24. 
$$(E + m_e c^2 - E')^2 = c^2 (p^2 - 2pp' \cos \theta + p'^2) + m_e^2 c^4$$

$$E^{2} + E'^{2} + m_{e}^{2}c^{4} + 2Em_{e}c^{2} - 2EE' - 2E'm_{e}c^{2} = c^{2}p^{2} - 2c^{2}pp'\cos\theta + c^{2}p'^{2} + m_{e}^{2}c^{4}$$

With 
$$E^2 = c^2 p^2$$
 and  $E'^2 = c^2 p'^2$ ,

$$Em_{e}c^{2} - EE' - E'm_{e}c^{2} = -EE'\cos\theta$$

$$m_{e}c^{2}(E - E') = EE'(1 - \cos\theta)$$

$$\frac{E - E'}{EE'} = \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_{e}c^{2}}(1 - \cos\theta)$$

25. (a) 
$$\frac{1}{E'} = \frac{1}{E} + \frac{1 - \cos \theta}{m_e c^2} = \frac{1}{11.32 \text{ keV}} + \frac{1 - \cos 62.9^\circ}{511.0 \text{ keV}} = 0.08940 \text{ keV}^{-1}$$

so 
$$E' = 1/0.08727 \text{ keV}^{-1} = 11.19 \text{ keV}$$
.

(b) 
$$K_{\rm e} = E_{\rm e} - m_{\rm e}c^2 = E - E' = 11.32 \text{ keV} - 11.19 \text{ keV} = 0.13 \text{ keV}$$

26. (a) 
$$\lambda' = \lambda + (h/m_e c)(1 - \cos \theta)$$

$$= 0.02218 \text{ nm} + (0.002426 \text{ nm})(1 - \cos 90^{\circ}) = 0.02461 \text{ nm}$$

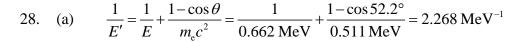
(b) The momenta of the incident and scattered photons are

$$p = \frac{h}{\lambda} = \frac{1}{c} \frac{hc}{\lambda} = \frac{1}{c} \frac{1240 \text{ eV} \cdot \text{nm}}{0.02218 \text{ nm}} = 5.591 \times 10^4 \text{ eV}/c \quad (x \text{ direction})$$
$$p' = \frac{h}{\lambda} = \frac{1}{c} \frac{hc}{\lambda} = \frac{1}{c} \frac{1240 \text{ eV} \cdot \text{nm}}{0.02461 \text{ nm}} = 5.039 \times 10^4 \text{ eV}/c \quad (y \text{ direction})$$
(c)  $K_e = E - E' = cp - cp' = 5.591 \times 10^4 \text{ eV} - 5.039 \times 10^4 \text{ eV} = 5.52 \times 10^3 \text{ eV}$ 

(d) Because momentum must be conserved, the *x* component of the electron's momentum must equal *p*, and the *y* component must equal -p':

$$p_{\rm ex} = p = 5.591 \times 10^4 \text{ eV}/c$$
 and  $p_{\rm ey} = -p' = -5.039 \times 10^4 \text{ eV}/c$   
 $p_{\rm e} = \sqrt{p_{\rm ex}^2 + p_{\rm ey}^2} = \sqrt{(5.591 \times 10^4 \text{ eV}/c)^2 + (5.039 \times 10^4 \text{ eV}/c)^2} = 7.527 \times 10^4 \text{ eV}/c$ 

in the direction given by  $\theta = \tan^{-1} \frac{p_{ey}}{p_{ex}} = \tan^{-1} \frac{-5.039 \times 10^4 \text{ eV}/c}{5.591 \times 10^4 \text{ eV}/c} = -42.0^\circ$ 



so 
$$E' = 1/2.268 \text{ MeV}^{-1} = 0.441 \text{ MeV}$$
.

(b) 
$$K_{\rm e} = E - E' = 0.662 \,\,{\rm MeV} - 0.441 \,\,{\rm MeV} = 0.221 \,\,{\rm MeV}$$

## 30. The energy of the original photon can be found from the sum of the energies of the scattered electron and photon:

$$E = E' + K_{e} = 2.302 \text{ MeV} + 0.239 \text{ MeV} = 2.541 \text{ MeV}$$

## The scattering angle can be found from Eq. 3.46:

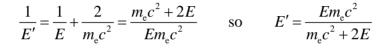
$$1 - \cos\theta = mc^2 \left(\frac{1}{E'} - \frac{1}{E}\right) = (0.511 \,\mathrm{MeV}) \left(\frac{1}{0.239 \,\mathrm{MeV}} - \frac{1}{2.541 \,\mathrm{MeV}}\right) = 1.937$$

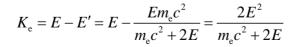
## from which

$$\theta = \cos^{-1}(-0.937) = 160^{\circ}$$

## 38. $K_{\rm e}$ is largest when E' is smallest (because $K_{\rm e} = E - E'$ ) and thus when 1/E' is largest,

which occurs when 
$$\cos \theta = -1$$
 (that is, when  $\theta = 180^{\circ}$ ).





43. The initial speed of the atom can be expressed as  $v = c(125.0 \text{ m/s})/(2.997 \times 10^8 \text{ m/s}) = 4.171 \times 10^{-7}c$ . The initial momentum (which is nonrelativistic at this low speed) is

$$p_{\rm i} = mv = \frac{1}{c}mc^2 \frac{v}{c} = \frac{1}{c}(1.007825 \text{ u})(931.5 \times 10^6 \text{ eV/u})(4.171 \times 10^{-7}) = 391.6 \text{ eV/c}$$

The photon momentum, which is in the opposite direction, has magnitude

$$p = \frac{1}{c} \frac{hc}{\lambda} = \frac{1}{c} \frac{1240 \text{ eV} \cdot \text{nm}}{97 \text{ nm}} = 12.8 \text{ eV}/c$$

The atom's final momentum is  $p_f = p_i - p = 391.6 \text{ eV}/c - 12.8 \text{ eV}/c = 378.8 \text{ eV}/c$  and its speed is

$$v_{\rm f} = \frac{p_{\rm f}}{m} = c \frac{p_{\rm f} c}{mc^2} = c \frac{378.8 \text{ eV}}{(1.007825 \text{ u})(931.5 \times 10^6 \text{ eV/u})} = 4.035 \times 10^{-7} c = 120.9 \text{ m/s}$$

So the change in the speed of the atom is 125.0 m/s - 120.9 m/s = 4.1 m/s.