**Introduction**

This is the second of two labs on simple harmonic motion (SHM). In the first lab you studied elastic forces and elastic energy, and you measured the net force on a pendulum bob held at an angle from vertical. This force tries to make the pendulum return to its equilibrium position and is called a “restoring force”. When the restoring force varies linearly with displacement from the equilibrium position, the position vs. time of the periodic motion is a sine or cosine function and is called “simple harmonic motion”, usually abbreviated to SHM. Other functional forms of restoring forces also result in periodic motion, but not SHM. We pay particular attention to SHM since it occurs frequently in nature and has mathematical simplicity and elegance. In this lab you will use a position sensor to document the motion of a mass bouncing up and down on a spring. You will analyze this situation using your knowledge of dynamics from Physics 1A to determine if the mass should be moving with SHM. You will then investigate with your lab data if the mass actually does move with SHM.

Please review sections 13.3 to 13.6 from Chapter 13 of Serway & Faughn.

**Pre-Lab Questions:**

1. a) Suppose you have a block resting on a horizontal smooth surface. The block (with a mass of \( m \)) is attached to a horizontal spring (spring constant \( k \) (N/m)), which is fixed at one end. The spring can be compressed and stretched. The mass is pulled to one side then released. What is the formula required to calculate the period of the oscillation of the mass? If \( m = 250 \) g, and \( k = 150 \) N/m, what are the period and frequency of the oscillation?
   
b) If the block of mass \( m \) were replaced with a block five times more massive, by what factor would the period change? Would the period be longer or shorter?

2. For the same block and spring configuration as in question 1(a), the graph shows the displacement of the block as a function of time. The maximum displacement is 15 cm. (The graph for this exercise is provided as the last page of this lab manual. For your records, trace your solutions from your completed graphs into your lab notebook, or print the page twice, copy your solution, and fasten the copy into your notebook)
   
a) If \( k = 150 \) N/m, calculate the mass of the block.
   b) Draw the graphs for velocity and acceleration of the motion.
   c) Draw the graph for kinetic energy and calculate and label on the graph the maximum value of the KE.
d) Draw the graph for the elastic potential energy and calculate and label on the graph the maximum value of the elastic PE.

e) Calculate the numerical value and draw the graph for the total energy.

3. Suppose you attach a mass \( m \) on a spring with a spring constant \( k \). You gently lower the mass so it hangs on the spring in equilibrium as shown in the figure.

   a) Draw a force diagram for the mass in this situation and calculate how much the spring is stretched.

   b) If you now pull the mass down an additional distance \( y \) so it is below its equilibrium position, then let go, the mass will bounce up and down. The equilibrium position will be the midpoint of the motion. Draw the force diagram for the mass just after you let go.

   c) At this instant, what is the net force on the mass? Is the force upwards or downwards? Is the force linear with the distance \( y \)?

   d) Will the periodic bouncing of the mass be SHM? Why?

**Group Activity:**

A horizontal block-spring system, like that shown for pre-lab questions #1(a) and #2, is moving with SHM. The equation of the motion is

\[
x(t) = A \sin\left(\sqrt{\frac{k}{m}} t\right) + D_0
\]

where \( t \) is time in seconds, \( A \) is the amplitude. For this motion:

- What is the numerical value of the amplitude of the motion?
- What are the numerical values for the frequency and the period?
- Make a plot (graph) of the position as a function of time, starting at \( t=0 \), for the first two periods of the motion. Label your axes, and show the motion to-scale.
- How long does it take the block to pass through the equilibrium position for both the first and second time?
Experiment A: Measure the Spring Constant for your Spring

In order to understand the SHM of a mass supported by a spring, you need to know the value of \( k \) for the spring you are using. In this section of the lab, you will repeat the measurement of \( k \) that you made last week. The springs we use in the lab are not identical and can suffer a change in their spring constants if stretched too close to their elastic limits. Hence this re-measurement is required. Check your spring to be sure it shows no signs of abuse. The following is an abbreviated version of the week 1 procedure for measuring the spring constant. You should refer to your own notes from week 1 to help you complete this work efficiently.

- Attach one end of the spring to the support and the other end to the holder for the masses. Place the position sensor on the floor vertically beneath the holder. Check that the position sensor measures the position of the holder and not the lab table or any other nearby object.
- Using the position sensor, measure the height to the base of the holder. Add masses as described on the whiteboard in the lab (depends on availability of masses) and measure the height after each mass addition. (Typically 30 g, 50 g, 80 g, and 100 g are appropriate). Include the mass of the holder which is labeled on the holder itself.
- Make a graph of force vs. spring stretch. Draw the best straight line through your points and from the slope of the line calculate the spring constant.

Experiment B: Measure the period of oscillation as a function of mass

In Experiment A, you took care to hang each mass on the spring so that it was still, with no swinging or bouncing. In this experiment, you want the mass to oscillate up and down (but not swing side-to-side). You will measure how the period of the motion changes as you use larger masses. You will need the same physical setup as for Experiment A, with the position sensor looking up at the base of the mass holder.

B1. Starting with a 30 g mass in the holder, gently pull the holder down only about an inch (not more) and let go. Set the Logger Pro software to display only a position vs. time plot. Use the position sensor to record a few seconds worth of data of the oscillation. Your data should show a smooth oscillation with several entire periods on the plot. If necessary repeat this a few times until you get a clean plot. Now you are ready to measure the period so proceed to B2.

B2. Select the “curve fit” button from the menu bar. On the “curve fit” menu be sure the default fit type of “automatic” is selected. Now you must choose the function that best represents the data from the functions available. Scroll down to find the function \( [A \sin(Bt + C)] + D \). Select this function and push the “try fit” button. This will show you the best fit that Logger Pro can make to your data with the selected function. Observe how well (or poorly) the calculated curve fits your data.

B3. To be sure that the function you selected is the best, try other functions and
observe how they fit the data. Does any other function do better? Does any other function make an acceptable fit? When you are certain you have the best fitting function select “OK”. This should take you back to the main screen and display the best-fit parameters.

B4. Think about the meaning of each constant \((A, B, C, D)\) in the fitted equation 
\[
[A \cdot \sin(Bt + C)] + D
\]
They are \(A =\) amplitude, \(B = 2\pi f\) – angular frequency, \(C\) = phase angle, and \(D =\) offset of the equilibrium position. In this case, you have fitted an equation that is a sine function, not a cosine function. This results in a different selection of the constant \(C\) for the phase angle, which we can ignore for this experiment. The phase angle tells you about the starting conditions, which in our case are arbitrary and not important. But we can tell the amplitude directly from the value of \(A\), and we can calculate the frequency from the fitted value of \(B\). Calculate the frequency and period from this. Record in a table in your lab book the mass (30g), the values of \(A, B\) and \(D\), and the period and frequency calculated from the fitted value of \(B\). (The fitted values of \(A\) or \(B\) may be negative. If so, you should ignore the negative sign(s) and use the magnitudes of \(A\) and \(B\).)\(^1\)

B5. Repeat B1 through B4 for different masses in increments as described on the whiteboard in the lab.

B6. For each row in your table calculate the total mass (weights plus holder), and using the spring constant from Experiment A calculate and enter into the table the quantity \(\sqrt{m/k}\)

B7. In your lab notebook, make a plot of the period (vertically) vs. \(\sqrt{m/k}\) (on the horizontal axis). Calculate the period from the \(B\) values. Set your graph scales so the point (0,0) is on the graph. Fit a best straight line through these data points and calculate the slope of this line. If possible, choose a line that passes through the point (0,0).

B8. From pre-lab question 1(a), would you expect your data to form a straight line? The equation \(T = 2\pi \sqrt{m/k}\) suggests you should have a straight line of slope \(2\pi\). Compare the slope of the line to that expected, and comment on any discrepancies.

B9. Examine how the value of the fitted parameter \(D\) changes as you add mass. Does \(D\) systematically increase or decrease with increasing mass? How and why would you expect \(D\) to change?

B10. What determines the size of the fitted parameter \(A\)?

B11. What critical parts of these activities help you to decide if the motion you observed is (or is not) the particular type of oscillation called simple harmonic motion?

\(^1\) Any negative signs indicated are due to a flaw in the software -- Amplitude and frequency are always positive quantities.
Experiment C: Position, Velocity and Acceleration plots

Save the last data set you acquired in Experiment B, or repeat a position measurement with just one mass value to get a clean data set.

- Compare the motion with the position graph in pre-lab question #2. Using the work you did for the prelab, predict what the velocity and acceleration plots will be like for the motion you have recorded.
- Now move to the next page of the Logger Pro program and examine the velocity and acceleration plots. Are they consistent with your predictions?
- What is the value of the velocity (max., min. or zero) when the mass is at its maximum displacement?
- What is the value of the acceleration (max., min. or zero) when the mass is at its maximum displacement?
- What is the value of the force (max., min. or zero) when the mass is at its maximum displacement?

Post-Lab Questions:

Your adventurous friend Lola goes bungee jumping. She leaps from a bridge that is 150 m above a river. Her bungee cord has an unstretched length of 85 m and a spring constant \( k = 700 \) N/m. Lola has a mass of 54 kg.

1. How fast is she falling when she just starts to stretch the cord? How long does it take for Lola to reach this point?

2. Lola stretches the bungee cord and it brings her to a stop. She then bounces back up again. What type(s) of mechanical energy does the system (Lola and the bungee cord) have just before she jumps? (I.e., gravitational PE, elastic PE, kinetic energy, etc.) What type(s) of energy does the system have at the instant she comes to rest at her lowest point?

3. What is her height above the river at her lowest point?  
   \[ \text{[Hint: use energy for this, assume no energy is lost, and remember that the energy stored in a stretched bungee cord (or spring) is } \frac{1}{2} k(\Delta x)^2 \text{, where } \Delta x \text{ is the amount of stretch.]} \]

4. What would be the period of her oscillation if we assume the bungee cord acts like a perfect spring?

5. Please do a write-up for the section of the lab that your TAs specified. You can download an example off the class website.
Graph for pre-lab question #2