PHYSICS 210A : EQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #5

(1) Consider a two-dimensional gas of fermions which obey the dispersion relation

$$\varepsilon(\boldsymbol{k}) = \varepsilon_0 \Big((k_x^2 + k_y^2)\, a^2 + \tfrac{1}{2} (k_x^4 + k_y^4)\, a^4 \Big) \ .$$

Sketch, on the same plot, the Fermi surfaces for $\varepsilon_{\mathrm{F}} = 0.1\,\varepsilon_0$, $\varepsilon_{\mathrm{F}} = \varepsilon_0$, and $\varepsilon_{\mathrm{F}} = 10\,\varepsilon_0$.

(2) Using noninteracting quantum statistics for fermions, answer the following:

(a) For ideal Fermi gases in $d = 1$, 2, and 3 dimensions, compute at $T = 0$ the average energy per particle E/N in terms of the Fermi energy ε_{F}.

(b) Using the Sommerfeld expansion, compute the heat capacity for a two-dimensional electron gas, to lowest nontrivial order in the temperature T.

(3) Consider a three-dimensional Fermi gas of $S = \frac{1}{2}$ particles obeying the dispersion relation $\varepsilon(\boldsymbol{k}) = A\,|\boldsymbol{k}|^4$.

(a) Compute the density of states $g(\varepsilon)$.

(b) Compute the molar heat capacity.

(c) Compute the lowest order nontrivial temperature dependence for $\mu(T)$ at low temperatures. *I.e.* compute the $\mathcal{O}(T^2)$ term in $\mu(T)$.

(4) In an n-type semiconductor, the donor levels lie a distance Δ below the bottom of the conduction band. Suppose there are M such donor levels. Due to the fact that such donor levels are spatially localized, one can ignore the possibility of double occupancy. Thus, each donor level can be occupied by at most one electron, but of either spin polarization. Assume the conduction band dispersion is isotropic, given by $\varepsilon_{\boldsymbol{k}} = \hbar^2 \boldsymbol{k}^2 / 2m^*$. (Set the conduction band minimum to $\varepsilon = 0$.)

(a) Assuming that the conduction band is very sparsely populated, find an expression for the conduction electron density $n_{\mathrm{c}}(T,\mu)$.

(b) Suppose there are N_{d} electrons sitting on the donor sites, *i.e.* N_{d} of the M donor levels are singly occupied. Find the entropy of these electrons.

(c) Find the chemical potential of the donor electrons.

(d) Use the fact that the donor electrons and the conduction band electrons are in thermal equilibrium to eliminate μ from the problem, and find the conduction electron density $n_{\mathrm{c}}(T)$ and the fraction $\nu_{\mathrm{d}}(T)$ of occupied donor sites. Assume that the donor concentration is ρ_{d}, and that all conduction electrons are due to singly ionized donors.