Heat Engines

Heat Engines, Heat Pumps, and Refrigerators

Getting something useful from heat

(Many slides are from prof. Tom Murphy)

Heat can be useful

• Normally heat is the end-product of the flow/transformation of energy
  – remember examples from lecture
  – heat regarded as waste: as useless end result
• Sometimes heat is what we want, though
  – hot water, cooking, space heating
• Heat can also be coerced into performing “useful” (e.g., mechanical) work
  – this is called a “heat engine”
Heat Engine Concept

- Any time a temperature difference exists between two bodies, there is a potential for heat flow
- Examples:
  - heat flows out of a hot pot of soup
  - heat flows into a cold drink
  - heat flows from the hot sand into your feet
- Rate of heat flow depends on nature of contact and thermal conductivity of materials
- If we’re clever, we can channel some of this flow of energy into mechanical work

Heat → Work

- We can see examples of heat energy producing other types of energy
  - Air over a hot car roof is lofted, gaining kinetic energy
  - That same air also gains gravitational potential energy
  - All of our wind is driven by temperature differences
  - We already know about radiative heat energy transfer
  - Our electricity generation thrives on temperature differences: no steam would circulate if everything was at the same temperature
Heat flows from $T_h$ to $T_c$, turning turbine along the way

**Heat Engine Nomenclature**

- The symbols we use to describe the heat engine are:
  - $T_h$ is the temperature of the hot object
  - $T_c$ is the temperature of the cold object
  - $\Delta T = T_h - T_c$ is the temperature difference
  - $\Delta Q_h$ is the amount of heat that flows out of the hot body
  - $\Delta Q_c$ is the amount of heat flowing into the cold body
  - $\Delta W$ is the amount of “useful” mechanical work
  - $\Delta S_h$ is the change in entropy of the hot body
  - $\Delta S_c$ is the change in entropy of the cold body
  - $\Delta S_{tot}$ is the total change in entropy (entire system)
  - $\Delta E$ is the entire amount of energy involved in the flow
What’s this *Entropy* business?

- Entropy is a measure of disorder (and actually quantifiable on an atom-by-atom basis)
  - Ice has low entropy, liquid water has more, steam has a lot

![Example of entropy states](image)

The Laws of Thermodynamics

1. **Energy is conserved**
2. **Total system entropy, $S$, can never decrease**
   \[ \Delta S_{\text{tot}} \geq 0 \]
3. **As the temperature goes to zero, the entropy approaches a constant value**—this value is zero for a perfect crystal lattice
   - The concept of the “total system” is very important: entropy can decrease locally, but it must increase elsewhere by *at least* as much
   - no energy flows into or out of the “total system”; if it does, there’s more to the system than you thought
Quantifying heat energy

- We’ve already seen many examples of quantifying heat
  - 1 Calorie is the heat energy associated with raising 1 kg (1 liter) of water 1 °C
  - In general, \( \Delta Q = c_P m \Delta T \), where \( c_P \) is the heat capacity
- We need to also point out that a change in heat energy accompanies a change in entropy:
  \[ \Delta Q = T \Delta S \]
- Adding heat increases entropy
  - more energy goes into random motions \( \rightarrow \) more randomness (entropy)

How much work can be extracted from heat?

Hot source of energy \( T_h \)

heat energy delivered from source \( \Delta Q_h \)

externally delivered work:

\[ \Delta W = \Delta Q_h - \Delta Q_c \]

conservation of energy

heat energy delivered to sink \( \Delta Q_c \)

Cold sink of energy \( T_c \)

efficiency = \( \frac{\Delta W}{\Delta Q_h} \) = work done

heat supplied
Let’s crank up the efficiency

Let’s extract a lot of work, and deliver very little heat to the sink

In fact, let’s demand 100% efficiency by sending no heat to the sink: all converted to useful work

\[ \Delta W = \Delta Q_h - \Delta Q_c \]

\[ \text{efficiency} = \frac{\Delta W}{\Delta Q_h} = \frac{\text{work done}}{\text{heat supplied}} \]

Not so fast…

• The second law of thermodynamics imposes a constraint on this reckless attitude: total entropy must never decrease

• The entropy of the source goes down (heat extracted), and the entropy of the sink goes up (heat added): remember that \( \Delta Q = T \Delta S \)
  – The gain in entropy in the sink must at least balance the loss of entropy in the source
  \[ \Delta S_{\text{tot}} = \Delta S_h + \Delta S_c = -\frac{\Delta Q_h}{T_h} + \frac{\Delta Q_c}{T_c} \geq 0 \]
  \[ \Delta Q_c \geq \left( \frac{T_c}{T_h} \right) \Delta Q_h \text{ sets a minimum on } \Delta Q_c \]
What does this entropy limit mean?

- $\Delta W = \Delta Q_h - \Delta Q_c$, so $\Delta W$ can only be as big as the minimum $\Delta Q_c$ will allow
  
  $\Delta W_{\text{max}} = \Delta Q_h - \Delta Q_{c,\text{min}} = \Delta Q_h - \Delta Q_h(T_c/T_h) = \Delta Q_h(1 - T_c/T_h)$

- So the maximum efficiency is:

  \[
  \text{maximum efficiency} = \frac{\Delta W_{\text{max}}}{\Delta Q_h} = \frac{\Delta Q_h}{\Delta Q_h(T_c/T_h)} = \frac{1 - T_c/T_h}{T_h}
  \]

  (THIS IS CALLED THE CARNOT EFFICIENCY

  \[
  \text{Carnot Eff} = \left( T_h - T_c \right)/T_h
  \]

- So perfect efficiency is only possible if $T_c$ is zero (in °K)
  - In general, this is not true

- As $T_c \rightarrow T_h$, the efficiency drops to zero: no work can be extracted

Examples of Maximum Efficiency

- A coal fire burning at 825 °K delivers heat energy to a reservoir at 300 °K
  - max efficiency is $(825 - 300)/825 = 525/825 = 64\%$
  - this power station can not possibly achieve a higher efficiency based on these temperatures

- A car engine running at 400 °K delivers heat energy to the ambient 290 °K air
  - max efficiency is $(400 - 290)/400 = 110/400 = 27.5\%$
  - not too far from reality
Example efficiencies of power plants

Power plants these days (almost all of which are heat-engines) typically get no better than 33% overall efficiency.

Types of heat engines

- External combustion engine
- Internal combustion engine (gas/diesel)
- Gas turbine (aka jet engine)
- Rocket
What to do with the waste heat ($\Delta Q_e$)?

- One option: use it for space-heating locally

![Diagram of cogeneration plant](https://example.com/cogeneration-diagram.png)

**Figure 3.13** A small cogeneration plant that uses the combustion of natural gas to drive a gas turbine coupled to an electric generator. The hot exhaust gases boil water to steam for use in space heating and cooling. *(Source: Exxon Corporation.)*

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**Overall efficiency greatly enhanced by cogeneration**

<table>
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<th>Table 3.1 Cogeneration Plant, University of Colorado, Boulder</th>
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<td>Overall efficiency</td>
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Heat Pumps provide a means to very efficiently move heat around, and work both in the winter and the summer.

**Heat Pump Diagram**

Figure 3.12  An electrically-driven heat pump using Freon as a working fluid. In principle, the system becomes an air conditioner if the fluid flow direction is reversed. In practice, the reversal of function is more complex.
Heat Engines

### Heat Pumps and Refrigerators: Thermodynamics

**Just a heat engine run backwards…**

- **Hot entity (indoor air)**: $T_h$
- **heat energy delivered**: $\Delta Q_h$
- **heat energy extracted**: $\Delta Q_c$
- **Cold entity (outside air or refrigerator)**: $T_c$

**delivered work:**

$$\Delta W = \Delta Q_h - \Delta Q_c$$

**conservation of energy**

**efficiency (heat pump):**

$$\frac{\Delta Q_h}{\Delta W} = \text{heat delivered}$$

**efficiency (refrigerator):**

$$\frac{\Delta Q_c}{\Delta W} = \text{heat extracted}$$

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### Heat Pump/Refrigerator Efficiencies

- Can work through same sort of logic as before to see that:
  - **heat pump efficiency is:** $T_h/(T_h - T_c) = T_h/\Delta T$ in °K
  - **refrigerator efficiency is:** $T_c/(T_h - T_c) = T_c/\Delta T$ in °K
- **Note that heat pumps and refrigerators are most efficient for small temperature differences**
  - hard on heat pumps in very cold climates
  - hard on refrigerators in hot settings
Example Efficiencies

• A heat pump maintaining 20 °C when it is –5 °C outside has a maximum possible efficiency of:
  \[
  \frac{293}{25} = 11.72
  \]
  – note that this means you can get almost 12 times the heat energy than you are supplying in the form of work!
  – this factor is called the C.O.P. (coefficient of performance)

• A freezer maintaining –5 °C in a 20 °C room has a maximum possible efficiency of:
  \[
  \frac{268}{25} = 10.72
  \]
  – called EER (energy efficiency ratio)