

PHYSICS 221A : NONLINEAR DYNAMICS
HW ASSIGNMENT #1

(1) Do all the recommended exercises in §1.2.4 of the notes.

(2) Using the method of partial fractions, solve the ODE

$$\frac{du}{dt} = (u-1)(u-2)(u-3)$$

for $t(u)$. Sketch the phase flow along the real u line, and the integral curves in the (t, u) plane.

(3) Consider two particles interacting on a line, with Lagrangian

$$L = \frac{1}{2}m(\dot{x}_1^2 + \dot{x}_2^2) - \frac{1}{2}k(x_1^2 + x_2^2) - \frac{1}{2}k'(x_1 - x_2)^2 .$$

Write the equations of motion as a dynamical system. What is the dimension N of phase space?

(4) Consider the harvested population dynamics

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - H(N) ,$$

where

$$H(N) = \begin{cases} H_0 N/L & \text{if } N \leq L \\ H_0 & \text{if } N \geq L . \end{cases}$$

Show that this four parameter model can be rescaled to yield a two-parameter one,

$$\dot{\nu} = \nu(1 - \nu) - \begin{cases} h\nu/\sigma & \text{if } \nu \leq \sigma \\ h & \text{if } \nu \geq \sigma . \end{cases}$$

Plot the ‘phase diagram’ for this model in the (σ, h) plane, labeling each region according to the number and type of fixed points present. Identify the bifurcations associated with the boundaries between different phases. Is hysteresis possible in this model?

(5) Any nonlinear ODE of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n ,$$

with $n \neq 1$, is known as *Bernoulli’s equation*. Show that upon dividing by y^n and changing variables from y to $z \equiv y^{1-n}$ that the equation becomes linear, and can be solved by finding the appropriate integrating factor. Recall that an integrating factor $u(x, y)$ for the inexact differential

$$dF = M(x, y) dx + N(x, y) dy$$

is such that the product $u(x, y) dF$ is exact. For the case

$$dF = dy + p(x) y dx ,$$

the integrating factor is

$$u(x) = \exp \left(\int^x dx' p(x') \right) ,$$

since

$$du(x) = p(x) u(x) dx \quad \Rightarrow \quad u(x) \cdot (dy + p(x) y dx) = d(u(x) y) .$$