h/25\text{ cm}) = 1 + (25\text{ cm}/f)$. This is the maximum magnification obtainable with a simple magnifier, because the image can't be seen clearly if it's closer than the near point (i.e., $\beta \approx h'/|\ell'|$ and $|\ell'| \geq 25$ cm). Incidentally, the most effective way to use a magnifier is to hold it close to your eye, moving the object up to it until a sharp image is seen.



Problem 53 Solution.

Solution

Equation 36-10 gives $M = -(83/6.1)(25/1.7) = -200$.

Problem

55. A 300-power compound microscope has a 4.5-mm-focal-length objective lens. If the distance from objective to eyepiece is 10 cm, what should be the focal length of the eyepiece?

Solution

For a 300×microscope, with $f_0 = 4.5$ mm and $L = 100$ mm, we can solve Equation 36-10 for $f_e = (100/4.5)(25\text{ cm}/300) = 1.85$ cm.

Problem

56. To the unaided eye, the planet Jupiter has an angular diameter of 50 arc seconds. What will be its angular size when viewed through a 1-m-focal-length refracting telescope with an eyepiece whose focal length is 40 mm?

Solution

The angular magnification of an astronomical telescope is given by Equation 36-11, so the apparent size is this times the actual size, or $\beta = \alpha(f_0/f_e) = 50''(1\text{ m}/40\text{ mm}) = 1250'' = 20.8' \approx \frac{1}{3}^\circ$.

Problem

57. A Cassegrain telescope like that shown in Fig. 36-42*b* has 1.0-m focal length, and the convex secondary mirror is located 0.85 m from the primary. What should be the focal length of the secondary in order to put the final image 0.12 m behind the front surface of the primary mirror?

Solution

Reference to Fig. 36-42*b* shows that parallel rays reflected by the objective mirror converge toward a point, 1 m − 0.85 m = 15 cm behind the secondary mirror, and behave as if they came from a virtual object, with $\ell = -15$ cm in the mirror equation. The final image is located a distance $\ell' = 85\text{ cm} + 12\text{ cm} = 97$ cm from the secondary mirror, whose required focal length is therefore $f^{-1} = \ell^{-1} + \ell'^{-1} = (-15\text{ cm})^{-1} + (97\text{ cm})^{-1} = (-17.7\text{ cm})^{-1}$. (Recall that a convex mirror has negative focal length.)