PHYSICS 140B : STATISTICAL PHYSICS
MIDTERM EXAMINATION

(1) Consider the single particle density of states

$$g(\varepsilon) = A\,\varepsilon\,\Theta(\varepsilon) + B\,\Theta(\varepsilon - \Delta) = \begin{cases} A\,\varepsilon & \text{if } 0 \le \varepsilon < \Delta \\ A\,\varepsilon + B & \text{if } \varepsilon \ge \Delta \end{cases} \quad .$$

Here, A, B, and Δ are positive constants. The following integrals may be useful:

$$\int\limits_0^\infty \! d\varepsilon\, \frac{\varepsilon^\alpha}{z^{-1}\,e^{\varepsilon/k_\mathrm{B}T} - 1} = (k_\mathrm{B}T)^{1+\alpha}\,\mathrm{Li}_{1+\alpha}(z) \quad , \quad \int\limits_0^\infty \! d\varepsilon\,\varepsilon^\alpha \ln\!\big[1 - z\,e^{-\varepsilon/k_\mathrm{B}T}\big] = -(k_\mathrm{B}T)^{1+\alpha}\,\mathrm{Li}_{2+\alpha}(z)$$

where $\mathrm{Li}_s(z) = \sum_{m=1}^\infty z^m/m^s$. Note $\mathrm{Li}_1(z) = -\ln(1-z)$.

(a) Assuming the particles have photon statistics, find an expression for the number density $n(T)$. [10 points]

(b) Assuming the particles are bosons, find an equation which relates the critical temperature T_c for Bose condensation to the number density n. [10 points]

(c) For $T > T_\mathrm{c}$, find a closed form expression for $n(T, z)$, where $z = \exp(\mu/k_\mathrm{B}T)$ is the fugacity. [10 points]

(d) For $T < T_\mathrm{c}$, find an expression for $n(T, n_0)$, where n_0 is the condensate number density. [10 points]

(e) For $T < T_\mathrm{c}$, find $p(T)$. [10 points]

(f) For $T > T_\mathrm{c}$, find an expression for $p(T, n)$. [50 quatloos extra credit]

(2) Consider $S = \frac{1}{2}$ fermions with the relativistic dispersion

$$\varepsilon(\boldsymbol{p}) = \sqrt{c^2 p^2 + m^2 c^4} \quad ,$$

in $d = 2$ space dimensions, where $\boldsymbol{p} = \hbar\boldsymbol{k}$ is the momentum.

(a) Find the density of states $g(\varepsilon)$. Don't forget to include the appropriate step function to indicate the energy below which $g(\varepsilon)$ vanishes. [10 points]

(b) Find the Fermi momentum $p_\mathrm{F}(n)$. [10 points]

(c) Find the second virial coefficient $B_2(T)$. [15 points]

The following integral may be useful: $\int\limits_a^\infty \! dx\, x\, e^{-x} = (1 + a)\, e^{-a}$.

(d) Find the chemical potential $\mu(T, n)$, valid to order T^2. [15 points]

(e) Find $n(T, z)$. [50 quatloos extra credit]